8 Areas of Plane Figures between Parallel Lines

By studying this lesson you will be able to

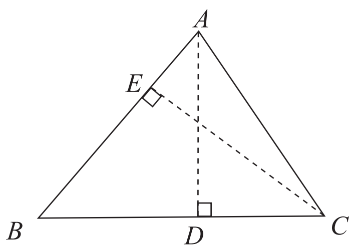
identify the theorems on the relationships between the areas of triangles and parallelograms on the same base and between the same pair of parallel lines, and solve problems related to them.

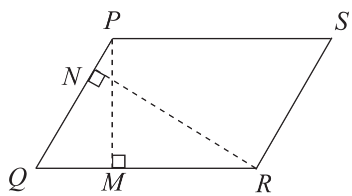
Introduction

You have already learnt about various plane figures and how the areas of certain special plane figures are found. Let us now recall how the areas of triangles and parallelograms are found.

When finding the areas of triangles and parallelograms, the terms altitude and base are used. Let us first recall what these terms mean.

Let us consider the given triangle *ABC* and the parallelogram *PQRS*.





When finding the area of a triangle, any one of its sides can be considered as the base. For example, the side *BC* of the triangle *ABC* can be considered as the base. Then *AD* is the corresponding altitude; that is, the perpendicular dropped from the vertex *A* to the side *BC*.

We know that,

area of triangle $ABC = \frac{1}{2} \times BC \times AD.$

Similarly, if we consider the side *AB* to be the base, the corresponding altitude is *CE*. Accordingly, we can also write,

area of triangle $ABC = \frac{1}{2} \times AB \times CE.$

We can similarly find the area of the triangle *ABC* by taking *AC* as the base and drawing the corresponding altitude from the vertex *B*.

Now let us consider the parallelogram *PQRS*. Here too, the area can be found by considering any one of the sides as the base. If we consider the side *QR* as the base, the corresponding altitude is the line segment *PM*. The length of *PM* is the distance between the two parallel straight line segments *QR* and *PS*, the side opposite *QR*.

We know that,

the area of parallelogram $PQRS = QR \times PM$.

Similarly, if we consider the side *PQ* as the base, the corresponding altitude is *RN*. Therefore we can also write,

the area of parallelogram $PQRS = PQ \times RN$.

Note

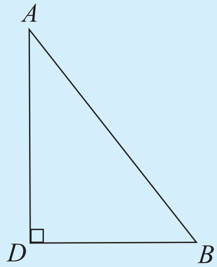
The length of the altitude of a triangle or a parallelogram is also often called the altitude.

To recall what has been learnt earlier regarding finding the areas of parallelograms and triangles, do the following exercise by applying the above facts.

Review Exercise

1. Complete the given table by using the data in each of the figures given below.

(i)



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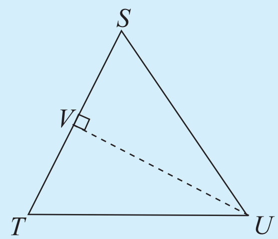

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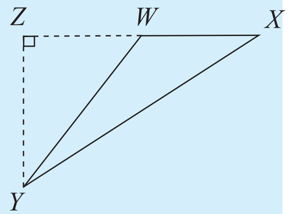

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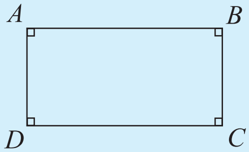

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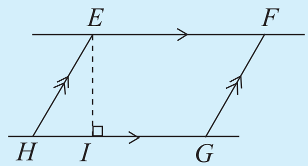


(vi)

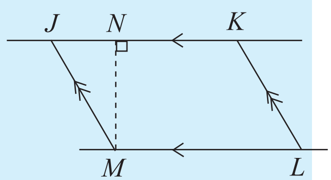


Figure	Base	Corresponding Altitude	Area (As a product of lengths)
(i) Triangle *ABD*			
(ii) Triangle *STU*			
(iii) Triangle *WXY*			
(iv) Rectangle *ABCD*			
(v) Parallelogram *EFGH*			
(v) Parallelogram *JKLM*			

8.1 Parallelograms and triangles on the same base and between the same pair of parallel lines

Let us first see what is meant by parallelograms and triangles on the same base and between the same pair of parallel lines. Consider the following figures.

