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NEW

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts;* **Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17).
* **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Using the **Principle of Mathematical Induction**, prove that $\sum_{r=1}^{n}(4r+1) = n(2n+3)$ for all $n \in \mathbb{Z}^{+}$.

2. Sketch the graphs of $y = 3|x-1|$ and $y = |x| + 3$ in the same diagram.
Hence or otherwise, find all real values of x satisfying the inequality $3|2x-1| > 2|x| + 3$.

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3. Sketch, in the same Argand diagram, the loci of the points that represent the complex numbers z satisfying

(i) $\text{Arg}(z+1-3i) = -\frac{\pi}{4}$ and

(ii) $|z-2| = \sqrt{2}$.

Hence, write down the complex numbers represented by the points of intersection of these loci.

4. Let $n \in \mathbb{Z}^+$. Write down the binomial expansion of $(1 + x)^n$ in ascending powers of x.
Show that if the coefficients of two consecutive terms of the above expansion are equal, then n is odd.

[see page four

5. Show that $\lim_{x \to \frac{\pi}{3}} \dfrac{\sin\left(x - \frac{\pi}{3}\right)}{\left(\sqrt{3x} - \sqrt{\pi}\right)} = \dfrac{2\sqrt{\pi}}{3}$.

6. The region enclosed by the curves $y = \dfrac{e^x}{1+e^x}$, $x = 0$, $x = \ln 3$ and $y = 0$ is rotated about the x–axis through 2π radians. Show that the volume of the solid thus generated is $\frac{\pi}{4}(4\ln 2 - 1)$.

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7. Show that the equation of the normal line to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ at the point $P \equiv (5\cos\theta,\ 3\sin\theta)$ on it, is $5\sin\theta\, x - 3\cos\theta\, y = 16\sin\theta\cos\theta$.

Find the y−intercept of the normal line drawn to the above ellipse at the point $\left(\frac{5}{2}, \frac{3\sqrt{3}}{2}\right)$ on it.

8. Let $m \in \mathbb{R}$ and l be the straight line passing through the point $A \equiv (1, 2)$ with gradient m. Write down the equation of l in terms of m.
It is given that the perpendicular distance from the point $B \equiv (2, 3)$ to the line l is $\frac{1}{\sqrt{5}}$ units. Find the values of m.

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9. Find the equation of the circle S having the centre at the point $(-2, 0)$ and passing through the point $(-1, \sqrt{3})$. Write down the equation of the chord of contact of the tangents drawn from the point $A \equiv (1, -1)$ to the circle S.

Hence, show that the x–coordinates of the points of contact of the tangents drawn to S from A satisfies the equation $5x^2 + 8x + 2 = 0$.

10. Let $\theta \neq (2n + 1)\frac{\pi}{2}$ for $n \in \mathbb{Z}$.

Using the identity $\cos^2\theta + \sin^2\theta = 1$, show that $\sec^2\theta = 1 + \tan^2\theta$.

It is given that $\sec\theta + \tan\theta = \frac{4}{3}$. **Deduce** that $\sec\theta - \tan\theta = \frac{3}{4}$.

Hence, show that $\cos\theta = \frac{24}{25}$.

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
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10 E I

Part B

* Answer **five** questions only.

11. (*a*) Let $f(x)=x^2+px+c$ and $g(x)=2x^2+qx+c$, where $p, q\in\mathbb{R}$ and $c>0$. It is given that $f(x)=0$ and $g(x)=0$ have a common root α. Show that $\alpha=p-q$.

Find c in terms of p and q, and **deduce** that

(i) if $p>0$, then $p<q<2p$,

(ii) the discriminant of $f(x)=0$ is $(3p-2q)^2$.

Let β and γ be the other roots of $f(x)=0$ and $g(x)=0$ respectively. Show that $\beta=2\gamma$.

Also, show that the quadratic equation whose roots are β and γ is given by

$2x^2+3(2p-q)x+(2p-q)^2=0$.

