නව නිර්දේශය / புதிய பாடத்திட்டம் / New Syllabus

NEW

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

ගණිතය I
கணிதம் I
Mathematics I

07 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය	-	මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம்	-	10 நிமிடங்கள்
Additional Reading Time	-	**10 minutes**

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts;* **Part A** (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
* **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(07) Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Let $A = \{x \in \mathbb{R} : |x + 1| \leq 2\}$ and $B = \{x \in \mathbb{R} : |x - 1| > 1\}$. Find $A \cap B$, $A \cup B$ and $A \cap B'$.

2. Let A and B be subsets of a universal set S. Show that $(A \cup B) \cap (A \cap B)' = (A \setminus B) \cup (B \setminus A)$.

[see page three

0240

3. Show that the compound proposition $(p \wedge q) \vee r$ and the compound proposition $(p \Rightarrow \sim q) \Rightarrow r$ are logically equivalent.

4. Using the method of contradiction, prove that if $n^2 + 6n + 3$ is even, then n is odd.

[see page four

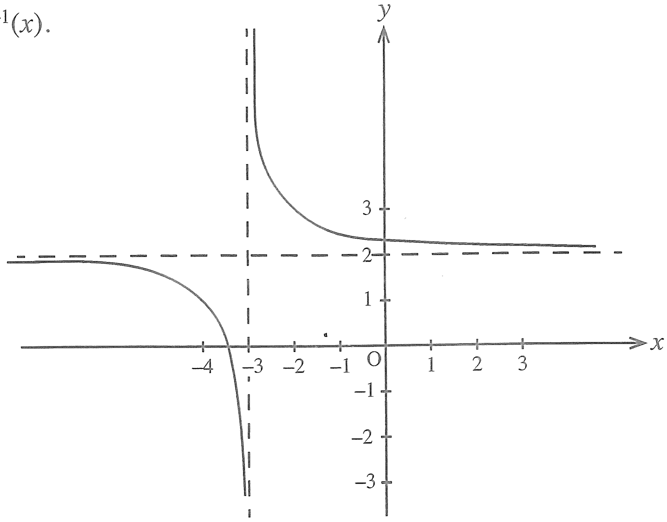
5. Solve the equation $\log_3 x = 2 - \log_3(6-x)$ for x.
When x takes this value, solve the equation $x^y = 2 - x^{-y}$ for y. (Hint: Use the substitution $u = x^y$.)

6. Find all real values of x satisfying the inequality $x + \frac{6}{x+1} > 4$.

[see page five

7. The graph of $f(x)=\dfrac{1}{x+a}+b$ is shown in the diagram. Using the information given there, write down the values of the constants a and b, and find $f^{-1}(x)$.

Given that $g(x) = x - 5$, solve $f^{-1}(g(x)) = 4$.



8. Write down the equation of the straight line l passing through the point $A\equiv(0, 3)$ with gradient -2. The line l meets the line $y = mx$ at the point B, where $m\ (\neq -2)$ is a constant. Find the x–coordinate of B in terms of m.

Given that the area of the triangle OAB is $\dfrac{9}{2}$ square units, where O is the origin, find the possible values of m.

[see page six

9. The radius r m of a right circular cylinder is increasing at a rate of 0.5 m s^{-1} and its height h m is decreasing at a rate of 0.2 m s^{-1}. Show that the rate of change of the volume V m^3 of the cylinder is given by $\frac{dV}{dt} = \frac{\pi r}{5}(5h - r)$.

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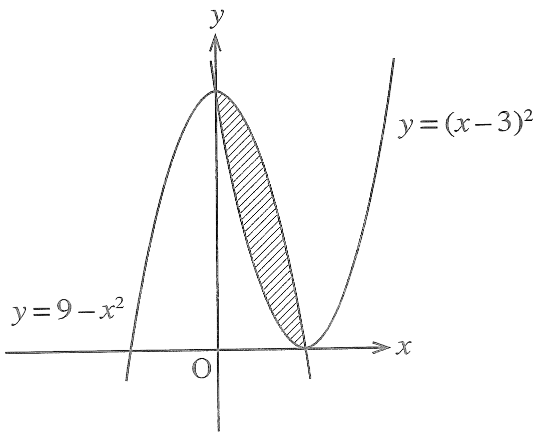
10. Find the area of the region enclosed by the curves $y = 9 - x^2$ and $y = (x-3)^2$.



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නව නිර්දේශය / புதிய பாடத்திட்டம் / New Syllabus

NEW

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

ගණිතය I
கணிதம் I
Mathematics I

07 E I

Part B

* Answer five questions only.

11. (*a*) A survey was carried out using 100 students in a class to find out which branches of mathematics they liked from amongst Algebra and Geometry. It was found that the number of students who liked Geometry was 10 more than twice the number of students who liked Algebra. It was also found that 80 students liked only one branch and 10 students did not like both.

Find the number of students who liked

(i) Algebra

(ii) Geometry

(iii) both Algebra and Geometry.