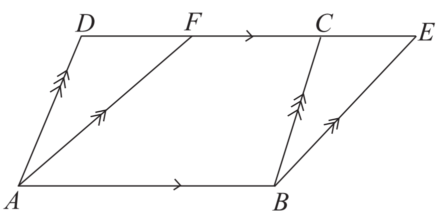


Figure (i)

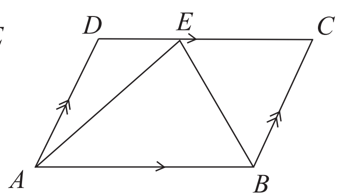


Figure (ii)

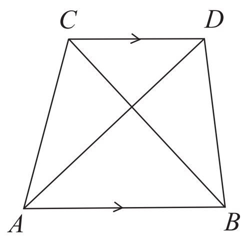


Figure (iii)

Both the parallelograms *ABCD* and *ABEF* in figure (i) lie between the pair of straight lines *AB* and *DE*. What is meant here by the word "between" is that a pair of opposite sides of each of the parallelograms lies on the straight lines *AB* and *DE*. Further, the side *AB* is common to both parallelograms. In such a situation, we say that the two parallelograms are on the same base and between the same pair of parallel lines. Here, the common side *AB* has been considered as the base. It is clear that corresponding to this common base, both the parallelograms have the same altitude. This is equal to the perpendicular distance between the two parallel lines *AB* and *DE*.

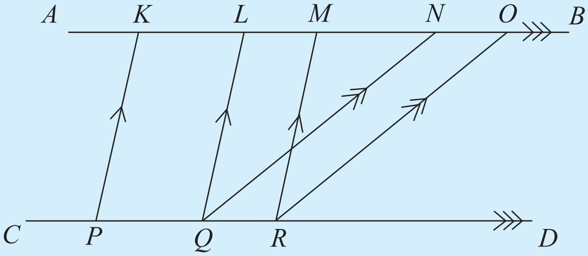
Figure (ii) depicts a parallelogram and a triangle which lie on the same base and between the same pair of parallel lines *AB* and *DC*. The parallelogram is *ABCD* and the triangle is *ABE*. The common base is *AB*. Observe that in this case, one side of the triangle lies on one of the parallel lines while the opposite vertex lies on the other line.

Figure (iii) depicts two triangles on the same base and between the same pair of parallel lines. The two triangles are *ABC* and *ABD*.

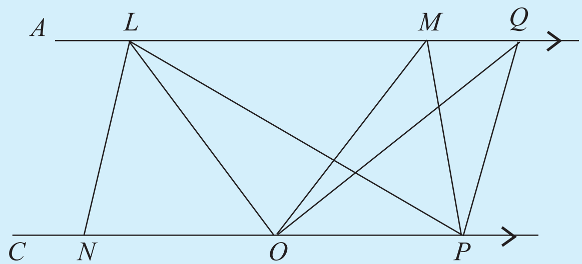
Exercise 8.1

1. Based on the information in the figure,

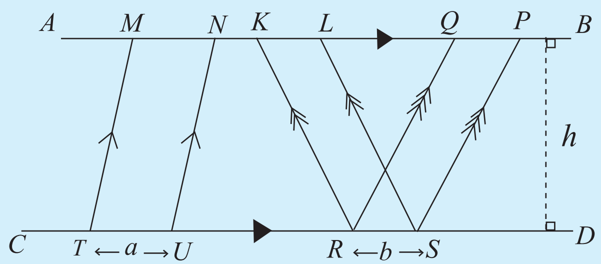
(i) name four parallelograms.
(ii) name the two parallelograms with the same base *QR* which lie between the pair of parallel lines *AB* and *CD*.



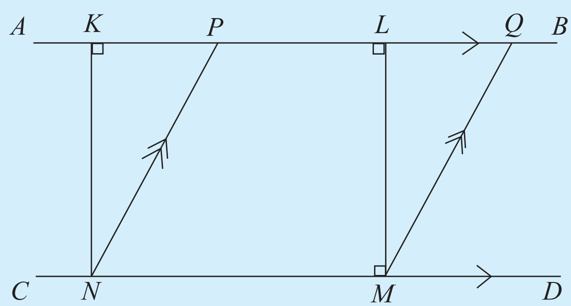
2. Write down all the triangles with the same base *OP* that lie between the pair of parallel straight lines *AQ* and *CP* in the given figure.



3. In the given figure, the perpendicular distance between the pair of parallel straight lines *AB* and *CD* is denoted by *h* and the base lengths of the parallelograms by *a* and *b*. Write down the areas of the parallelograms *PQRS*, *KLSR* and *MNUT* in terms of these symbols.



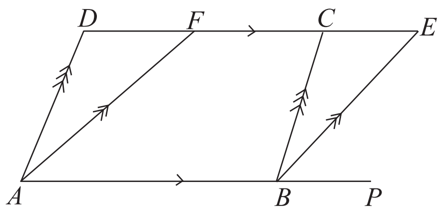
4. The rectangle *KLMN* and the parallelogram *PQMN* in the given figure lie between the pair of parallel straight lines *AB* and *CD*. $NM = 10$ cm and $LM = 8$ cm.



(i) Find the area of the rectangle *KLMN*.
(ii) Find the area of the parallelogram *PQMN*.
(iii) What is the relationship between the area of the rectangle *KLMN* and the parallelogram *PQMN* ?

8.2 The areas of parallelograms on the same base and between the same pair of parallel lines

Next we look at the relationship between the areas of parallelograms on the same base and between the same pair of parallel lines. Consider the given parallelograms.



Let us see whether the areas of the parallelograms *ABCD* and *ABEF* are equal.

