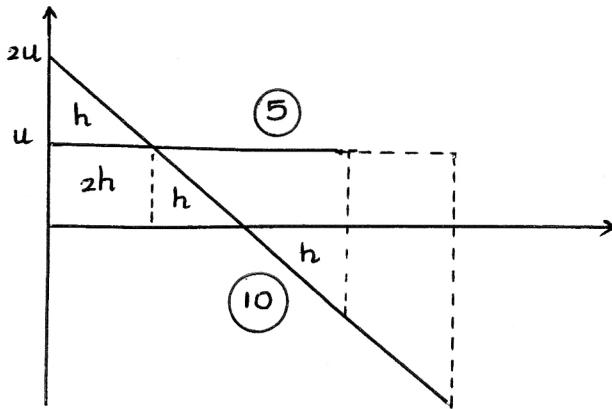


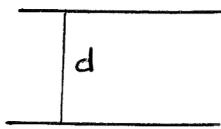
01.



The required ratio is 2:3. (10)

25

02.



River - R

Man - M

$$v_{R,E} = v$$

Since the man sets off from one bank to the other bank of the river at time $\frac{d}{u}$,

$$v_{M,R} = \uparrow u \quad (5)$$

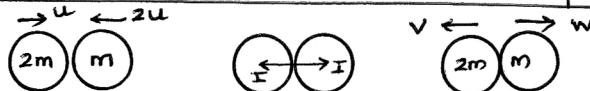
$$\begin{aligned} v_{M,E} &= v_{M,R} + v_{R,E} \quad (5) \\ &= \uparrow u + \rightarrow \quad (5) \end{aligned}$$

$$= \sqrt{u^2 + v^2} \quad (5)$$

$$\tan \theta = \frac{u}{v} \quad (5)$$

25

03.



Conservation of momentum:

$$mw - 2mv = 2mu - m \times 2u \quad (10)$$

$$w = 2v \dots (1)$$

Newton's law of restitution:

$$w+v = \frac{5}{6} \times 3u \quad (5)$$

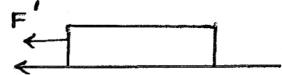
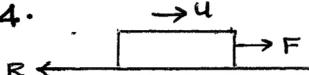
$$w+v = \frac{5}{2}u \dots (2)$$

$$(1), (2) \Rightarrow v = \frac{5}{6}u \quad (5)$$

$$w = \frac{5}{6}(2u) \quad (5)$$

25

04.



$$F \times \frac{1000u}{3600} = P \times 1000 \quad (5)$$

$$F = \frac{3600P}{u}$$

$$\rightarrow F - R = 0 \quad (5)$$

$$\begin{aligned} R &= F \\ &= \frac{3600P}{u} \end{aligned}$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0 &= \frac{25u^2}{324} + 2ah \quad (5) \\ a &= -\frac{25u^2}{648h} \end{aligned}$$

$$\rightarrow F = mg$$

$$-F' - R = ma \quad (5)$$

$$F' = -R - ma$$

$$= \frac{-3600P}{u} + \frac{25mu^2}{648h} \quad (5)$$

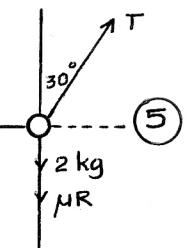
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$$05. \rightarrow T \sin 30 = R \quad (5)$$

$$\frac{T}{2} = R$$

$$\begin{aligned} \uparrow T \cos 30 - 2 - \mu R &= 0 \quad (5) \\ T \frac{\sqrt{3}}{2} - 2 - \frac{1}{4} \cdot \frac{T}{2} &= 0 \quad (5) \\ T \left(\frac{\sqrt{3}}{2} - \frac{1}{8} \right) &= 2 \end{aligned}$$

$$T = \frac{8}{4\sqrt{3}-1} \quad (5)$$



25

$$06. \underline{P} = \lambda \underline{i}, \underline{Q} = \mu (\underline{i} + \underline{j}) \quad \} \quad (5)$$

$$\underline{R} = \underline{P} + \underline{Q}$$

$$6\underline{i} + 2\underline{j} = \lambda \underline{i} + \mu(\underline{i} + \underline{j})$$

$$(\lambda - 6 + \mu)\underline{i} + (\mu - 2)\underline{j} = \underline{0} \quad (5)$$

