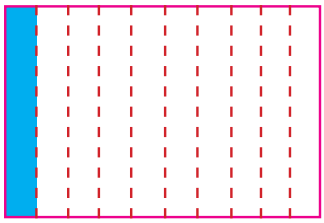
# 11 Decimals

By studying this lesson you will be able to

- represent a fraction with a denominator that can be written as a power of ten, as a decimal number,
- represent a decimal number as a fraction, and
- multiply and divide a decimal number by a whole number.

## 11.1 Writing a proper fraction with a denominator which is a power of ten, as a decimal number

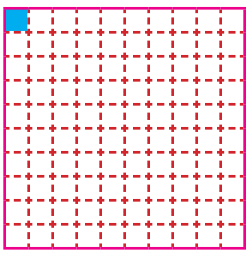
In Grade 6 we learnt how to write a proper fraction with 10 or 100 as the denominator, as a decimal number.



When a unit is divided into 10 equal parts, then one part is equal to $\frac{1}{10}$.

This is denoted as a decimal number, by 0.1.

That is, $0.1 = \frac{1}{10}$.



When a unit is divided into 100 equal parts, then one part is equal to $\frac{1}{100}$.

This is denoted as a decimal number, by 0.01.

That is, $0.01 = \frac{1}{100}$.

We have learnt that when a unit is divided into 1000 equal parts, then one part is equal to $\frac{1}{1000}$.

$\frac{1}{1000}$ is written in decimal form as 0.001. That is, $0.001 = \frac{1}{1000}$.

The number 0.001 is read as zero point zero zero one. The position where 1 is written after the second decimal place in 0.001 is defined as the third decimal place. The place value of the third decimal place is $\frac{1}{1000}$.

Since $\frac{7}{1000}$ is seven $\frac{1}{1000}$ s, we obtain $\frac{7}{1000} = 0.007$ The number 0.007 is read as zero point zero zero seven.

Let us consider $\frac{24}{1000}$ .

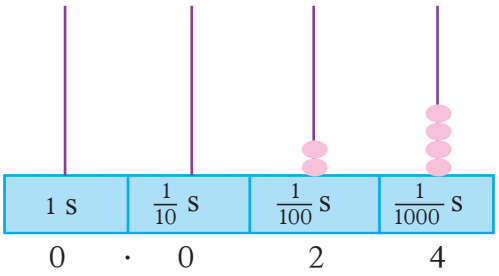
$\frac{24}{1000} = \frac{20}{1000} + \frac{4}{1000}$

Since, $\frac{20}{1000} = \frac{20 \div 10}{1000 \div 10} = \frac{2}{100}$

$\frac{24}{1000}$ = two $\frac{1}{100}$ s + four $\frac{1}{1000}$ s.

Accordingly, $\frac{24}{1000} = 0.024$.

0.024 is read as zero point zero two four.
Let us represent 0.024 on an abacus.




**Example 1**

(1) Write each of the following fractions as a decimal number.

(i) $\frac{4}{1000}$ (ii) $\frac{97}{1000}$ (iii) $\frac{751}{1000}$

(i) $\frac{4}{1000} = 0.004$ (ii) $\frac{97}{1000} = 0.097$ (iii) $\frac{751}{1000} = 0.751$

## Exercise 11.1

(1) Express each of the following fractions as a decimal number. Represent them on an abacus.

(i) $\frac{9}{10}$ (ii) $\frac{75}{100}$ (iii) $\frac{9}{1000}$ (iv) $\frac{25}{1000}$ (v) $\frac{275}{1000}$

## 11.2 Writing a proper fraction with a denominator which is not a power of ten, as a decimal number

Let us learn how to express a proper fraction with a denominator which is not a power of 10, as a decimal number.

- The given fraction can easily be written as a decimal number if it can be converted into an equivalent fraction which has a power of 10 as its denominator.

Let us express $\frac{1}{2}$ as a decimal number.

10 can be divided by 2 without remainder. $10 \div 2 = 5$. Therefore, by multiplying the numerator and the denominator of $\frac{1}{2}$ by 5, it can be converted into an equivalent fraction with 10 as the denominator.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

$$\frac{5}{10} = 0.5$$

Therefore, $\frac{1}{2} = 0.5$.