Observe that,
area of parallelogram *ABCD* = area of trapezium *ABCF* + area of triangle *AFD*
area of parallelogram *ABEF* = area of trapezium *ABCF* + area of triangle *BEC*

Therefore it is clear that, if
the area of triangle *AFD* = the area of triangle *BEC*,
then the areas of the two parallelograms will be equal.

In fact, these two triangles are congruent. Therefore their areas are equal. The congruence of the two triangles under the conditions of SAS can be shown as follows.

In the two triangles *AFD* and *BEC*,

$AD = BC$ (opposite sides of a parallelogram)
$AF = BE$ (opposite sides of a parallelogram)

Also, since $D\hat{A}B = C\hat{B}P$ (corresponding angles) and $F\hat{A}B = E\hat{B}P$ (corresponding angles), by subtracting these equations we obtain

$D\hat{A}F = C\hat{B}E$.

Accordingly, the two triangles *AFD* and *BEC* are congruent under the conditions of SAS.

Therefore we obtain,
area of parallelogram *ABCD* = area of parallelogram *ABEF*.

We can write this as a theorem as follows.

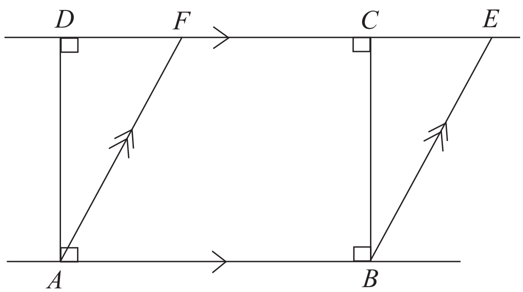
Theorem: Parallelograms on the same base and between the same pair of parallel lines are equal in area.

Now let us obtain an important result using this theorem. You have used the following formula when finding the area of a parallelogram in previous grades and in the above exercise.

Area of a parallelogram = Base $\times$ Perpendicular height

Have you ever thought about how this result was obtained? We can now use the above theorem to prove this result.

The figure depicts a rectangle *ABCD* (that is, a parallelogram) and a parallelogram *ABEF* on the same base and between the same pair of parallel lines. According to the above theorem, their areas are equal.



We know that,

area of the parallelogram *ABEF* = Area of the rectangle *ABCD*

$= AB \times AD$

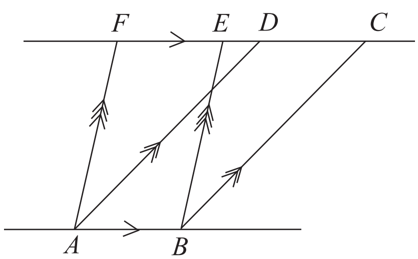
$= AB \times$ perpendicular distance between the two parallel lines

= base of the parallelogram $\times$ perpendicular height

Let us now consider how calculations are done using this theorem.

Example 1

The area of the parallelogram *ABEF* in the figure is 80 cm^2 while $AB = 8$ cm.



(i) Name the parallelograms in the figure that lie on the same base and between the same pair of parallel lines.
(ii) What is the area of the parallelogram *ABCD*?
(iii) Find the perpendicular distance between the parallel lines *AB* and *FC*.

Now let us answer these questions.

(i) *ABEF* and *ABCD*.
(ii) Since the parallelograms *ABEF* and *ABCD* lie on the same base *AB* and between the same pair of parallel lines *AB* and *FC*, their areas are equal. Therefore, the area of $ABCD = 80\text{ cm}^2$.
(iii) Let us take the perpendicular distance between the pair of parallel lines as h centimetres.

Then,

area of $ABEF = AB \times h.$

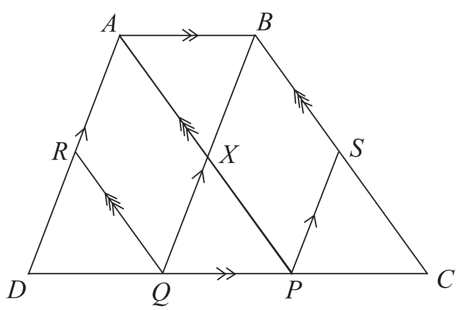
$80 = 8 \times h$

$\therefore\ h = 10$

$\therefore$ the perpendicular distance between the parallel lines is 10 cm.

Now, by considering an example, let us see how riders are proved using this theorem.

Example 2



According to the information in the above figure,
(i) show that *ABQD* and *ABCP* are parallelograms.
(ii) show that the parallelograms *ABQD* and *ABCP* are of the same area.
(iii) prove that $\Delta SPC \equiv \Delta RDQ$.
(iv) prove that, area of parallelogram *AXQR* = area of parallelogram *BXPS*.

(i) In the quadrilateral *ABQD*

AB//*DQ* (given)

AD//*BQ* (given)

Since a quadrilateral with pairs of opposite sides parallel, is a parallelogram, *ABQD* is a parallelogram. Similarly, since *AB* // *PC* and *AP* // *BC*, we obtain that *ABCP* is a parallelogram.