$$\lambda + \mu - 6 = 0 \quad \& \quad \mu = 2 \quad (5)$$

$$\lambda = 4 \quad (5)$$

$$\therefore |\underline{P}| = 4, |\underline{Q}| = 2\sqrt{2} \quad (5)$$

25

$$07. P(2HT)$$

$$= \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \right) 3 \quad (5) \quad (5) \quad (5) \quad (5)$$

$$= \frac{3}{8} \quad (5)$$

25

$$08. P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(A \cup B) = \frac{7}{12}$$

$$(i) P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (5)$$

$$\frac{7}{12} = \frac{1}{2} + \frac{1}{3} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{4} \quad (5)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (5)$$

$$= \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4} \quad (5)$$

$$(ii) P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} \quad (5)$$

25

$$09. \bar{x} = 5 \quad (5)$$

$$S^2 = (2-5)^2 + (3-5)^2 + (8-5)^2 + (7-5)^2 + (5-5)^2 \quad (10)$$

$$= 9+4+9+4 \\ = 26 \quad (5)$$

$$s = \sqrt{26} \quad (5)$$

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$$10. 50 + 5x - 5y = 80 \quad (5)$$

$$x - y = 6 \dots (1) \quad (5)$$

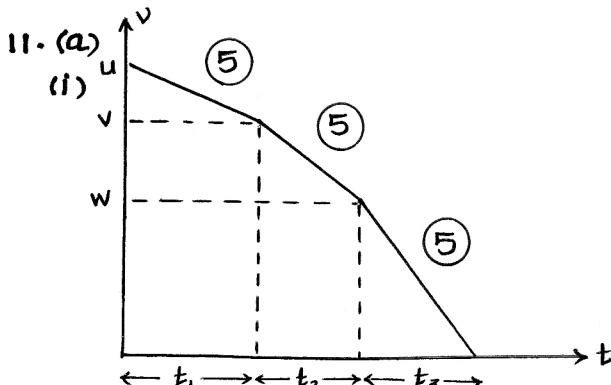
Since $x < 19$,

$$4x - 5y = 19 \dots (2) \quad (5)$$

$$(1), (2) \Rightarrow x = 11 \quad (5)$$

$$y = 5 \quad (5)$$

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$$(ii) u-v = ft_1 \dots (1) \quad (5)$$

$$v-w = 2ft_2 \dots (2) \quad (5)$$

$$w = 3ft_3 \dots (3) \quad (5)$$

$$\left(\frac{u+v}{2}\right)t_1 = \frac{a}{3} \dots (4) \quad (5)$$

$$\left(\frac{v+w}{2}\right)t_2 = \frac{a}{3} \dots (5) \quad (5)$$

$$\frac{w}{2}t_3 = \frac{a}{3} \dots (6) \quad (5)$$

$$(1) + (4) \Rightarrow u^2 - v^2 = \frac{2af}{3} \quad (3) + (6) \Rightarrow$$

$$(2) + (5) \Rightarrow v^2 - w^2 = \frac{4af}{3} \quad w^2 = \frac{6af}{3} \quad (5)$$

$$u^2 = \frac{12af}{3} \quad (5)$$

$$u^2 = 4af$$

40

$$(iii) v^2 = u^2 - \frac{2af}{3}$$

$$= 4af - \frac{2af}{3} \quad (5)$$

$$= \frac{10af}{3}$$

$$v = \sqrt{\frac{10af}{3}} \quad (5)$$

$$w = \sqrt{2af} \quad (5)$$

$$(iv) T = t_1 + t_2 + t_3$$

$$= \frac{u-v}{f} + \frac{v-w}{2f} + \frac{w}{3f}$$

$$= \frac{6u - 3v - w}{6f} \quad (5)$$

$$= \frac{u}{12f} (12 - (\sqrt{30} + \sqrt{2})) \quad (5)$$

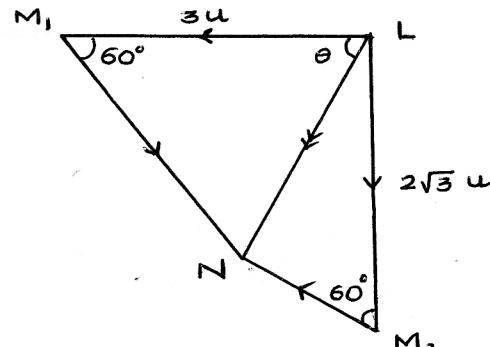
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$$(b) \sqrt{c,E} = \sqrt{c,A} + \sqrt{A,E}$$

