Algebraic Expressions

By studying this lesson you will be able to

- construct algebraic expressions,
- simplify algebraic expressions, and
- find the value of algebraic expressions by substituting numbers.

12.1 Constructing algebraic expressions

Kavin buys the same amount of milk every day. If this amount is not known, then we cannot represent it by a number although it is a constant value.

As in the above situation, when the numerical value of a constant amount is not known, it is defined as an "unknown constant".

The daily income of a certain shop takes different values depending on its daily sales. Since the daily income is not a fixed value, it is a variable.

Simple letters of the English alphabet such as $a, b, c, ..., x, y, z$ are used to represent unknown constants and variables.

Accordingly, considering the above two examples, the amount of milk bought each day can be denoted by the letter a and the daily income of the shop can be denoted by x.

Let us denote the number of bananas in a bunch in a shop by a. When a comb of 12 bananas is sold, the number of bananas remaining in the bunch can be denoted by $a - 12$.

The expression $a - 12$ is an **algebraic expression**. a and 12 are defined as the "**terms**" of this expression.

For free distribution

If the price of a banana is 8 rupees, then $8 \times a$ rupees can be gained by selling all the bananas in the bunch. This is written as $8a$. The coefficient of a in the term $8a$ is 8. There is only one algebraic term in the expression $8a$.

Let us take the number of rice packets sold daily by a vendor as x. If the price of a rice packet is 80 rupees, then the vendor's daily income is $80 \times x$ rupees. We write this as $80x$ rupees.

If the vendor receives a new order to supply 10 more packets daily, then the number of rice packets sold daily will be $x + 10$.

The terms in the expression $x + 10$, are x and 10.

Example 1

The letter m represents a number of unknown value.
(i) Write in terms of m, the number which is three times the given number.
(ii) Write in terms of m, the number which is 15 more than twice the given number.

↬ (i) The number which is three times as large as m is $3m$.
   (ii) The number which is twice as large as m is $2\,m$.
        Therefore, the number that is greater than $2\,m$ by 15 is $2\,m + 15$.

Exercise 12.1

(1) (i) Construct an algebraic expression for the price of 5 apples by taking the price of one apple as a rupees.

(ii) The price of a pineapple is 10 rupees more than the price of 5 apples. Construct an algebraic expression for the price of a pineapple in terms of a.

(2) A shop owner buys 12 loaves of bread from a bakery at *b* rupees per loaf. He then sells these loaves so that each loaf brings him a profit of 3 rupees.

   (i) What is the total amount the shop owner pays for the loaves of bread?

   (ii) What is the selling price of a loaf of bread?

   (iii) A customer buys a loaf of bread and 500g of sugar. The price of 1 kilogram of sugar is 80 rupees. What is the total amount the customer spends?

(3) 1 m = 100 cm.

   (i) The length of a table is *k* centimeters more than 2 meters. Express the length of the table in centimeters in terms of *k*.

   (ii) The width of this table is 50 cm less than its length. Write the width as an expression of *k*.
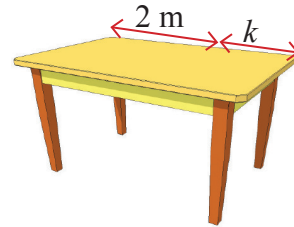
12.2 More on constructing algebraic expressions

The algebraic expressions we have constructed so far contain one algebraic symbol, one or more mathematical operations and numbers.

The following table describes algebraic expressions containing one unknown term.

Algebraic Expression	Unknown constant or variable in the algebraic expression	Coefficient of the unknown constant or variable	Terms of the algebraic expression	Mathematical operations in the order they appear in the algebraic expression
$4x$	x	4	$4x$	$\times$
$y+4$	y	1	y , 4	$+$
$p-10$	p	1	p , 10	$-$
$20+3m$	m	3	20 , $3m$	$+$, $\times$
$3a+5$	a	3	$3a$, 5	$\times$, $+$

The mathematical operations of addition, subtraction and multiplication are used in the above expressions. The coefficient of the unknown in each of the expressions is a positive whole number. The operation division is not used in any of these algebraic expressions.