(ii) Since the parallelograms $ABQD$ and $ABCP$ lie on the same base AB and between the same pair of parallel lines AB and DC, by the above theorem, their areas are equal.

$\therefore$ area of parallelogram $ABQD$ = area of parallelogram $ABCP$.

(iii) In the triangles SPC and RDQ in the figure,

$$S\hat{P}C = R\hat{D}Q \quad \text{(since } SP \parallel AD\text{, corresponding angles)}$$

$$S\hat{C}P = R\hat{Q}D \quad \text{(since } SC \parallel RQ\text{, corresponding angles)}$$

Further, $AB = PC$ (opposite sides of the parallelogram $ABCP$)

$AB = DQ$ (opposite sides of the parallelogram $ABQD$)

Therefore, $PC = DQ$.

$\therefore \quad \Delta SPC \equiv \Delta RDQ$. (AAS)

(iv) Area of parallelogram $ABQD$ = area of parallelogram $ABCP$ (proved)

Area of ΔRDQ = area of ΔSPC (since $\Delta RDQ \equiv \Delta SPC$)

Therefore,

area of $ABQD$ – area of ΔRDQ = area of $ABCP$ – area of ΔSPC.

Then, according to the figure,

area of trapezium $ABQR$ = area of trapezium $ABSP$.

Therefore,

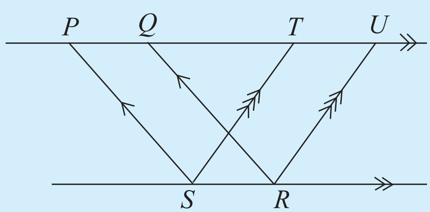
by subtracting the area of the triangle ABX from both sides, we get

area of trapezium $ABQR$ – area of Δ ABX = area of trapezium $ABSP$ – area of Δ ABX

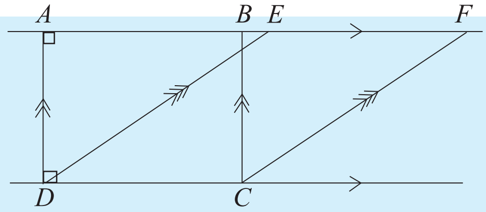
$\therefore$ area of parallelogram $AXQR$ = area of parallelogram $BXPS$.

Exercise 8.2

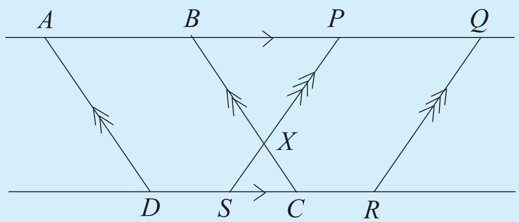
1. The figure shows two parallelograms that lie between the pair of parallel lines PU and SR. The area of the parallelogram $PQRS$ is 40 cm^2. With reasons, write down the area of the parallelogram $TURS$.



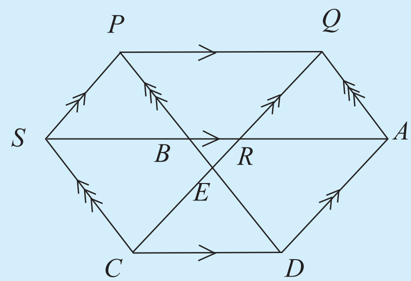
2. A rectangle $ABCD$ and a parallelogram $CDEF$ are given in the figure. If AD = 7 cm and CD = 9 cm, with reasons, write down the area of the parallelogram $CDEF$.



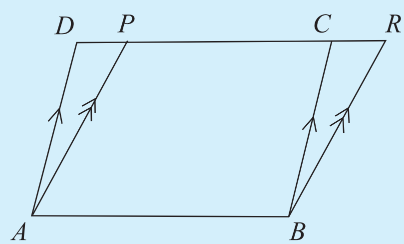
3. The figure shows two parallelograms $ABCD$ and $PQRS$ that lie between the pair of parallel lines AQ and DR. It is given that $DS = CR$.
   (i) Show that $DC = SR$.
   (ii) Prove that the area of the pentagon $ABXSD$ is equal to the area of the pentagon $PQRCX$.
   (iii) Prove that the area of the trapezium $APSD$ is equal to the area of the trapezium $BQRC$.



4. Based on the information in the figure,
   (i) name two parallelograms which are equal in area to the area of the parallelogram $PQRS$.
   (ii) name two parallelograms which are equal in area to the area of the parallelogram $ADCR$.
   (iii) prove that the area of the parallelogram $PECS$ is equal to the area of the parallelogram $QADE$.



5. Based on the information in the figure, prove that the area of triangle ADP is equal to the area of triangle BRC.



6. Construct the parallelogram $ABCD$ such that AB = 6 cm, $D\hat{A}B = 60^0$ and AD = 5 cm. Construct the rhombus $ABEF$ equal in area to the area of $ABCD$ and lying on the same side of AB as the parallelogram. State the theorem that you used for your construction.