$$= \overrightarrow{P_{80^\circ}} + \overleftarrow{3u} \quad (5)$$

$$\sqrt{c,E} = \sqrt{c,B} + \sqrt{B,E}$$

$$= \overleftarrow{60^\circ} + \downarrow 2\sqrt{3} \quad (5)$$



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In ΔLM_1N , ΔLM_2N
using sin rule

$$\frac{w}{\sin 60} = \frac{3u}{\sin(60+\theta)} \dots\dots (1) \quad (5)$$

$$\frac{w}{\sin 60} = \frac{2\sqrt{3}u}{\sin(30+\theta)} \dots\dots (2) \quad (5)$$

$$3 \sin(30+\theta) = 2\sqrt{3} \sin(60+\theta) \quad (5)$$

$$3 \left(\frac{1}{2} \cos\theta + \frac{\sqrt{3}}{2} \sin\theta\right) = 2\sqrt{3} \left(\frac{\sqrt{3}}{2} \cos\theta + \frac{1}{2} \sin\theta\right)$$

$$\frac{3}{2} \cos\theta = \frac{\sqrt{3}}{2} \sin\theta \quad (5)$$

$$\tan\theta = \sqrt{3}$$

$$\theta = 60^\circ \quad (5)$$

$$\therefore w = 3u \quad (5)$$

70

12. (a)

$$(i) v^2 = u^2 - 2gl(1-\cos\theta) \quad (10)$$

$$(ii) T = \frac{m}{l} (u^2 - 2gl + 3gl \cos\theta) \quad (10)$$

$$(iii) \omega_1 = \frac{u}{l}, \quad \omega_2 = \sqrt{\frac{u^2 - 4gl}{l}} \quad (5)$$

$$(iv) T_1 = \frac{m}{l} (u^2 + gl), \quad T_2 = \frac{m}{l} (u^2 - 5gl) \quad (5)$$

$$T_1 = \frac{m}{l} (\ell^2 \omega_1^2 + gl), \quad T_2 = \frac{m}{l} (\ell^2 \omega_2^2 - gl) \quad (5)$$

$$v^2 = u^2 - 2gl(1-\cos\theta)$$

$$\ell^2 \dot{\theta}^2 = u^2 - 2gl(1-\cos\theta) \quad (5)$$

$$\dot{\theta}^2 = \frac{u^2}{\ell^2} - \frac{2g}{\ell} (1-\cos\theta)$$

$$= \omega_1^2 - \frac{4g}{\ell} \sin^2 \frac{\theta}{2} \quad (5)$$

$$= \omega_1^2 + (\omega_2^2 - \frac{u^2}{\ell^2}) \sin^2 \frac{\theta}{2} \quad (5)$$

$$= \omega_1^2 + (\omega_2^2 - \omega_1^2) \sin^2 \frac{\theta}{2} \quad (5)$$

$$= \omega_1^2 (1 - \sin^2 \frac{\theta}{2}) + \omega_2^2 \sin^2 \frac{\theta}{2} \quad (5)$$

$$= \omega_1^2 \cos^2 \frac{\theta}{2} + \omega_2^2 \sin^2 \frac{\theta}{2}$$

$$\dot{\theta} = \sqrt{\omega_1^2 \cos^2 \frac{\theta}{2} + \omega_2^2 \sin^2 \frac{\theta}{2}} \quad (3)$$

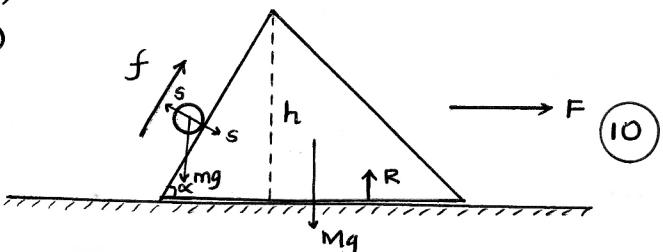
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$$\begin{aligned} (B) T &= \frac{m}{l} (u^2 - 3gl(1-\cos\theta) + gl) \\ &= T_1 - 3mg(1-\cos\theta) \quad (5) \\ &= T_1 - 6mg \sin^2 \frac{\theta}{2} \quad (5) \\ &= T_1 + (T_2 - T_1) \sin^2 \frac{\theta}{2} \quad (5) \\ &= T_1 \cos^2 \frac{\theta}{2} + T_2 \sin^2 \frac{\theta}{2} \end{aligned}$$