Let us now consider algebraic expressions that have a fraction as the coefficient of the unknown.

There are x number of marbles in a bottle. They are placed in three containers such that each container has the same number of marbles. Then the number of marbles in one container is $x \div 3$. That is, $\frac{x}{3}$.

The width of a hostel room is half its length. If the length is l meters, let us write the width in meters.

The width of the room is $l \div 2$ meters. That is, the width of the room is $\frac{l}{2}$ m.

The length of the adjoining room is one meter more than the width of this room. Let us write the length of the adjoining room as an algebraic expression.

The length of the adjoining room $= \dfrac{l}{2} + 1$ meters.

Example 1

(1) If more than one meter of cloth is bought, then the price of one meter is p rupees. If less than one meter of cloth is bought, then an additional 10 rupees is charged. Write the price of $\dfrac{1}{2}$ a meter of cloth as an algebraic expression.

Price of 1m of cloth $= p$ rupees

Since the quantity which is bought is less than 1 m,

the price of $\dfrac{1}{2}$ m of cloth $= \dfrac{p}{2} + 10$ rupees.

Example 2

(1) A father sells the 3 plots of land he owns at p rupees per plot. He then divides the money he receives equally among his four children. Write the amount of money received by each child as an algebraic expression.

The money obtained by selling the three plots of land $= 3p$ rupees

The amount of money received by each child $= \dfrac{3p}{4}$ rupees

Exercise 12.2

(1) Complete the following table.

Algebraic expression	Unknown constant or variable in the algebraic expression	Terms of the algebraic expression
$\dfrac{a}{2} + 5$	a	$\dfrac{a}{2}, 5$
$\dfrac{p}{4} - 8$		
$\dfrac{x}{5} + 10$		
$25 + \dfrac{y}{3}$		

For free distribution

(2) Construct an algebraic expression for each of the following situations.

(i) The value of a number is denoted by a. What is the value of the number that is greater by 4 than half the value of the given number?

(ii) In a restaurant, a loaf of bread is sold for p rupees. A person buys $\frac{1}{4}$ of a loaf of bread and a dish of dhal. The dish of dhal costs 30 rupees. Write an algebraic expression for the total amount of money the person has to pay.

(iii) The height of a building is 5 meters less than $\frac{1}{2}$ of its length. If its length is l meters, write the height as an expression of l.

(iv) The price of 1kg of sugar is y rupees. If a 100 rupee note is tendered when $\frac{1}{2}$ kg of sugar is bought, write the balance as an algebraic expression of y.

(3) The price of a box of pencils containing 12 pencils is x rupees.

(i) Write the price of one pencil as an algebraic expression.
(ii) If the price of an eraser is 10 rupees, write the amount of money required to buy 2 pencils and an eraser as an algebraic expression.

(4) Write the expressions given below in words.
The expression $5a - 8$ can be expressed in words as follows.

If a denotes a given value, then $5a - 8$ denotes the value which is 8 less than the value of five times a.

(i) $2a + 8$ (ii) $3x - 15$ (iii) $2(p + 5)$

(iv) $\frac{p}{4} - 4$ (v) $20 - 5p$ (vi) $\frac{x}{2} + 14$

(vii) $\frac{y}{5} - 1$ (viii) $30 + \frac{p}{2}$ (ix) $45 - \frac{y}{3}$

12.3 Constructing algebraic expressions having two unknown terms

The price of a pencil is x rupees and the price of an eraser is y rupees. Let us write the price of 5 pencils and 2 erasers as an algebraic expression.