Let us express $\frac{1}{4}$ as a decimal number.

Although 10 cannot be divided by 4 without remainder, 100 can be divided by 4 without remainder. $100 \div 4 = 25$.

Therefore, by multiplying the numerator and the denominator of $\frac{1}{4}$ by 25, it can be converted into an equivalent fraction with 100 as the denominator.

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100}$$

$$\frac{25}{100} = 0.25$$

Therefore, $\frac{1}{4} = 0.25$.

Let us express $\frac{1}{8}$ as a decimal number.

Although 10 and 100 cannot be divided by 8 without remainder, 1000 can be divided by 8 without remainder. $1000 \div 8 = 125$.

Therefore, by multiplying the numerator and the denominator of $\frac{1}{8}$ by 125, it can be converted into an equivalent fraction with 1000 as the denominator.

$$\frac{1}{8} = \frac{1 \times 125}{8 \times 125} = \frac{125}{1000}$$

$$\frac{125}{1000} = 0.125$$

Therefore, $\frac{1}{8} = 0.125$.

According to the above description, the proper fractions that can be converted into equivalent fractions with a power of 10 as the denominator, can easily be expressed as decimal numbers.

That is, if 10, 100, 1000 or any other power of 10 can be divided without remainder by the denominator of a given fraction, then that fraction can be written as a decimal number with one or more decimal places.

**Example 1**

Express each of the fractions $\frac{1}{5}$, $\frac{13}{25}$ and $\frac{77}{125}$ as a decimal number.

$\frac{1}{5} = \frac{2}{10} = 0.2$

$\frac{13}{25} = \frac{52}{100} = 0.52$

$\frac{77}{125} = \frac{77 \times 8}{125 \times 8} = \frac{616}{1000} = 0.616$

## 11.3 Writing a mixed number as a decimal number

Now let us consider how a mixed number is expressed as a decimal number.

Let us write $3\frac{5}{20}$ as a decimal number.

$3\frac{5}{20} = 3 + \frac{5}{20}$

$= 3 + \frac{5 \times 5}{20 \times 5} = 3 + \frac{25}{100}$

$= 3 + 0.25$

$= 3.25$

Let us write $7\frac{11}{40}$ as a decimal number.

$7\frac{11}{40} = 7 + \frac{11}{40}$

$= 7 + \frac{11 \times 25}{40 \times 25}$

$= 7 + \frac{275}{1000}$

$= 7.275$

## 11.4 Writing an improper fraction as a decimal number

Let us consider how an improper fraction is written as a decimal number.

Let us write $\frac{17}{5}$ as a decimal number.

Method I

$\frac{17}{5} = 3\frac{2}{5} = 3 + \frac{2}{5}$

$= 3 + \frac{4}{10} = 3 + 0.4$

$= 3.4$

Method II

$\frac{17}{5} = \frac{34}{10} = \frac{30}{10} + \frac{4}{10}$

$= 3 + 0.4$

$= 3.4$

**Example 1**

Express $\frac{9}{8}$ as a decimal number.

Method I

$\frac{9}{8} = 1 + \frac{1}{8}$

$\frac{9}{8} = 1 + \frac{125}{1000}$

$= 1 + 0.125$

$= 1.125$

Method II

$\frac{9}{8} = \frac{9 \times 125}{8 \times 125}$

$= \frac{1125}{1000} = \frac{1000}{1000} + \frac{125}{1000}$

$= 1 + 0.125$

$= 1.125$

**Exercise 11.2**

Express the following fractions and mixed numbers as decimal numbers.