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(b)

(i)



$$\text{For the system, } F = ma \rightarrow 0 = MF + m(F + f \cos\alpha) \quad (15) \dots\dots (1)$$

For the particle,

$$-mg \sin\alpha = m(f + F \cos\alpha) \quad (15) \dots\dots (2)$$

40

$$(i) (M+m)F = -mf \cos\alpha$$

$$\begin{aligned} \frac{F}{m \cos\alpha} &= \frac{-f}{M+m} = \frac{F \cos\alpha}{m \cos^2\alpha} \quad (5) \\ &= \frac{F \cos\alpha + f}{m \cos^2\alpha - M - m} = \frac{g \sin\alpha}{M + m \sin^2\alpha} \quad (5) \end{aligned}$$

$$f = -\frac{(M+m)g \sin\alpha}{M + m \sin^2\alpha} \quad (5)$$

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$$(ii) \rightarrow v^2 = u^2 + 2fs$$

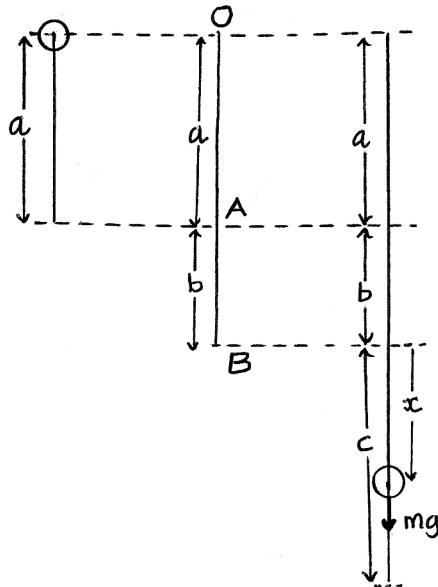
$$0 = v^2 - \frac{2g(M+m)\sin\alpha}{M + m \sin^2\alpha} \times$$

$$h \csc\alpha \quad (10)$$

$$\Rightarrow h = \frac{v^2 (M+m \sin^2\alpha)}{2g(M+m)} \quad (10)$$

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13. (i)