The price of 5 pencils = 5 × x rupees = $5x$ rupees
The price of 2 erasers = 2 × y rupees = $2y$ rupees
The price of 5 pencils and 2 erasers = $(5x + 2y)$ rupees

The price of 1 kg of sugar is x rupees, the price of 1kg of wheat flour is y rupees and the price of a box of matches is 3 rupees. Let us write the amount of money required to buy 500 g of sugar, 2 kg of wheat flour and 3 boxes of matches as an algebraic expression.

$$\left.\begin{array}{r}\text{The price of 500 g of sugar,}\\\text{when the price of 1 kg of sugar is } x \text{ rupees}\end{array}\right\} = \frac{x}{2} \text{ rupees}$$

$$\left.\begin{array}{r}\text{The price of 2 kg of wheat flour}\\\text{when the price of 1 kg of wheat flour is } y \text{ rupees}\end{array}\right\} = 2y \text{ rupees}$$

$$\left.\begin{array}{r}\text{The price of 3 boxes of matches,}\\\text{when the price of one box of matches is 3 rupees}\end{array}\right\} = 9 \text{ rupees}$$

Therefore, the required amount of money $= (\frac{x}{2} + 2y + 9)$ rupees

Example 1

(i) There are a number of boys and b number of girls in a class. Write the total number of students in the class as an algebraic expression.

The total number of students in the class = $a + b$

(ii) Write the algebraic expression $\frac{x}{2} + \frac{y}{2}$ in words.

"Add one half of the value represented by y to one half of the value represented by x"

Example 2

25 coconuts were bought at a rupees each and all 25 fruits were sold at b rupees each. Assume that b is greater than a. Write an algebraic expression for the profit.

$$\text{The price of a coconut} = a \text{ rupees}$$

The amount of money spent on buying 25 coconuts $= 25a$ rupees

The amount of money gained by selling 25 coconuts $= 25b$ rupees

$$\text{Profit} = (25b - 25a) \text{ rupees}$$

Exercise 12.3

(1) Construct algebraic expressions for the following.

(i) A number is represented by a. What is the number that is greater than a by b?

(ii) A number is represented by p. Write the number that is less than p by q.

(iii) The price of a coconut is x rupees.
The price of 1kg of rice is y rupees.
Write an expression in terms of x and y for the price of 4 coconuts and 3 kg of rice.

(iv) The price of 1 kg of sugar is x rupees and the price of a 250 g packet of tea is y rupees. Find the amount of money required to buy 2 kg and 500 g of sugar and 2 packets of tea.

(v) 250 g $= \frac{1}{4}$ kg. 1kg of potatoes is x rupees. A bundle of green leaves is y rupees. Write an algebraic expression for the amount paid if 250 g of potatoes and a bundle of green leaves are bought.

(vi) There are x number of Sinhala books and y number of English books in the school library. $\frac{1}{2}$ the Sinhala books and $\frac{1}{2}$ the English books are Literature books. If the library has issued 23 Sinhala Literature books and 18 English Literature books, then express the number of literature books remaining in the library as an algebraic expression.

(2) Write the following expressions in words.

    (i) $3x + 5y$ (ii) $2a - 7b$ (iii) $\frac{x}{4} - y + 5$ (iv) $2k + 3p - 8$

12.4 Simplifying the terms of an algebraic expression

Let us consider an algebraic expression similar to one we constructed earlier.

The price of an orange is a rupees. Nimal bought 5 oranges and Deepani bought 8.

Nimal   Deepani

Nimal spent $5a$ rupees and Deepani spent $8a$ rupees. So the total amount of money spent by both of them is $5a + 8a$.

Since the number of oranges bought by both of them is 13, the total amount spent is $13 \times a$ rupees. That is $13a$ rupees.

This shows that $5a + 8a = 13a$.

Algebraic terms such as $5a$ and $8a$ which have the same unknown are called "**like terms**". By adding or subtracting several such terms, we can simplify them to one term.

There are no like terms in the algebraic expression $4x + 3y + 5$. Such an expression cannot be simplified further. The terms $4x$, $3y$, 5 of this expression are called "**unlike terms**".

    Let us simplify $4x + 3y + x + 2y$.

    Let us write the like terms together.
$$4x + 3y + x + 2y = 4x + 1x + 3y + 2y$$
$$= 5x + 5y$$

    Let us simplify $10p + 4k + p - k$.
$$10p + 4k + p - k = 10p + 1p + 4k - 1k$$
$$= 11p + 3k$$

Example 1

Simplify the following.

