නව/පැරණි නිර්දේශය – புதிய/பழைய பாடத்திட்டம் – New/Old Syllabus

NEW/OLD

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

උසස් ගණිතය I
உயர் கணிதம் I
Higher Mathematics I

11 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions **1 - 10**) and **Part B** (Questions **11 - 17**).
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(11) Higher Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Factorize: $(a+b-c)(b+c-a)(c+a-b)-8abc$.

2. Let a relation R be defined on the set of all integers $\mathbb{Z}$ by aRb if $a+3b$ is divisible by 4. Show that R is an equivalence relation on $\mathbb{Z}$ and write down the equivalence class of 0.

[see page three

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3. Let $f(x)=\frac{x-1}{2x+1}$ for $x \neq -\frac{1}{2}$.

Find $f^{-1}(x)$. Also, find $f\left(3f^{-1}(0)\right)$.

4. Find the values of the constant α such that

$$\begin{vmatrix} a+p\alpha & b+q\alpha & c+r\alpha \\ a\alpha+p & b\alpha+q & c\alpha+r \\ x & y & z \end{vmatrix} + 3\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 0.$$

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5. Two variable points $P \equiv (ap^2, 2ap)$ and $Q \equiv (aq^2, 2aq)$ lie on the parabola $y^2 = 4ax$ such that PQ subtends a right angle at the origin O.
Show that $pq = -4$ and that the mid-point of PQ lies on the parabola $y^2 = 2a(x - 4a)$.

6. Let $a, b \in \mathbb{R}$ and let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} \dfrac{a\sin 2x}{x} & \text{if} \quad x < 0, \\ (b-1)x + a & \text{if} \quad 0 \leq x \leq 1, \\ \dfrac{b(x-1)}{|x-1|} & \text{if} \quad 1 < x. \end{cases}$$

If f is continuous, find the values of a and b.

[see page five

7. Let $f(x)=\begin{cases} x^2+1, & \text{if} \quad x\le 0, \\ -x^2+1, & \text{if} \quad 0<x<1, \\ x-1, & \text{if} \quad 1\le x. \end{cases}$

Show that $f(x)$ is differentiable at $x = 0$ and non-differentiable at $x = 1$.
Write down $f'(x)$ for $x \neq 1$.

8. Solve the differential equation $\frac{dy}{dx}+2y=x$, subject to the condition $y = 1$ when $x = 0$.

[see page six

9. Let f be a real-valued function on $[0, 1]$ such that f' is continuous on $[0, 1]$.

Also, let $g(x) = 3x^2 f(x^3) + xf'(x)$ for $x \in [0, 1]$. Show that $\int_0^1 g(x)\,dx = f(1)$.

10. Sketch the curves whose polar equations are given by $r = \sqrt{3} \cos \theta$ and $r = 2 \sin \theta - \sqrt{3} \cos \theta$ in the same diagram, and find the polar coordinates of their points of intersection.

[see page seven

නව/පැරණි නිර්දේශය – புதிய/பழைய பாடத்திட்டம் – New/Old Syllabus

NEW/OLD

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
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උසස් ගණිතය I
உயர் கணிதம் I
Higher Mathematics I

11 E I

Part B

✻ Answer **five** questions only.

11. (*a*) Let A, B and C be subsets of a universal set S. Stating clearly the Laws of Algebra of sets that you use, show that

(i) $A'\cup\big((A\cup B)-B\big)=(A\cap B)'$,

(ii) $(A\cup B\cup C)-\big((A-C)-B\big)=B\cup C$,

where $A-B$ is defined by $A\cap B'$.

(*b*) In a music class of 100 students, 85 students like to play violin, 20 like to play piano and 45 like to play guitar. Also, 10 like to play violin and piano, 15 like to play piano and guitar, and 30 like to play guitar and violin. Find the number of students who like to play

(i) all three instruments,

(ii) violin and guitar, but not piano,

(iii) violin or guitar,

assuming that every student like to play at least one of the three instruments.

12. (*a*) Let $a, b, c > 0$.

(i) Show that $\dfrac{a+b}{2}\geq\sqrt{ab}$ and **deduce** that $(a+b)\,(b+c)\,(c+a)\geq 8abc$.

(ii) Using $\dfrac{a+b+c}{3}\geq\sqrt[3]{abc}$, show that if $a+b+c=2$, then $(1-a)\,(1-b)\,(1-c)\leq\dfrac{1}{27}$.