$$mg = \frac{\lambda b}{a} \dots (1) \quad (5)$$

$$mg - T = m\ddot{x} \dots (2) \quad (5)$$

$$T = \frac{\lambda(b+x)}{a} \dots (3) \quad (5)$$

$$mg - \frac{\lambda(b+x)}{a} = m\ddot{x} \quad (5)$$

$$mg - \frac{\lambda b}{a} - \frac{\lambda x}{a} = m\ddot{x}$$

$$mg - mg - \frac{\lambda x}{a} = m\ddot{x} \quad (5)$$

$$- \frac{mg}{b} x = m\ddot{x} \quad (5)$$

$$\ddot{x} = - \frac{g}{b} x$$

30

$$(ii) x = A \cos \omega t + B \sin \omega t$$

$$\dot{x} = -A\omega \sin \omega t + B\omega \cos \omega t \quad (5)$$

$$\ddot{x} = -A\omega^2 \cos \omega t - B\omega^2 \sin \omega t \quad (5)$$

$$= -\omega^2 x \quad (5)$$

$$-\frac{g}{b} x = -\omega^2 x \quad (5)$$

$$\omega^2 = \frac{g}{b}$$

$$\omega = \sqrt{\frac{g}{b}} \quad (5)$$

$$\text{At } t=0, x=c \text{ and } \dot{x}=0 \quad (5)$$

$$c = A + 0$$

$$c = A \quad (5)$$

$$0 = 0 + B$$

$$B = 0 \quad (5)$$

$$\therefore x = c \cos \sqrt{\frac{g}{b}} t \quad (5)$$

$$(iii) x = c \cos \omega t \dots (1) \quad (5)$$

$$\dot{x} = -c\omega \sin \omega t \dots (2) \quad (5)$$

$$\frac{x^2}{c^2} = c^2 \cos^2 \omega t \quad (5)$$

$$\frac{\dot{x}^2}{c^2} = c^2 \sin^2 \omega t \quad (5)$$

$$\Rightarrow \frac{\dot{x}^2}{c^2} + \frac{x^2}{c^2} = c^2$$

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(iv) For $x = -b$

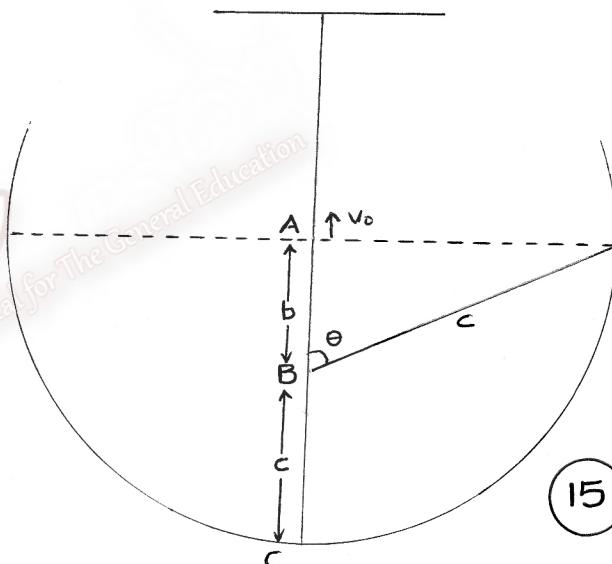
$$\dot{x}^2 = v_0^2 \quad (5)$$

$$v_0^2 = \omega^2 (c^2 - b^2) \quad (5)$$

$$v_0 = \omega \sqrt{c^2 - b^2} \quad (5)$$

15

(v)



15

$$\cos \theta = \frac{b}{c}$$

$$\theta = \cos^{-1} \left(\frac{b}{c} \right) \quad (5)$$

Time from C to A :

$$t_1 = \frac{\pi - \theta}{\omega} \quad (5)$$

Time from A to point of equilibrium :

$$t_2 = \frac{v_0}{g} \quad (5)$$

Time to reach C again :

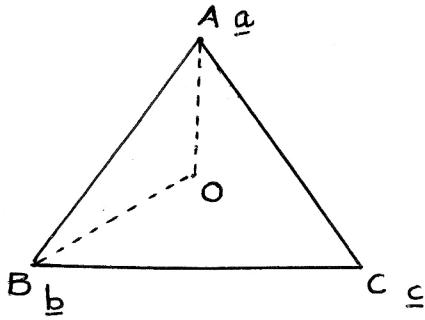
$$T = 2(t_1 + t_2) \quad (5)$$

$$= 2 \left(\frac{\pi - \theta}{\omega} + \frac{v_0}{g} \right) \quad (5)$$

45

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14. (a)



$$\vec{OA} \perp \vec{BC} \Rightarrow \vec{OA} \cdot \vec{BC} = 0 \quad (5)$$

$$\underline{a} \cdot (\underline{c} - \underline{b}) = 0 \quad (5)$$

$$\underline{a} \cdot \underline{c} - \underline{a} \cdot \underline{b} = 0 \quad \dots \dots (1)$$

$$\vec{OB} \perp \vec{AC} \Rightarrow \vec{OB} \cdot \vec{AC} = 0 \quad (5)$$

$$\underline{b} \cdot (\underline{c} - \underline{a}) = 0 \quad (5)$$

$$\underline{b} \cdot \underline{c} - \underline{b} \cdot \underline{a} = 0 \quad \dots \dots (2)$$

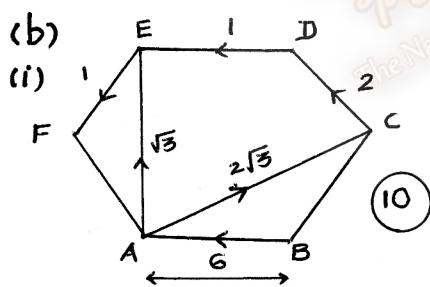
$$\underline{a} \cdot \underline{c} - \underline{b} \cdot \underline{c} = 0 \quad (5)$$

$$(\underline{a} - \underline{b}) \cdot \underline{c} = 0 \quad (5)$$

$$\vec{BA} \cdot \vec{OC} = 0 \quad (5)$$

$$\therefore AB \perp OC \quad (5)$$

(b)



$$\rightarrow x = 2\sqrt{3} \cos 30 - 6 - 2 \cos 60 \quad (10)$$

$$= 3 - 6 - 1 - 1 - \frac{1}{2} \quad (5)$$

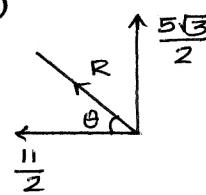
$$= -\frac{11}{2} \quad (5)$$

$$(iii) \uparrow Y = \sqrt{3} + 2\sqrt{3} \cos 60 + 2 \sin 60 - 1 \sin 60 \quad (10)$$