(i) $\frac{3}{5}$ (ii) $\frac{3}{4}$ (iii) $\frac{8}{25}$ (iv) $\frac{321}{500}$ (v) $\frac{39}{40}$

(vi) $13\frac{1}{2}$ (vii) $2\frac{7}{50}$ (viii) $2\frac{1}{8}$ (ix) $3\frac{7}{40}$ (x) $5\frac{14}{125}$

(xi) $\frac{13}{10}$ (xii) $\frac{27}{20}$ (xiii) $\frac{7}{5}$ (xiv) $\frac{97}{8}$ (xv) $\frac{251}{250}$

## 11.5 Writing a decimal number as a fraction

**Let us write 0.5 as a fraction.**

$0.5 = \frac{5}{10}$

To express $\frac{5}{10}$ in its simplest form, let us divide the numerator and the denominator by 5.

$$0.5 = \frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

**Let us write 0.375 as a fraction.**

There fore, $0.375 = \frac{375}{1000}$

To express $\frac{375}{1000}$ in its simplest form, let us divide the numerator and the denominator by 125.

$$\frac{375}{1000} = \frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$$

$0.375 = \frac{3}{8}$

**Let us write 1.75 as a fraction.**

$1.75 = 1 + 0.75 = 1 + \frac{75}{100} = 1\frac{75}{100}$

To express $\frac{75}{100}$ in its simplest form, let us divide the numerator and the denominator by 25.

$$\frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$$

Therefore, $1.75 = 1\frac{3}{4}$.

**Example 1**

Express 1.625 as a fraction in its simplest form.

$$1.625 = 1 + 0.625 = 1 + \frac{625}{1000} = 1 + \frac{625 \div 25}{1000 \div 25} = 1 + \frac{25}{40} = 1 + \frac{25 \div 5}{40 \div 5}$$
$$= 1 + \frac{5}{8}$$
$$= 1\frac{5}{8}$$

## Exercise 11.3

Write each of the following decimal numbers as a fraction and express it in the simplest form.

(i) 0.7 (ii) 1.3 (iii) 0.45 (iv) 8.16

(v) 6.75 (vi) 0.025 (vii) 4.225 (viii) 8.625

## 11.6 Multiplying a decimal number by a whole number

$2 \times 3 = 2 + 2 + 2 = 6$

This illustrates the fact that the product of two whole numbers can be obtained by writing it as a sum.

Now let us find the value of $0.1 \times 3$.

$0.1 \times 3 = 0.1 + 0.1 + 0.1$

$= 0.3$

Let us find the value of $0.8 \times 2$.

$0.8 \times 2 = 0.8 + 0.8$

$= 1.6$

Let us find the value of $0.35 \times 4$.

$0.35 \times 4 = 0.35 + 0.35 + 0.35 + 0.35$

$= 1.40$

$= 1.4$

Let us examine the above answers by considering the following table.

| | |
|---|---|
| $0.1 \times 3 = 0.3$ | $1 \times 3 = 3$ |
| $0.8 \times 2 = 1.6$ | $8 \times 2 = 16$ |
| $0.35 \times 4 = 1.40$ | $35 \times 4 = 140$ |

It will be clear to you from observing the above table that, when multiplying a decimal number by a whole number, the answer can be obtained by following the steps given below too.

- Consider the decimal number as a whole number by disregarding the decimal point and multiply it by the given whole number.

- Place the decimal point in the answer that is obtained such that the final answer has the same number of decimal places as the original decimal number.

Now let us find the value of 24.31 × 6.
First let us multiply the numbers without taking the decimal places into consideration.

$$\begin{array}{r} 2431 \\ \times \quad 6 \\ \hline 14586 \end{array}$$

Since 24.31 has two decimal places, place the decimal point such that the final answer too has two decimal places. Then 24.31 × 6 = 145.86

It must be clear to you that, when the whole number by which the decimal number has to be multiplied is large, the method given above is much easier to use than the method of repeatedly adding the decimal number.

**Example 1**

Find the value of 4.276 × 12.

$$\begin{array}{r} 4276 \\ \times \quad 12 \\ \hline 8552 \\ 4276 \\ \hline 51312 \end{array}$$

Since 4.276 has three decimal places, the decimal point is placed such that the answer too has three decimal places.

Then, 4.276 × 12 = 51.312

## Exercise 11.4

Evaluate the following.