(*b*) Let $h(x)=x^3+ax^2+bx+c$, where $a, b, c\in\mathbb{R}$. It is given that x^2-1 is a factor of $h(x)$. Show that $b=-1$.

It is also given that the remainder when $h(x)$ is divided by x^2-2x is $5x+k$, where $k\in\mathbb{R}$. Find the value of k and show that $h(x)$ can be written in the form $(x-\lambda)^2(x-\mu)$, where $\lambda, \mu\in\mathbb{R}$.

12. (*a*) It is required to select a musical group consisting of eleven members from among five pianists, five guitarists, three female singers and seven male singers such that it includes **exactly** two pianists and **at least** four guitarists. Find the number of different such musical groups that can be selected.

Find also the number of musical groups among these, having exactly two female singers.

(*b*) Let $U_r=\dfrac{3r-2}{r(r+1)(r+2)}$ and $V_r=\dfrac{A}{r+1}-\dfrac{B}{r}$ for $r\in\mathbb{Z}^+$, where $A, B\in\mathbb{R}$.

Find the values of A and B such that $U_r=V_r-V_{r+1}$ for $r\in\mathbb{Z}^+$.

Hence, show that $\displaystyle\sum_{r=1}^{n}U_r=\frac{n^2}{(n+1)(n+2)}$ for $n\in\mathbb{Z}^+$.

Show that the infinite series $\displaystyle\sum_{r=1}^{\infty}U_r$ is convergent and find its sum.

Now, let $W_r=U_{r+1}-2U_r$ for $r\in\mathbb{Z}^+$. Show that $\displaystyle\sum_{r=1}^{n}W_r=U_{n+1}-U_1-\sum_{r=1}^{n}U_r$.

Deduce that the infinite series $\displaystyle\sum_{r=1}^{\infty}W_r$ is convergent and find its sum.

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13. (a) Let $\mathbf{A} = \begin{pmatrix} a+1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ a & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} a & 1 \\ a & 2 \end{pmatrix}$, where $a \in \mathbb{R}$.

Show that $\mathbf{A}^{\mathrm{T}}\mathbf{B} - \mathbf{I} = \mathbf{C}$; where $\mathbf{I}$ is the identity matrix of order 2.

Show also that $\mathbf{C}^{-1}$ exists **if and only if** $a \neq 0$.

Now, let $a = 1$. Write down $\mathbf{C}^{-1}$.

Find the matrix $\mathbf{P}$ such that $\mathbf{CPC} = 2\mathbf{I} + \mathbf{C}$.

(b) Let $z, w \in \mathbb{C}$. Show that $|z|^2 = z\bar{z}$ and applying it to $z - w$,

show that $|z-w|^2 = |z|^2 - 2\,\mathrm{Re}\, z\bar{w} + |w|^2$.

Write a similar expression for $|1 - z\bar{w}|^2$ and show that $|z-w|^2 - |1-z\bar{w}|^2 = -\left(1-|z|^2\right)\left(1-|w|^2\right)$.

Deduce that if $|w| = 1$ and $z \neq w$, then $\left|\dfrac{z-w}{1-z\bar{w}}\right| = 1$.

(c) Express $1+\sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 < \theta < \frac{\pi}{2}$.
It is given that $(1+\sqrt{3}i)^m(1-\sqrt{3}i)^n = 2^8$, where m and n are positive integers.
Applying De Moivre's theorem, obtain equations sufficient to determine the values of m and n.

14. (a) Let $f(x) = \dfrac{x(2x-3)}{(x-3)^2}$ for $x \neq 3$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \dfrac{9(1-x)}{(x-3)^3}$ for $x \neq 3$.

Hence, find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

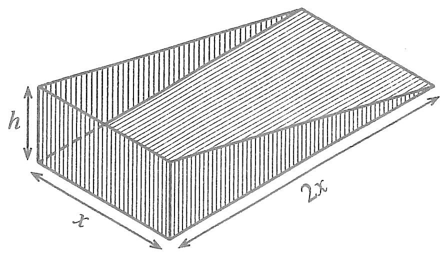
Also, find the coordinates of the turning point of $f(x)$.