$$= \sqrt{3} + \sqrt{3} + \sqrt{3} - \frac{\sqrt{3}}{2} \quad (5)$$

$$= \frac{5\sqrt{3}}{2} \quad (5)$$

(iii)



$$\begin{aligned} R^2 &= \frac{121}{4} + \frac{75}{4} \quad (5) \\ &= \frac{196}{4} \end{aligned}$$

$$R = 7 \quad (5)$$

$$\tan \theta = \frac{5\sqrt{3}}{11} \quad (5)$$

15

$$(iv) A) Y \cdot x = 2 \cdot 8 \cos 30 +$$

$$1 \cdot 8 \cos 30 + 1 \cdot 2 \sin 60 \quad (10)$$

$$\frac{5\sqrt{3}}{2} x = 2 \cdot 4 \cdot \frac{\sqrt{3}}{2} + 1 \cdot 8 \cdot \frac{\sqrt{3}}{2}$$

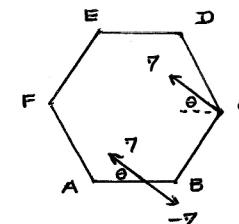
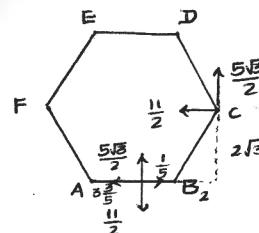
$$+ 1 \cdot 2 \cdot \frac{\sqrt{3}}{2} \quad (5)$$

$$5x = 8 + 8 + 2$$

$$x = \frac{18}{5} \quad (5)$$

20

(v)



$$G) = \frac{5\sqrt{3}}{2} \cdot \frac{11}{2} + \frac{11}{2} \cdot 2\sqrt{3} \quad (10)$$

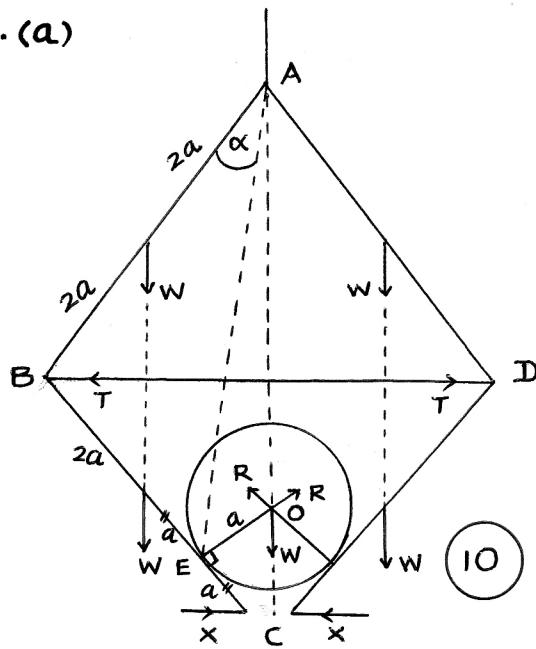
$$= \frac{99\sqrt{3}}{4} \quad (5)$$

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15. (a)



$$\text{circular disc : } \uparrow 2R \cos 45^\circ = W$$

$$2R \frac{1}{\sqrt{2}} = W$$

$$R = \frac{W}{2\sqrt{2}} \quad (5)$$

In $\triangle OCE$, $\hat{EOC} = 45^\circ$

$$\therefore CE = OE = a \quad (5)$$

$$BC : B \uparrow \times 4a \cos 45^\circ = R \cdot 3a +$$

$$(10) \quad W \cdot 2a \cos 45^\circ$$

$$4x \cdot \frac{1}{\sqrt{2}} = \frac{W}{\sqrt{2}} 3 + 2W \cdot \frac{1}{\sqrt{2}} \quad (5)$$