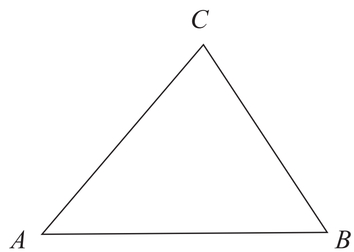
8.3 The areas of parallelograms and triangles on the same base and between the same pair of parallel lines

You have used the following formula in previous grades to find the area of a triangle.

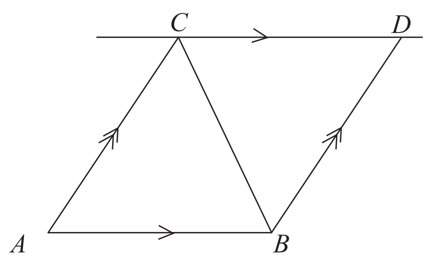
$$\text{Area of the triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

Now we will explain why this formula is valid.

Let us consider the following triangle *ABC*.



Now let us draw a line parallel to *AB* through the point *C*, as shown in the figure, and mark the point *D* on this line such that *ABDC* is a parallelogram. In other words let us mark the intersection point of the line drawn through *B* parallel to *AC* and the line drawn through *C* parallel to *AB*, as *D*.



The area of the triangle *ABC* is exactly half the area of the parallelogram *ABDC*. This is because the diagonals of a parallelogram divide the parallelogram into two congruent triangles. We learnt this in the lesson on parallelograms in Grade 10.

Therefore,

$$\text{area of triangle } ABC = \frac{1}{2} \text{ the area of parallelogram } ABDC$$

$$= \frac{1}{2} \times AB \times \text{perpendicular distance between } AB \text{ and } CD$$

$$= \frac{1}{2} \times AB \times \text{perpendicular height}$$

We have obtained the familiar formula for the area of a triangle.

Consider again the result that we observed here;

area of triangle $ABC = \frac{1}{2}$ the area of parallelogram $ABDC$.

In section 8.2 of this lesson, we learnt that the areas of parallelograms on the same base and between the same pair of parallel lines are equal. Therefore, in relation to the above figure, the area of any parallelogram that lies on the same base AB and between the same pair of parallel lines AB and CD is equal to the area of $ABDC$. Therefore,

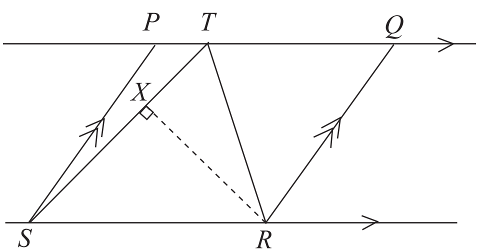
area of triangle $ABC = \frac{1}{2} \times$ (area of any parallelogram with base AB lying between the parallel lines AB and CD).

This result is given below as a theorem.

Theorem: If a triangle and a parallelogram lie on the same base and between the same pair of parallel lines, then the area of the triangle is exactly half the area of the parallelogram.

Let us now consider how calculations are performed using this theorem.

Example 1



The figure illustrates a parallelogram $PQRS$ and a triangle STR on the same base and between the same pair of parallel lines. The area of the parallelogram $PQRS$ is 60 cm^2.

(i) Find the area of the triangle STR. Give reasons for your answer.

(ii) If $ST = 6$ cm, find the length of the perpendicular RX from R to ST.

(i)The parallelogram $PQRS$ and the triangle STR lie on the same base and between the same pair of parallel lines. Therefore the area of triangle STR is half the area of parallelogram $PQRS$.

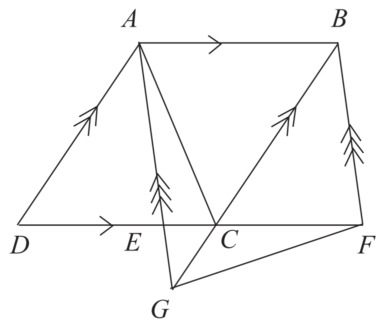
$\therefore$ area of $\Delta\, STR = 30$ cm^2

(ii) Area of $\Delta\, STR = \frac{1}{2} \times ST \times RX$

$\therefore\; 30 = \frac{1}{2} \times 6 \times RX$

$\therefore\; RX = \underline{\underline{10 \text{ cm}}}$

Example 2



E is a point on the side *DC* of the parallelogram *ABCD*. The straight line drawn through *B* parallel to *AE,* meets *DC* produced at *F.* *AE* produced and *BC* produced meet at *G.*

Prove that,

(i) *ABFE* is a parallelogram.
(ii) the areas of the parallelograms *ABCD* and *ABFE* are equal.
(iii) the area of $\Delta\, ACD$ = the area of $\Delta\, BFG$.

