නව/පැරණි නිර්දේශය – புதிய/பழைய பாடத்திட்டம் – *New/Old Syllabus*

NEW/OLD

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

උසස් ගණිතය II
உயர் கணிதம் II
Higher Mathematics II

11 E II

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

- *This question paper consists of two parts;* **Part A** (Questions **1 - 10**) and **Part B** (Questions **11 - 17**).
- **Part A:** *Answer* ***all*** *questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- **Part B:** *Answer* ***five*** *questions only. Write your answers on the sheets provided.*
- *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove* ***only*** **Part B** *of the question paper from the Examination Hall.*

- *Statistical Tables will be provided.*
- *g denotes the acceleration due to gravity.*

For Examiners' Use only

(11) Higher Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

Part A

1. Let the position vectors of three points A, B and C with respect to a fixed origin O be $\mathbf{i}+\mathbf{j}-\mathbf{k}$, $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ and $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$, respectively. Find $\overrightarrow{AB} \times \overrightarrow{AC}$ and **hence**, find the area of the triangle ABC.

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2. A system of forces consists of the forces $\mathbf{F}_1 = 2\mathbf{i}+3\mathbf{j}-\mathbf{k}$ and $\mathbf{F}_2 = \mathbf{i}-\mathbf{j}+\mathbf{k}$ both acting at the origin O, and $\mathbf{F}_3 = -3\mathbf{i}-2\mathbf{j}$ acting at the point $(1, 0, 1)$. Show that the system of forces reduces to a couple and find its vector moment.

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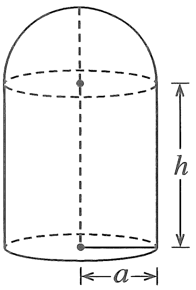
[*see page three*

3. A solid object S is formed by fixing rigidly a uniform hemisphere of radius a and density ρ to a uniform right circular cylinder of radius a, height h and density 2ρ, as shown in the figure.

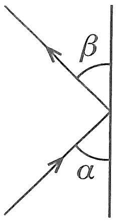


S is immersed in a homogeneous liquid of density ρ_1 with its axis vertical. When the cylinder is above the hemisphere, it floats in the liquid with only the hemisphere totally immersed and when the hemisphere is above the cylinder, it floats in the liquid with only the cylinder totally immersed.

Show that $h = \frac{2a}{3}$ and $\rho_1 = 3\rho$.

4. The position vector of a particle P at time t is given by $\mathbf{r} = t\mathbf{i} + 2\cos t\,\mathbf{j} - 2\sin t\,\mathbf{k}$. Find the velocity and the speed of P at time t, and show that the velocity makes a constant angle with the x–axis. Also, find the acceleration of P at time t.

5. A smooth uniform sphere A of mass m moving on a smooth horizontal floor collides with a smooth vertical wall. Just before the collision the velocity of A is of magnitude u and makes an angle α with the wall. Just after the impact the velocity of A makes an angle β with the wall. Show that $\tan\beta = e\tan\alpha$, where e is the coefficient of restitution between A and the wall.
Also, find the loss of kinetic energy of A due to the collision.



6. A uniform rod AB of mass m and length $2a$ with a particle of mass m fixed to it at the point B performs small oscillations about a smooth horizontal axis through A.

Show that the period of small oscillations is $\frac{8\pi}{3}\sqrt{\frac{a}{g}}$.

[see page five

7. The probability that a certain team wins a match is 0.4. Find the probability that, in 5 matches, team wins

(i) exactly 4 matches,

(ii) less than 4 matches.

8. It is reported that a certain insurance company receives 2 claims per day, on average. Assuming that the number of claims received per day follows a Poisson distribution, find the probability that the company receives

(i) exactly 2 claims,

(ii) at least 1 claim,

on a randomly selected day.

[*see page six*

9. The probability density function $f(x)$ of a continuous random variable X is given by

$$f(x)=\begin{cases} ax-bx^2 & , \text{ for } 0\le x\le 2,\\ 0 & , \text{ otherwise,}\end{cases}$$

where a and b are constants. It is given that $E(X)=\frac{1}{3}$. Find the values of a and b.

10. The data collected for a period of 30 days, from a quality control process conducted by a company manufacturing toy cars are summarized as follows:

Number of toy cars rejected	0	1	2	3	4
Number of days	4	6	7	10	3

Let X be the number of toy cars rejected on a randomly selected day. Obtain the probability mass function of X and **hence**, find $E(X)$ and Var(X).

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නව/පැරණි නිර්දේශය – புதிய/பழைய பாடத்திட்டம் – New/Old Syllabus

NEW/OLD

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

උසස් ගණිතය II
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11 E II

Part B

✻ Answer **five** questions only.

11. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act at the points with the position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively are given below:

Point of action	Force
$\mathbf{r}_1 = \mathbf{i} + \mathbf{k}$	$\mathbf{F}_1 = \mathbf{j} - \mathbf{k}$
$\mathbf{r}_2 = \mathbf{i} + \mathbf{j}$	$\mathbf{F}_2 = -\mathbf{i} + \mathbf{k}$
$\mathbf{r}_3 = \mathbf{j} + \mathbf{k}$	$\mathbf{F}_3 = \mathbf{i} - \mathbf{j}$

Show that this system of forces is equivalent to a couple and find its vector moment.

Now, the force $\mathbf{F}_3$ is replaced by a force $\mathbf{F}_4$ such that the system of forces consisting of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ is in equilibrium. Find $\mathbf{F}_4$ and its line of action in the form $\mathbf{r} = \mathbf{r}_0 + \lambda\mathbf{F}$, where $\mathbf{r}_0$ and $\mathbf{F}$ are to be determined and λ is a parameter.