(i) 2.45 × 6 (ii) 0.75 × 4 (iii) 3.47 × 15

(iv) 15.28 × 13 (v) 0.055 × 3 (vi) 1.357 × 41

## • Multiplying a decimal number by 10, 100 and 1000

Let us consider the following products.

| | | |
|---|---|---|
| 2.1 × 10 = 21.0 | 2.1 × 100 = 210.0 | 2.1 × 1000 = 2100.0 |
| 3.75 × 10 = 37.50 | 3.75 × 100 = 375.00 | 3.75 × 1000 = 3750.00 |
| 23.65 × 10 = 236.50 | 23.65 × 100 = 2365.00 | 23.65 × 1000 = 23650.00 |
| 43.615 × 10 = 436.150 | 43.615 × 100 = 4361.500 | 43.615 × 1000 = 43615.000 |

The following facts are discovered by examining the above products.

- The number that is obtained when a decimal number is multiplied by 10 can be obtained by moving the decimal point in the original decimal number by one place to the right. $37.16 \times 10 = 371.6$
- The number that is obtained when a decimal number is multiplied by 100 can be obtained by moving the decimal point in the original decimal number by two places to the right. $37.16 \times 100 = 3716$
- The number that is obtained when a decimal number is multiplied by 1000 can be obtained by moving the decimal point in the original decimal number by three places to the right. $37.160 \times 1000 = 37\ 160$

### Exercise 11.5

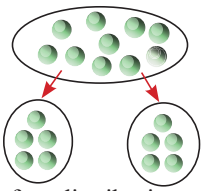
Evaluate the following.

| | | |
|---|---|---|
| (i) $4.74 \times 10$ | (ii) $0.503 \times 10$ | (iii) $0.079 \times 10$ |
| (iv) $5.83 \times 100$ | (v) $5.379 \times 100$ | (vi) $0.07 \times 100$ |
| (vii) $1.2 \times 100$ | (viii) $0.0056 \times 10$ | (ix) $0.0307 \times 100$ |
| (x) $3.7 \times 1000$ | (xi) $8.0732 \times 1000$ | (xii) $6.0051 \times 1000$ |

## 11.7 Dividing a decimal number by 10, 100 and 1000

$10 = 5 \times 2$ means that there are 2 heaps of five in 10. Therefore, when 10 is divided into two equal heaps there are 5 in each heap.

That is $10 \div 2 = 5$.
You have learnt this in Grade 6.



Now let us find the value of 32.6 ÷ 10.

32.6 ÷ 10 is how many 10s there are in 32.6.
We know that 3.26 × 10 = 32.6.
Therefore, 32.6 ÷ 10 = 3.26

Similarly,
145.56 ÷ 100 is how many 100s there are in 145.56.
Since 1.4556 × 100 = 145.56,
we obtain 145.56 ÷ 100 = 1.4556

6127.3 ÷ 1000 is how many 1000s there are in 6127.3.
Since 6.1273 × 1000 = 6127.3,
we obtain 6127.3 ÷ 1000 = 6.1273.

Let us consider the following divisions.

| | | |
|---|---|---|
| 7871.8 ÷ 10 = 787.18 | 7871.8 ÷ 100 = 78.718 | 7871.8 ÷ 1000 = 7.8718 |
| 169.51 ÷ 10 = 16.951 | 169.51 ÷ 100 = 1.6951 | 169.51 ÷ 1000 = 0.16951 |
| 9.51 ÷ 10 = 0.951 | 9.51 ÷ 100 = 0.0951 | 9.51 ÷ 1000 = 0.00951 |

Accordingly,

- The number that is obtained by dividing a decimal number by 10 is equal to the number that is obtained by moving the decimal point in the original decimal number by one decimal place to the left.
  6.0 ÷ 10 = 0.60
- The number that is obtained by dividing a decimal number by 100 is equal to the number that is obtained by moving the decimal point in the original decimal number by two decimal places to the left.
  006.0 ÷ 100 = 0.060 = 0.06
- The number that is obtained by dividing a decimal number by 1000 is equal to the number that is obtained by moving the decimal point in the original decimal number by three decimal places to the left.
  006.0 ÷ 1000 = 0.0060 = 0.006

## Exercise 11.6

Evaluate the following.

(i) $27.1 \div 10$ (ii) $1.36 \div 10$ (iii) $0.26 \div 10$ (iv) $0.037 \div 10$
(v) $0.0059 \div 10$ (vi) $58.9 \div 100$ (vii) $3.7 \div 100$ (viii) $97.6 \div 100$
(ix) $0.075 \div 100$ (x) $0.0032 \div 100$ (xi) $4375.8 \div 1000$
(xii) $356.8 \div 1000$

## • Dividing a decimal number by a whole number

Let us find the value of $7.5 \div 3$.