(i) In the quadrilateral *ABFE,*
$AE \,//\, BF$ (data)
$AB \,//\, EF$ (data)
$\therefore$ *ABFE* is a parallelogram (since pairs of opposite sides are parallel)

(ii) The parallelograms *ABCD* and *ABFE* lie on the same base *AB* and between the same pair of parallel lines *AB* and *DF.*
$\therefore$ according to the theorem,
area of parallelogram *ABCD* = area of parallelogram *ABFE*

(iii) The parallelogram *ABCD* and the triangle *ACD* lie on the same base *DC* and between the same pair of parallel lines *AB* and *DC.*
$\therefore$ according to the theorem,
$\frac{1}{2}$ the area of parallelogram *ABCD* = area of triangle *ACD.*

Similarly,

the parallelogram $ABFE$ and the triangle BFG lie on the same base BF and between the same pair of parallel lines BF and AG.
Therefore,

$$\frac{1}{2} \text{ the area of parallelogram } ABFE = \text{ area of triangle } BFG$$

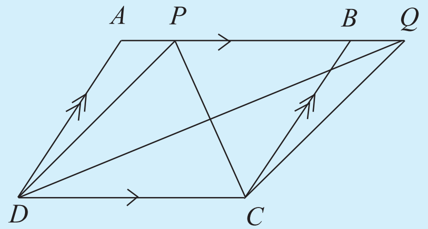
Since, area of parallelogram $ABCD$ = area of parallelogram $ABFE$,

$$\frac{1}{2} \text{ the area of parallelogram } ABCD = \frac{1}{2} \text{ the area of parallelogram } ABFE$$

$$\therefore \quad \text{area of } \Delta\, ACD = \text{area of } \Delta\, BFG$$

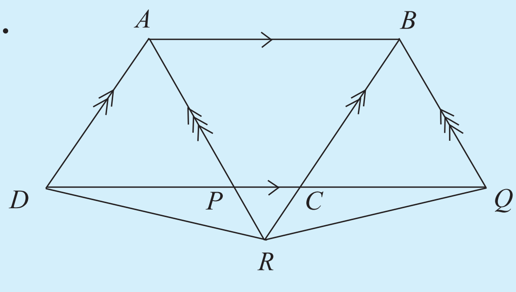
Exercise 8.3

1. The area of the parallelogram $ABCD$ in the figure is 50 cm^2.



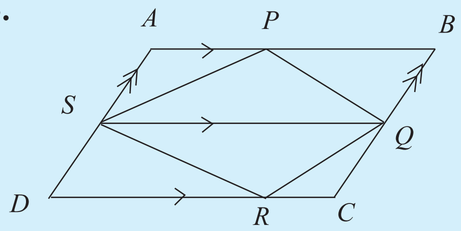
(i) What is the area of triangle PDC?
(ii) What is the area of triangle DCQ?

2.



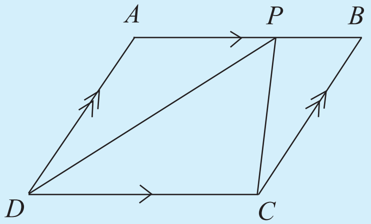
The point P lies on the side DC of the parallelogram $ABCD$. The straight line drawn through B parallel to AP meets DC produced at Q. Further, AP produced and BC produced meet at R. Prove that the area of triangle ADR is equal to the area of triangle BQR.

3.



In the figure, SQ has been drawn parallel to the side AB of the parallelogram $ABCD$, such that it meets the side AD at S and the side BC at Q. Prove that the area of the quadrilateral $PQRS$ is exactly half the area of the parallelogram $ABCD$.

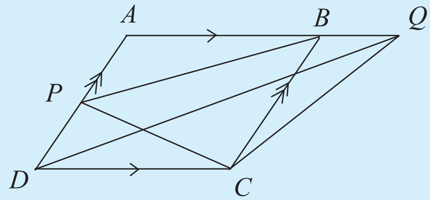
4.



P is any point on the side AB of the parallelogram $ABCD$. Prove that,

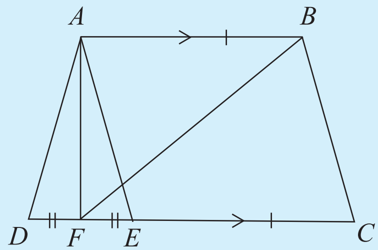
area of $\Delta\, APD$ + area of $\Delta\, BPC$ = area of $\Delta\, DPC$

5.



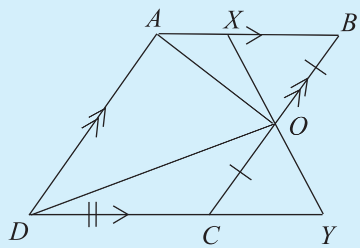
In the figure, the point P lies on the side AD of the parallelogram $ABCD$, and the point Q lies on AB produced. Prove that, area of $\Delta\, CPB$ = area of $\Delta\, CQD$.

6.