It is **given that** $f''(x) = \dfrac{18x}{(x-3)^4}$ for $x \neq 3$.

Find the coordinates of the point of inflection of the graph of $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection.

(b) The adjoining figure shows the portion of a dust pan without its handle. Its dimensions in centimetres, are shown in the figure. It is given that its volume x^2h cm^3 is 4500 cm^3. Its surface area S cm^2 is given by $S = 2x^2 + 3xh$. Show that S is minimum when $x = 15$.



[see page nine

15. (*a*) It is given that there exist constants A and B such that

$x^3 + 13x - 16 = A(x^2+9)(x+1) + B(x^2+9) + 2(x+1)^2$ for all $x \in \mathbb{R}$.

Find the values of A and B.

Hence, write down $\dfrac{x^3+13x-16}{(x+1)^2(x^2+9)}$ in partial fractions and

find $\displaystyle\int \frac{x^3+13x-16}{(x+1)^2(x^2+9)}\,dx$.

(*b*) Using integration by parts, evaluate $\displaystyle\int_0^1 e^x \sin^2 \pi x\,dx$.

(*c*) Using the formula $\displaystyle\int_0^a f(x)dx = \int_0^a f(a-x)dx$, where a is a constant,

show that $\displaystyle\int_0^\pi x\cos^6 x \sin^3 x\,dx = \frac{\pi}{2}\int_0^\pi \cos^6 x \sin^3 x\,dx$.

Hence, show that $\displaystyle\int_0^\pi x\cos^6 x \sin^3 x\,dx = \frac{2\pi}{63}$.

16. Let $A \equiv (1, 2)$ and $B \equiv (3, 3)$.

Find the equation of the straight line l passing through the points A and B.

Find the equations of the straight lines l_1 and l_2 passing through A, each making an acute angle $\frac{\pi}{4}$ with l.

Show that the coordinates of any point on l can be written in the form $(1+2t, 2+t)$, where $t \in \mathbb{R}$.

Show also that the equation of the circle C_1 lying entirely in the first quadrant with radius $\frac{\sqrt{10}}{2}$, touching both l_1 and l_2, and its centre on l is $x^2+y^2-6x-6y+\frac{31}{2}=0$.

Write down the equation of the circle C_2 whose ends of a diameter are A and B.

Determine whether the circles C_1 and C_2 intersect orthogonally.

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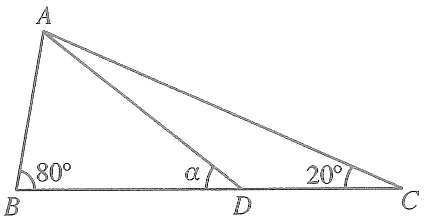
17. (*a*) Write down $\sin(A-B)$ in terms of $\sin A$, $\cos A$, $\sin B$ and $\cos B$.

Deduce that

(i) $\sin(90° - \theta) = \cos\theta$, and

(ii) $2\sin 10° = \cos 20° - \sqrt{3}\sin 20°$.

(*b*) In the usual notation, state the **Sine Rule** for a triangle *ABC*.



In the triangle *ABC* shown in the figure, $A\hat{B}C = 80°$ and $A\hat{C}B = 20°$. The point *D* lies on *BC* such that $AB = DC$. Let $A\hat{D}B = \alpha$.

Using the **Sine Rule** for suitable triangles, show that $\sin 80° \sin(\alpha - 20°) = \sin 20° \sin\alpha$.

Explain why $\sin 80° = \cos 10°$ and **hence**, show that $\tan\alpha = \dfrac{\sin 20°}{\cos 20° - 2\sin 10°}$.

Using the result in (*a*)(ii) above, **deduce** that $\alpha = 30°$.

(*c*) Solve the equation $\tan^{-1}(\cos^2 x) + \tan^{-1}(\sin x) = \frac{\pi}{4}$.

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