Divide the whole number part.
When long division is being performed, place the decimal point in the answer, when the number immediately to the right of the decimal point is being divided. Then continue with the division.

**Step 1**

$$\begin{array}{r} 2 \\ 3\overline{)7.5} \\ \underline{6} \\ 1 \end{array}$$

$2 \times 3 = 6$
$7 - 6 = 1$

$7 \div 3 = 2$ with a remainder of 1

Since the decimal part of 7.5 occurs after 7, place the decimal point after 2 in the answer.

**Step 2**

$$\begin{array}{r} 2.\downarrow \\ 3\overline{)7.5} \\ \underline{6} \\ 1\ 5 \end{array}$$

Bring 5 down

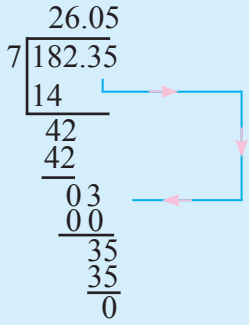
**Step 3**

$$\begin{array}{r} 2.5 \\ 3\overline{)7.5} \\ \underline{6} \\ 1\ 5 \\ \underline{1\ 5} \\ 0 \end{array}$$

$5 \times 3 = 15$
$15 - 15 = 0$

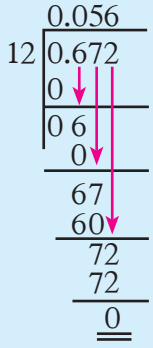
## Example 1

(i) Find the value of 182.35 ÷ 7.



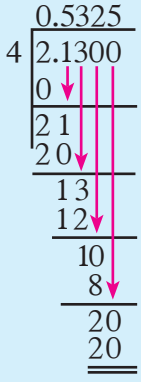

Place the decimal point, when the number 3, which is immediately to the right of the decimal place is being divided.

(ii) Find the value of 0.672 ÷ 12.



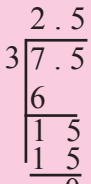

0.672 ÷ 12 = 0.056

(iii) Find the value of 2.13 ÷ 4.




2.13 ÷ 4 = 0.5325

## Further knowledge




The digit in the ones place of 7.5 is 7. This denotes 7 ones.
When 7 is divided by 3, we obtain 2 and a remainder of 1.
A remainder of one means 1 ones. That is, ten $\frac{1}{10}$ s.
The digit 5 in 7.5 denotes five $\frac{1}{10}$ s. Therefore, there are fifteen $\frac{1}{10}$ s in the first decimal place. Let us divide this fifteen $\frac{1}{10}$ s by 3. Then we obtain five $\frac{1}{10}$ s with no remainder. That is 7.5÷ 3 = 2.5

## Exercise 11.7

(1) Evaluate the following.

| | | |
|---|---|---|
| (i) 84.6 ÷ 2 | (ii) 167.2 ÷ 4 | (iii) 54.6 ÷ 3 |
| (iv) 98.58 ÷ 6 | (v) 74.5 ÷ 5 | (vi) 35.86 ÷ 2 |
| (vii) 0.684 ÷ 6 | (viii) 0.735 ÷ 7 | (ix)1.08 ÷ 4 |
| (x) 7.401 ÷ 3 | (xi) 8.04 ÷ 8 | (xii) 11.745 ÷ 9 |

(2) If the height of a child is 145 cm, express this height in metres.

### Summary

- When multiplying a decimal number by a whole number, consider the decimal number as a whole number by disregarding the decimal point and multiply the two numbers. Place the decimal point in the answer that is obtained so that it has the same number of decimal places as the original decimal number.
- When a decimal number is multiplied by a power of ten, the number of places the decimal point in the decimal number shifts to the right is equal to the number of zeros in the power of ten by which it is multiplied.
- When a decimal number is divided by a power of ten, the number of places the decimal point in the decimal number shifts to the left is equal to the number of zeros in the power of ten by which it is divided.