(*b*) Using truth tables, determine whether each of the following compound propositions is a tautology or a contradiction.

(i) $(p \wedge q) \wedge (q \Rightarrow \sim p)$

(ii) $(p \wedge q \wedge r) \vee (p \wedge q \wedge (\sim r)) \vee (\sim (p \wedge q))$

12. (*a*) Using the **Principle of Mathematical Induction**, prove that

$$\sum_{r=1}^{n} r(3r+2) = \frac{n}{2}(n+1)(2n+3) \quad \text{for all } n \in \mathbb{Z}^+.$$

(*b*) Let $U_r = \dfrac{r^2+r-1}{(r+1)^2(r+2)^2}$ for $r \in \mathbb{Z}^+$.

Verify that $U_r = \dfrac{r}{(r+1)^2} - \dfrac{(r+1)}{(r+2)^2}$ for $r \in \mathbb{Z}^+$.

Show that $\displaystyle\sum_{r=1}^{n} U_r = \frac{1}{4} - \frac{(n+1)}{(n+2)^2}$ for $n \in \mathbb{Z}^+$.

Hence, show that $\displaystyle\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Deduce that $\displaystyle\sum_{r=20}^{\infty} U_r = \frac{20}{441}$.

[See page eight

13. (*a*) Let $k\ (\neq 0)$ be a real constant. It is given that the quadratic equation $2kx^2+12x+2k-5=0$ has real roots. Show that $2k^2-5k-18\leq 0$.

Find the maximum and the minimum of possible values of k.

Let α and β be the roots of the equation $2kx^2+12x+2k-5=0$.

Find the quadratic equation whose roots are $2(\alpha+\beta)$ and $3\alpha\beta$.

(*b*) Let $f(x)=x^3+px^2+q$ and $g(x)=x^3+qx^2-p$, where p and q are real numbers. It is given that $(x+2)$ is a factor of $f(x)$ and that when $g(x)$ is divided by $(x+1)$, the remainder is -8.
Find the values of p and q.

For these values of p and q, find the least value of $f(x)-g(x)$.

14. (*a*) Let $a, b\in\mathbb{R}$. The expansion of $(1+ax)^8$, in ascending powers of x, discarding the terms involving powers of x greater than two is $1+24x+bx^2$. Show that $a=3$ and $b=252$.

Hence, find an approximate value for $(1.03)^8+(0.97)^8$.

(*b*) A person wants to take a loan of Rs. 2 000 000 from a bank, to be paid back in 10 years. The bank charges an annual interest of 6% compounded monthly. Let Rs. A_n be the outstanding amount after paying the n^{th} installment at the end of the n^{th} month, where $n\leq 120$.
Show that $A_1=1.005A-x$, where A is the loan amount and x is the monthly installment.
Obtain expressions for A_2 and A_3, and write down A_n in terms of A, x and n.
Hence, find the value of x.

15. Let $A\equiv(1,\ 1)$ and $B\equiv(5,\ 9)$.

Find the equation of the straight line AB and show that the point $C\equiv(4,\ 2)$ does not lie on the line AB.
The line perpendicular to AB and passing through C, intersects AB at the point D.
Find the coordinates of D and show that $AD{:}DB=1{:}3$.

Also, find the coordinates of the point E such that $ADCE$ is a rectangle.

Let F be the point of intersection of the line AB and the line $x+y=k$. The line passing through the point F and parallel to the line AC passes through the point E. Find the value of the constant k.

16. (*a*) Evaluate $\displaystyle\lim_{x\to 2}\frac{x^4-16}{\sqrt{x}-\sqrt{2}}$.

(*b*) Differentiate each of the following with respect to x:

(i) $(2+3x)^5(1+x^2)^{10}$ (ii) $\dfrac{\ln x}{3\ln x+1}$ (iii) $\sqrt{x}\,e^{-(x^2-1)}$

(*c*) A closed rectangular box needs to be constructed such that the length of the base is 3 times its width. It costs 100 rupees per square meter for the top and the bottom faces, and 60 rupees per square meter for the sides of the box. If the volume of the box must be 60 m^3, show that the cost C (in rupees) to make the box is given by $C=600x^2+\dfrac{9600}{x}$, where x m is the width of the base of the box.

Determine the value of x that minimizes the cost to make the box.

[See page nine

17. (*a*) Using the method of **integration by parts**, find $\int x^3 (\ln x)^2 \mathrm{d}x$.

(*b*) The following table gives the values of the function $f(x) = \ln(1 + x^2)$, correct to three decimal places, for values of x between 1 and 2.5 at intervals of length 0.25.

x	1.00	1.25	1.50	1.75	2.00	2.25	2.50
$f(x)$	0.693	0.941	1.179	1.402	1.609	1.802	1.981

Using **Simpson's rule**, find an approximate value for $I = \int_1^{2.5} \ln(1+x^2)\mathrm{d}x$.

Hence, find an approximate value for $\int_1^{2.5} \ln\left(e^{2x}\sqrt{1+x^2}\right)\mathrm{d}x$.

* * *