(*b*) The transformation $\begin{pmatrix} x' \\ y' \end{pmatrix}=\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ maps points in the xy-plane to the points in the $x'y'$-plane. Find the equations of the two straight lines in the xy-plane through the point $(0, 1)$ which are mapped onto themselves.

Let $A\equiv(1, 1)$ and $B\equiv(1, 0)$ be two points in the xy-plane. Show that their images lie on the line $2x'-3y'-5=0$ in the $x'y'$-plane.

[see page eight

13. State and prove **De Moivre's Theorem** for a positive integral index.

Using **De Moivre's Theorem**, show that

$$\frac{\cos 5\theta}{\cos\theta} = 16\cos^4\theta - 20\cos^2\theta + 5 \text{ for } \cos\theta \neq 0.$$

Using this result,

(i) evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos 5\theta \tan\theta \, d\theta$,

(ii) show that the roots of the quadratic equation $16x^2 - 20x + 5 = 0$ are $\cos^2\frac{\pi}{10}$ and $\cos^2\frac{3\pi}{4}$.

Deduce that $\sec^2\frac{\pi}{10} + \sec^2\frac{3\pi}{10} = \frac{1}{4}$.

14.(*a*) Let C_1 be the ellipse $x^2 + 6y^2 = 25$ and C_2 be the parabola $y^2 = 4x$. Sketch the graphs of C_1 and C_2 in the same diagram indicating the coordinates of their points of intersection.

Find the area of the region R in the **first quadrant** bounded by the curves C_1 and C_2.

Also, find the volume of the solid generated by rotating the region R through 2π radians about the x-axis.

(*b*) A family of curves satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{2x+4y-1}{x+2y-3}$.

Using the substitution $v = x + 2y$, show that the given differential equation gets transformed to $\dfrac{dv}{dx} = \dfrac{5(v-1)}{(v-3)}$.

Hence, find the equation satisfied by the given family of curves in terms of x and y.

Also, obtain the differential equation satisfied by the orthogonal trajectories of this family of curves.

15.(*a*) Let $I_n = \displaystyle\int \frac{dx}{(x^2 + a^2)^n}$, where $a > 0$.

Show that, $2(n-1)a^2 I_n = \dfrac{x}{(x^2 + a^2)^{n-1}} + (2n-3)I_{n-1}$ for $n \geq 2$.

Hence, find $\displaystyle\int_0^a \frac{dx}{(x^2 + a^2)^4}$.

(*b*) Let f be a function such that $(x^2 + 1) f''(x) + 2x f'(x) + f(x) = 0$.

Show that $(x^2 + 1) f'''(x) + 4x f''(x) + 3f'(x) = 0$.

It is given that $f(0) = 1$ and $f'(0) = 2$.

Find the Maclaurin series of $f(x)$ in ascending powers of x up to and including the term x^3.

Using this, find an approximate value for $\displaystyle\int_0^{0.1} f(x)\,dx$.

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16. Let S be the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Show that the equation of the chord joining the points $P\equiv(a\cos\theta,\ b\sin\theta)$ and $Q\equiv(a\cos\phi,\ b\sin\phi)$ is $\frac{x}{a}\cos\left(\frac{\theta+\phi}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right)=\cos\left(\frac{\theta-\phi}{2}\right)$.

Write down the equation of the tangent drawn to S at P.

The tangents drawn to S at the points P and Q intersect at a point R.

Show that $R\equiv\left(a\frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)},\ b\frac{\sin\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}\right)$.

Now, suppose that the points P and Q on S are such that $\phi=\theta-\frac{\pi}{3}$. Show that R lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{4}{3}$.

Find the equations of the tangents drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{4}{3}$ which are parallel to the tangent to S at P.

17. (*a*) Let, $f(x)=\frac{\cos x}{\sqrt{5}+\sin x}$ for $x\in\mathbb{R}$.

(i) Show that $-\frac{1}{2}\le f(x)\le\frac{1}{2}$ for $x\in\mathbb{R}$.

(ii) For $0\le x\le\pi$, sketch the graph of $y=f(x)$.

(*b*) The following table gives values of the function $f(x)=\ln(3+x^2)$ correct to four decimal places for values of x between 0 to 6 at intervals of length 1.

x	0	1	2	3	4	5	6
$f(x)$	1.0986	1.3863	1.9459	2.4849	2.9444	3.3322	3.6636

Using **Simpson's Rule**, find an approximate value for $I=\int_0^6 \ln(3+x^2)\,dx$.

Hence, find an approximate value for $\int_0^6 \ln(3e+ex^2)\,dx$.

* * *