(i) $3x + 6k + 5x + 3k + 7$ (ii) $5a + b + 8 + 3a - b - 5$

(i) $3x + 6k + 5x + 3k + 7 = 3x + 5x + 6k + 3k + 7$
$$= 8x + 9k + 7$$

(ii) $5a + b + 8 + 3a - b - 5 = 5\,a + 3a + b - b + 8 - 5$
$$= 8a + 0 + 3$$
$$= 8a + 3$$

Example 2

There are 25 boys and 15 girls in a Grade 4 class.
There are 28 boys and 11 girls in a Grade 5 class.
The price of a pen is p rupees and the price of an eraser is q rupees. Find the total amount of money needed to give a pen to each boy in Grade 4, an eraser to each girl in Grade 4, an eraser to each boy in Grade 5 and a pen to each girl in Grade 5.

The money needed to give pens and erasers
to the students in Grade 4 $\quad = 25p + 15q$

The money needed to give pens and
erasers to the students in Grade 5 $\quad = 11p + 28q$

The money needed to give pens and erasers
to the students in Grade 4 and Grade 5 $\quad = 25p + 15q + 11p + 28q$

$$= 25p + 11p + 15q + 28q$$
$$= 36p + 43q$$

Exercise 12.4

(1) Simplify the following.

(i) $4x + 5y + 3x + 7$ (ii) $3a + 4 + 6b + 3$

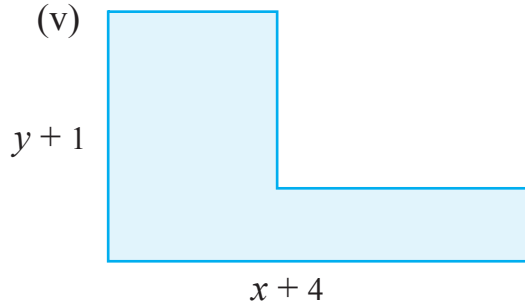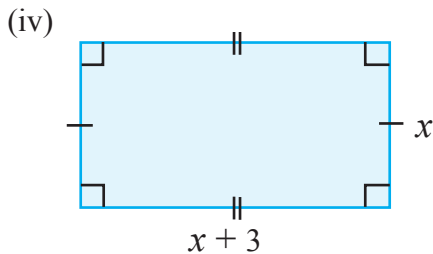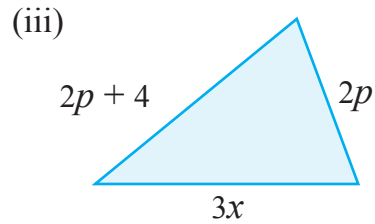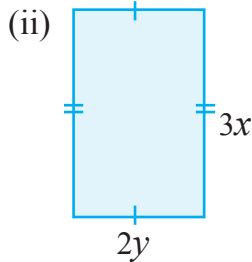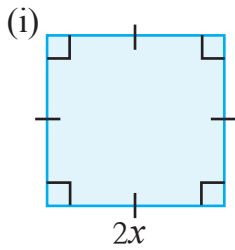(iii) $5p + 4q - 2p + q$ (iv) $10m - 7n + 10n - 4m$

(v) $3k + 5l + 10 + k + 4l - 5$ (vi) $8x - 4y - 11 + x + 7y + 13$

(2) Write an algebraic expression for the perimeter of each of the figures below. Simplify the expression.

(i)

$2x$

(ii)

$3x$

$2y$

(iii)

$2p + 4$ $2p$

$3x$

(iv)

x

$x + 3$

(v)

$y + 1$

$x + 4$

12.5 Substituting values for the unknowns in an algebraic expression

When $x = 2$, the expression $x + 3$ takes the value 5. You have learnt in grade 6 that giving a numerical value to the unknown term in an algebraic expression in this manner is called substitution. By substitution, an algebraic expression gets a value.