$$4x = 3W + 2W$$

$$x = \frac{5W}{4} \quad (5)$$

$$ABC : A \uparrow \times 8a \cos 45^\circ +$$

$$2W \cdot 2a \cos 45^\circ = R \cdot 5a \sin \alpha$$

$$(10) \quad + T \cdot 4a \cos 45^\circ$$

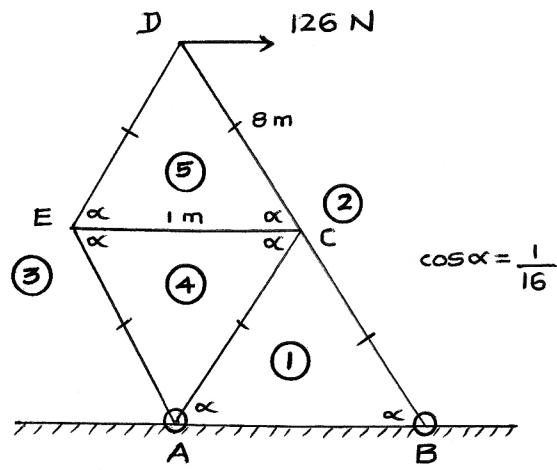
$$\frac{5W}{4} \cdot 8 \cdot \frac{1}{\sqrt{2}} + 2W \cdot 2 \cdot \frac{1}{\sqrt{2}} =$$

$$(5) \quad \frac{W}{\sqrt{2}} \cdot 5 \cdot \frac{3}{5} + T \cdot 4 \cdot \frac{1}{\sqrt{2}}$$

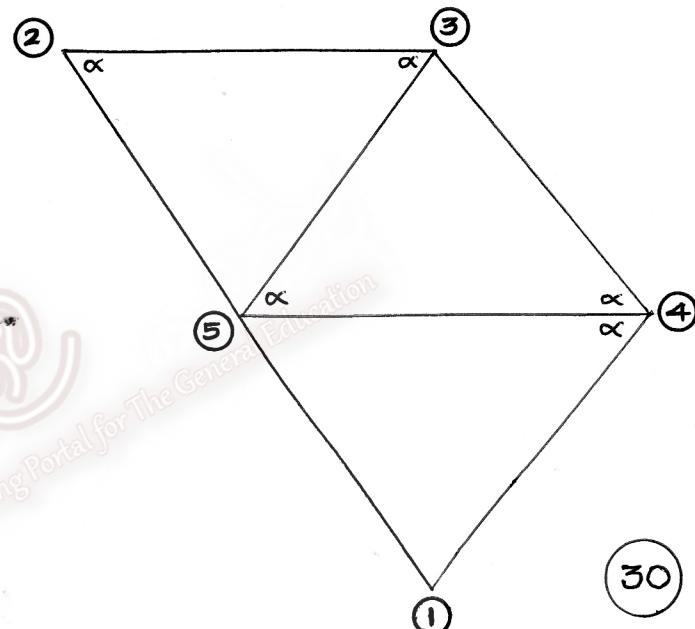
$$10W + 4W = 3W + 4T \quad (5)$$

$$T = \frac{11W}{4}$$

(b)



$$\cos \alpha = \frac{1}{16}$$



30

Rod	Tension	Thrust
CD	-	1008 N
ED	1008 N	-
AE	1008 N	-
EC	-	126 N
AC	1008 N	-
BC	-	2016 N

10
10
10
10
10
10
10

90

16. Theory $OG = \frac{a \sin \alpha}{\alpha}$

35

To deduce that $OG = \frac{4a}{\pi\sqrt{2}}$
 $= \frac{2\sqrt{2}a}{\pi}$

10

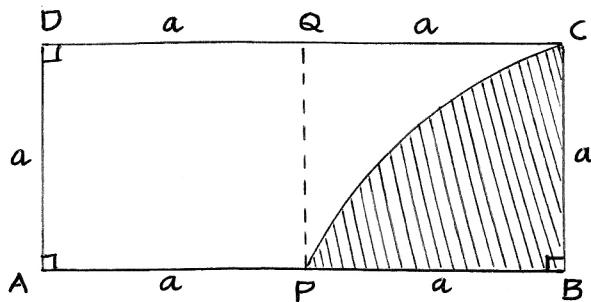


Figure Mass Centre of gravity
From AD From AB

ABCD $2a^2\rho$ a $\frac{a}{2}$

sector PBC $\frac{1}{4}\pi a^2\rho$ $2a - \frac{2a}{\pi}$ $\frac{2a}{\pi}$

Remaining $a^2\rho(2 - \frac{\pi}{4})$ \bar{x} \bar{y}

$$a^2\rho(2 - \frac{\pi}{4})\bar{x} = 2a^2\rho \cdot a - \frac{\pi a^2\rho}{4}(2a - \frac{2a}{\pi})$$
10 10 10