$AB//DC$ and $DC > AB$ in the trapezium $ABCD$.
The point E lies on the side CD such that $AB = CE$. The point F lies on the side DE such that the area of the triangle AFE is equal to the area of the triangle ADF. Prove that the area of the trapezium $ABFD$ is exactly half the area of the trapezium $ABCD$.

7.

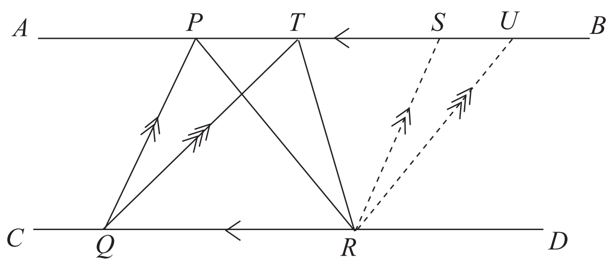


O is the midpoint of the side BC of the parallelogram $ABCD$ and X is an arbitrary point on AB. Also, XO produced and DC produced meet at Y.

Prove that,
(i) the area of $\Delta\, BOX$ = the area of $\Delta\, COY$
(ii) the area of trapezium $AXYD$ = the area of parallelogram $ABCD$.
(iii) the area of trapezium $AXYD$ is twice the area of triangle ADO.

8.4 Triangles on the same base and between the same pair of parallel lines

Now let us consider the two triangles *PQR* and *TQR* that lie on the same base *QR* and between the same pair of parallel lines *AB* and *CD*.



As discussed in section 8.3 the parallelogram related to the triangle *PQR* is *PQRS*, and the parallelogram related to the triangle *TQR* is *TQRU*.

Since the parallelogram related to the triangle *PQR* is *PQRS,*

area of triangle $PQR = \frac{1}{2}$ the area of parallelogram *PQRS*.

Since the parallelogram related to the triangle *TQR* is *TQRU,*

area of triangle $TQR = \frac{1}{2}$ the area of parallelogram *TQRU*.

However, since the parallelograms *PQRS* and *TQRU* lie on the same base *QR* and between the same pair of parallel lines, by the theorem,
area of parallelogram *PQRS* = area of parallelogram *TQRU*.

$\therefore\ \frac{1}{2}$ the area of parallelogram $PQRS = \frac{1}{2}$ the area of parallelogram *TQRU*

That is, area of triangle *PQR* = area of triangle *TQR*.

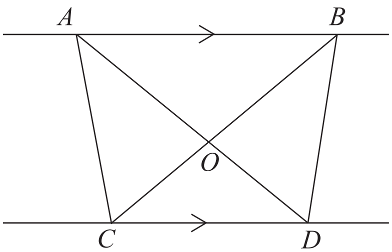
As stated previously, the areas of the two triangles *PQR* and *TQR* which lie on the same base *QR* and between the same pair of parallel lines *AB* and *CD* are equal in area.

Triangles which satisfy the above given conditions in this manner are equal in area. This is stated as a theorem as follows.

Theorem: Triangles on the same base and between the same pair of parallel lines are equal in area.

Let us now consider through the following examples how problems are solved using this theorem.

Example 1



In the given figure, $AB//CD$.

(i) Name a triangle that has the same area as triangle ACD. Write down the theorem that your answer is based on.

(ii) If the area of triangle ABC is 30 cm^2, find the area of triangle ABD.

(iii) Prove that the area of triangle AOC is equal to the area of triangle BOD.

(i) Triangle BCD.

Triangles on the same base and between the same pair of parallel lines are equal in area.

(ii) Area of triangle ABD = 30 cm^2.

(iii) Area of $\Delta\, ACD$ = Area of $\Delta\, BCD$. (On the same base CD and $AB//CD$.)

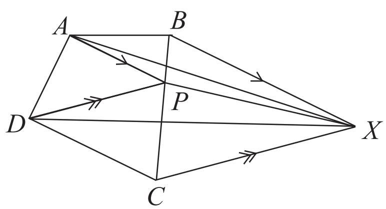
According to the figure, the triangle COD is common to both these triangles. When this portion is removed,

area of $\Delta\, ACD$ – area of $\Delta\, COD$ = area of $\Delta\, BCD$ – area of $\Delta\, COD$

$\therefore$ area of $\Delta\, AOC$ = area of $\Delta\, BOD$

Example 2

The point P lies on the side BC of the quadrilateral $ABCD$. The line drawn though B parallel to AP meets the line drawn through C parallel to DP at X. Prove that the area of triangle ADX is equal to the area of quadrilateral $ABCD$.



Proof: Since the triangles APB and APX lie on the same base AP and between the same pair of parallel lines AP and BX, according to the theorem,

$$\Delta APB = \Delta APX \text{———} (1)$$

Similarly, since $DP//CX$,

$$\Delta DPC = \Delta DPX \text{———} (2)$$

From (1) + (2), $\Delta ABP + \Delta DPC = \Delta APX + \Delta DPX.$

Let us add the area of triangle ADP to both sides.