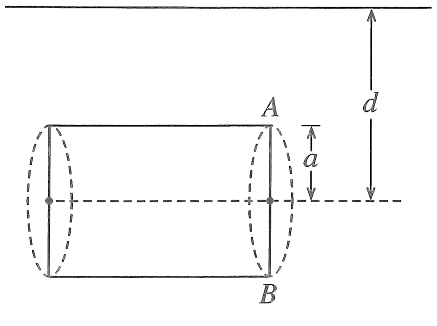
The system of forces consisting of $\mathbf{F}_1$, $2\mathbf{F}_2$ and $3\mathbf{F}_3$ acting at $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively, reduces to a single force $\mathbf{R}$ together with a couple of vector moment $\mathbf{G}$, when reduced at the origin O. Find $\mathbf{R}$ and $\mathbf{G}$.

Hence, show that this system of forces reduces to a single resultant force.

12. A circular lamina of radius a is immersed in a homogeneous liquid with its centre at a depth $h(> a)$ below the free surface of the liquid. Show that the centre of pressure of the lamina is on its vertical diameter at a distance $\frac{a^2}{4h}$ below the centre.

A right circular cylindrical tank of radius a with a circular lid of radius a, smoothly hinged at a point A on the circumference of the lid, is filled with a homogeneous liquid of density ρ and kept closed by a smooth lock at the point B diametrically opposite to A. This tank is immersed in a homogeneous liquid of density $\frac{\rho}{2}$ with AB vertical, A above B, and its axis horizontal and at a depth $d(> a)$ from the free surface of the liquid. (See the figure)

Now, the lock is released. Show that the lid remains closed if $d > \frac{9a}{4}$.



13. A particle P of mass m is projected vertically upwards with speed u from a point O. It is subject to a resistive force of magnitude mkv^2, where v is the speed of the particle.

Show that $\frac{dv}{dt} + g + kv^2 = 0$ for the upward motion of P.

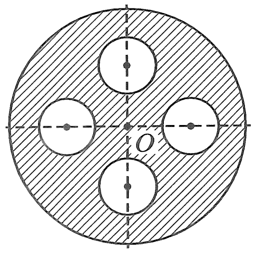
Show that the time taken by P to reach its greatest height H above O is $\frac{1}{\sqrt{gk}}\tan^{-1}\left(\sqrt{\frac{k}{g}}u\right)$ and that $H = \frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$.

Also, find the velocity of P, in terms of u, k and g, when it returns to O.

14. Two smooth uniform spheres A and B of equal mass and equal radius, moving on a smooth horizontal floor, collide with each other. Just before the collision, the velocities of A and B are $u(3\mathbf{i} + 4\mathbf{j})$ and $u(-\mathbf{i} + \frac{1}{2}\mathbf{j})$, respectively and the line joining the centres of A and B is parallel to $\mathbf{i}$. The coefficient of restitution between A and B is $\frac{\sqrt{3}}{2}$. Find the velocities of A and B just after the collision and show that they are perpendicular to each other.

Also, find the impulse on B from A and the loss of kinetic energy due to the collision.

15. A uniform wheel is in the shape of a disc of radius a, centre O with four identical small discs of radius $\frac{a}{4}$ removed from it. The centres of the four small discs lie on two perpendicular diameters of the wheel and all are at a distance $\frac{a}{2}$ from O as shown in the figure.



Show that the moment of inertia of the wheel about the axis through O perpendicular to its plane is $\frac{55}{96}Ma^2$, where M is the mass of the wheel.

The wheel is placed on a rough horizontal floor and given an impulse horizontally so that it starts sliding with speed u and no angular speed.

The wheel perform sliding and rolling for a period of time T and then, pure rolling begins. Find T in terms of u, g and μ where μ is the coefficient of friction between the wheel and the floor.

16. A discrete random variable X has probability distribution given below:

x	0	1	2	3	4
$P(X=x)$	p	q	r	0.2	0.1

where p, q and r are constants.

It is given that $E(X) = 1.5$ and $E(X^2) = 4.1$.

Find each of the following:

(i) The values of p, q and r.

(ii) $P\left(\frac{1}{2} < X < \frac{7}{2}\right)$

(iii) $\mathrm{Var}(X)$

(iv) $E(3-2X)$ and $\mathrm{Var}(3-2X)$

Let X_1 and X_2 be two independent discrete random variables having the same probability distribution as that of X given above, and let $Y = X_1 + 2X_2$.

(v) Find $P(Y = k)$ for $k = 0, 1, 2, 3, 4$, and **hence**, find $P(Y \geq 5)$.

(vi) Write down the value of $E(Y)$.

17. (*a*) A continuous random variable X has probability density function $f(x)$ given by

$$f(x)=\begin{cases}\frac{15}{2}x^2(1-x^2) & , \text{ for } 0\leq x\leq 1,\\ 0 & , \text{ otherwise.}\end{cases}$$

Find E(X) and Var(X).

Also, find $P\left(\frac{1}{2}<X<1\right)$.

Let Y be the random variable defined by $Y = 3X - 2$.

Find E(Y) and Var(Y).

(*b*) The heights of employees of a certain company are normally distributed with mean 160 cm and standard deviation 5 cm.

(i) Find the probability that the height of a randomly selected employee is greater than 165 cm and less than 170 cm.

(ii) Given that an employee selected at random has a height greater than 165 cm, find the probability that the employee has a height greater than 170 cm.

* * *