$$\left(\frac{8-\pi}{4}\right)\bar{x} = 2a - \frac{\pi a}{2} + \frac{a}{2}$$

$$= (5-\pi)\frac{a}{2}$$

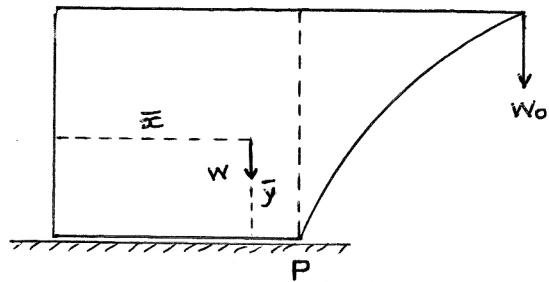
$$\bar{x} = \frac{(5-\pi)}{(8-\pi)} 2a$$
5

$$a^2\rho(2 - \frac{\pi}{4})\bar{y} = 2a^2\rho \cdot \frac{a}{2} - \frac{\pi}{4} a^2\rho \cdot \frac{2a}{\pi}$$
10 10 10

$$\left(\frac{8-\pi}{4}\right)\bar{y} = a - \frac{a}{2}$$
5

$$\bar{y} = \frac{2a}{8-\pi}$$
5

80



P) $a w_0 = w(a - \bar{x})$ 10

$$= w \left[a - \left(\frac{5-\pi}{8-\pi} \right) 2a \right]$$
5

$$= \frac{wa}{8-\pi} (8-\pi-10+2\pi)$$

$$= \frac{wa}{8-\pi} (\pi-2)$$
5

$$w_0 = \frac{w(\pi-2)}{(8-\pi)}$$
5

25

17. (a) If $P(A \cap B) = P(A) \cdot P(B)$

A and B are independent.

05

A and C are independent

$$\Rightarrow P(A \cap C) = P(A) \cdot P(C)$$
5

$$\frac{1}{20} = \frac{1}{5} \cdot P(C)$$
5

$$P(C) = \frac{1}{4}$$
5

15

$$P(B \cup C) = P(B) + P(C) - P(B \cap C)$$
5

$$\frac{3}{8} = \frac{1}{6} + \frac{1}{4} - P(B \cap C)$$
5

$$P(B \cap C) = \frac{1}{24}$$
5

$$P(B) \cdot P(C) = \frac{1}{6} \cdot \frac{1}{4}$$

$$= \frac{1}{24}$$
5

$$\therefore P(B \cap C) = P(B) \cdot P(C)$$
5

$\therefore B$ and C are independent.

5

30

(b)

Marks	Mid value	Frequency	$y = \frac{x_i - a}{c}$	fy	fy^2
10-20	15	18	-3	-54	162
20-30	25	34	-2	-68	136
30-40	35	58	-1	-58	58
40-50	45	42	0	0	0
50-60	55	24	1	24	24
60-70	65	10	2	20	40
70-80	75	6	3	18	54
80-90	85	8	4	32	128
		200	(5) $\frac{-180+94}{200} = -86$	602	
			(5) + (5)		
			(5) + (5)		

$$\begin{aligned}
 &= 10 \sqrt{3.01 - 0.185} \\
 &= 10 \sqrt{3.01 - 0.19} \\
 &= 10 \sqrt{2.82} \quad (5) \\
 &= 10 \times 1.679 \\
 &= 16.79 \quad (5)
 \end{aligned}$$

20

(iv) coefficient of skewness

$$\begin{aligned}
 &= \frac{3(\text{mean} - \text{median})}{\text{standard deviation}} \quad (5) \\
 &= \frac{3(40.7 - 38.2)}{14.2} \quad (10) \\
 &= \frac{3 \times 2.5}{14.2} \\
 &= 0.53 \quad (5)
 \end{aligned}$$

20

(i) modal class = 30 - 40

30

This is positive skewness.

The distribution is positive.

(ii) mean $\bar{x} = c\bar{y} + a$ (5)

05

10

$$\begin{aligned}
 &= 10 \times \frac{-86}{200} + 45 \quad (5) \\
 &= 45 - \frac{86}{20} \\
 &= 45 - 4.3 \\
 &= 40.7 \quad (5)
 \end{aligned}$$

15

(iii) standard deviation 6

$$= c \sqrt{\frac{\sum fy^2}{\sum fy} - y^2} \quad (5)$$

$$= 10 \sqrt{\frac{602}{200} - \left(\frac{-86}{200}\right)^2} \quad (5)$$