Then, $\Delta ABP + \Delta DPC + \Delta ADP = \Delta APX + \Delta DPX + \Delta ADP$

$\therefore$ area of quadrilateral $ABCD$ = area of the triangle ADX

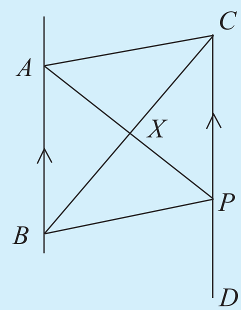
Exercise 8.4

1. The area of triangle ABP which lies between the parallel lines AB and CD in the figure is 25 cm^2.

(i) What is the area of triangle ABC ?

(ii) If the area of triangle ABX is 10 cm^2, what is the area of triangle ACX ?

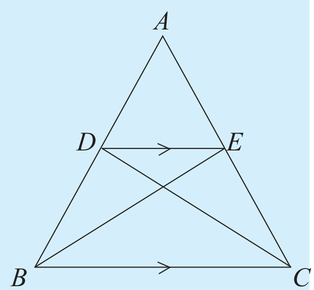
(iii) Explain with reasons what the relationship between the areas of the triangles ACX and BPX is.



2. In the figure, DE is drawn parallel to the side BC of the triangle ABC, such that it touches the side AB at D and the side AC at E.

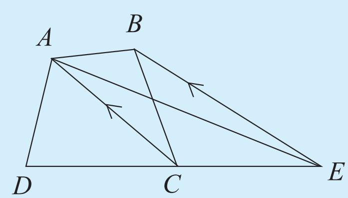
(i) Name a triangle which is equal in area to the triangle BED.

(ii) Prove that the triangles ABE and ADC are equal in area.



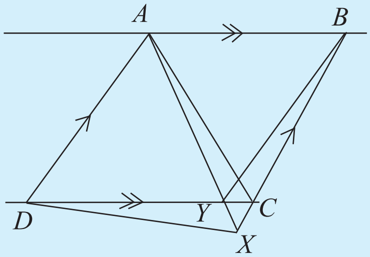
3. The straight line drawn through the point B parallel to the diagonal AC of the quadrilateral $ABCD$, meets the side DC produced at E.

(i) Name a triangle which is equal in area to the triangle ABC. Give reasons for your answer.



(ii) Prove that the area of the quadrilateral $ABCD$ is equal to the area of the triangle ADE.

4. $ABCD$ is a parallelogram. A straight line drawn from A intersects the side DC at Y and BC produced at X.
Prove that,
(i) the triangles DYX and AYC are equal in area.
(ii) the triangles BCY and DYX are equal in area.



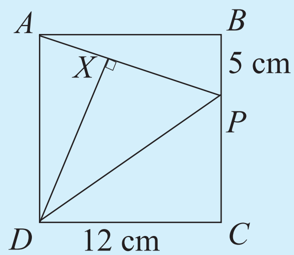
5. The point Y lies on the side BC of the parallelogram $ABCD$. The side AB produced and DY produced meet at X. Prove that the area of triangle AYX is equal to the area of triangle BCX.

6. BC is a fixed straight line segment of length 8 cm. With the aid of a sketch, describe the locus of the point A such that the area of triangle ABC is 40 cm^2.

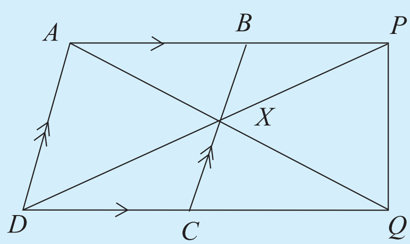
7. Construct the triangle ABC such that $AB = 8$ cm, $AC = 7$ cm and $BC = 4$ cm. Construct the triangle PAB which is equal in area to the triangle ABC, with P lying on the same side of AB as C, and $PA = PB$.

Miscellaneous Exercise

1. The length of a side of the square $ABCD$ in the figure is 12 cm. The point P lies on the side BC such that $BP = 5$ cm. Find the length of DX.



2. X is a point on the side BC of the parallelogram $ABCD$. The side AB produced and DX produced meet at P and the side DC produced and AX produced meet at Q. Prove that the area of the triangle PXQ is exactly half of the area of the parallelogram $ABCD$.



3. The diagonals of the parallelogram *PQRS* intersect at *O*. The point *A* lies on the side *SR*. Find the ratio of the area of the triangle *POQ* to that of the triangle *PAQ*.

4. *ABCD* and *ABEF* are two parallelograms, unequal in area, drawn on either side of *AB*.
Prove that,
(i) *DCEF* is a parallelogram.
(ii) the area of the parallelogram *DCEF* is equal to the sum of the areas of the parallelograms *ABCD* and *ABEF*.

5. *ABCD* is a parallelogram. *EF* has been drawn parallel to *BD* such that it intersects the side *AB* at *E* and the side *AD* at *F*.
Prove that,
(i) the triangles *BEC* and *DFC* are equal in area.
(ii) the triangles *AEC* and *AFC* are equal in area.