Let us consider the expression $x + 3$.
When $x = 2$,
$x + 3 = 2 + 3 = 5$.

Let us find the value of $3x - 5$ when $x = 4$.

$3x - 5 = 3 \times 4 - 5$
$= 12 - 5 = 7$

Let us find the value of $4a - 3$ when $a = 2$.
$4a - 3 = 4 \times 2 - 3$
$= 8 - 3$
$= 5$

Let us now substitute values for the unknowns in an algebraic expression which has two unknown terms and find its value.

Let us find the value of $3x + 4y$ when $x = 4$ and $y = 5$.

$$3x + 4y = 3 \times 4 + 4 \times 5$$
$$= 12 + 20$$
$$= 32$$

Example 1

Find the value of each of the algebraic expressions given below when $x = 4$ and $y = 2$.

(i) $x - y$
$$x - y = 4 - 2 = 2$$

(ii) $3x - y - 5$
$$3x - y - 5 = 3 \times 4 - 2 - 5$$
$$= 12 - 2 - 5$$
$$= 10 - 5$$
$$= 5$$

Exercise 12.5

(1) Find the value of each of the algebraic expressions given below when $a = 4$.

   (i) $3a - 5$ (ii) $5(a - 3)$ (iii) $15 - 2a$ (iv) $7a - 5$

(2) For each of the values given to x, find the value of $6x + 4$.

   (i) $x = 1$ (ii) $x = 2$ (iii) $x = 5$ (iii) $x = 12$

(3) Find the value of each of the given expressions by substituting the given values.

   (i) $4x - 13y + 5$ when $x = 4$ and $y = 1$

  (ii) $7a - 3b - 8$, when $a = 3$ and $b = 1$

  (iii) $2p + k - 5$, when $p = 6$ and $k = 2$

(1) The length of a room is x meters less than twice its width. The width of the room is 3 m. Write an expression in terms of x for the length of the room.

(2) The price of a pen is x rupees and the price of 12 books is y rupees. Nimal buys 2 pens and 3 books. Write an expression for the total amount of money spent by Nimal.

(3) Write each expression given below in words.

    (i) $8 + \frac{y}{2}$ (ii) $16 - \frac{a}{3}$

(4) Simplify the following.

    (i) $8a + 7b - 3 - 6b - 2a$ (ii) $6x + 5y - 6x - 3y$

(5) Find the value of each of the expressions given below when $x = 7$ and $y = 3$.

    (i) $6x - 5y$ (ii) $7x - 3 - 6y$

(6) A father's age was 35 years at the time his son was born.

    (i) Write the age of the father, when his son is x years old.

    (ii) The mother is 4 years younger to the father. Write the mother's age in terms of x when the son is x years old.

    (iii) How many years older is the mother than the son?

For free distribution

Summary

- In an algebraic expression, the number written together with an unknown is called the "coefficient of the unknown".
- Algebraic terms with the same unknown are called "like terms".
- Several like terms can be simplified into one term by adding or subtracting them.
- Algebraic terms with different unknowns are called "unlike terms".
- Two unlike terms cannot be simplified further into one term by adding or subtracting them.

Ponder

(1) A vendor sells 1kg of brinjals for 10 rupees more than twice the price he paid for 1 kg of brinjals. He sells 1kg of papaw for 8 rupees more than three times the price he paid for 1 kg of papaw.

The vendor buys 1kilogram of brinjals and 1kilogram of papaw for x rupees and y rupees respectively.

(i) Write an algebraic expression for the amount the vendor spent to buy 1 kg of brinjals and 1 kg of papaw.

(ii) Write an algebraic expression for the selling price of 1 kg of brinjals.

(iii) Write an algebraic expression for the selling price of 1 kg of papaw.

(iv) Write an algebraic expression for the amount he receives by selling 1 kg of brinjals and 1 kg of papaw.

(v) If the vendor bought 1 kg of brinjals for 35 rupees and 1 kg of papaw for 20 rupees, obtain values for the algebraic expressions in (i), (ii), (iii) and (iv) above.