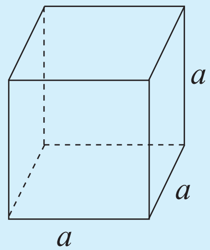
5 Volume of solids

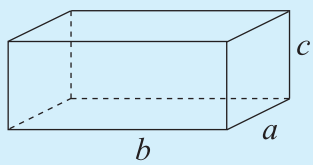
By studying this lesson you will be able to
- compute the volume of a square based right pyramid, right circular cone and a sphere.

Review Exercise

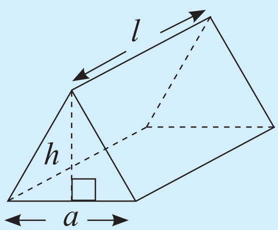
1. Shown below are figures of some solid objects that you have studied before. Complete the given table by recalling how their volume was computed.



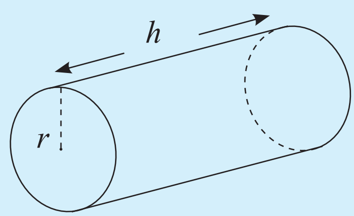
Cube



Cuboid



Triangular prism



Cylinder

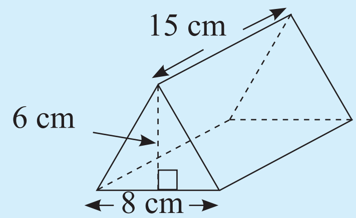
Object	Cross-sectional area	Volume
Cube		
Cuboid		
Triangular Prism		
Cylinder		

2. Find the volume of a cube of side length 10 cm.

3. Find the volume of a cuboid of length 15 cm, width 10 cm and height 8 cm.

4. Find the volume of a cylinder of radius 7 cm and height 20 cm.

5. Find the volume of the prism shown in the figure.

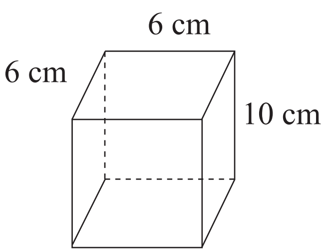


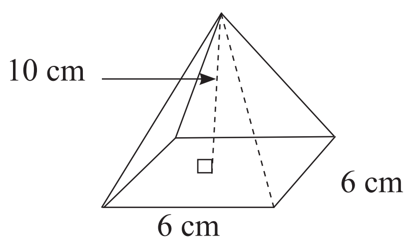
5.1 Volume of a square based right pyramid

Let us do the following activity to construct a formula to find the volume of a square based right pyramid.

Activity

Use a piece of cardboard to construct the hollow cuboid and the hollow pyramid, shown in the following figure. The cuboid has a square base with 6 cm sides and is of height 10 cm. It does not have a top. The right pyramid has a square base, again with 6 cm sides, and the height of the pyramid is also 10 cm. Do not include a base, so that you can fill it with sand.





Completely fill the pyramid with sand and empty it into the cuboid. Find how many times you need to do this to fill the cuboid.

You would have observed in the above activity, that filling the pyramid completely with sand and emptying it into the cuboid thrice, will completely fill the cuboid without any overflow.

Let us consider a square based cuboid with side length a and height h, and a square based right pyramid of side length a and height h.

According to the activity,

Volume of the Pyramid $\times$ 3 = Volume of the Cuboid

$\therefore$ Volume of the Pyramid $= \frac{1}{3} \times$ Volume of the Cuboid

$= \frac{1}{3} \times$ Area of the Base $\times$ Perpendicular Height

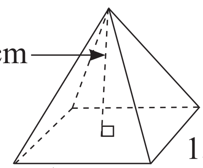
$= \frac{1}{3} \times (a \times a) \times h$

$= \frac{1}{3} a^2h$

Volume of the Pyramid $= \frac{1}{3}a^2h$

Example 1

Find the volume of a square based right pyramid, of height 10 cm and base length 15 cm.



Volume of the Pyramid $= \frac{1}{3} a^2h$

$= \frac{1}{3} \times 15 \times 15 \times 10$

$= 750$

$\therefore$ Volume of the Pyramid is 750 cm^3.

Example 2

The volume of a pyramid with a square base is 400 cm^3. Find the length of a side of the base, if its height is 12 cm.

Let us take the length of a side of the base to be "a" cm.

Volume of the Pyramid $= \frac{1}{3} a^2h$

$\therefore \frac{1}{3} a^2h = 400$

$\therefore \frac{1}{3} a^2 \times 12 = 400$

$\therefore 4a^2 = 400$

$\therefore a^2 = 100$

$= 10^2$

$\therefore a = 10$

$\therefore$ Length of a side of the base is 10 cm.

Exercise 5.1

1. The height of a pyramid with a square base is 9 cm. The length of a side of the base is 5 cm. Find the volume of the pyramid.

2. A square pyramid of height 10 cm has a base of area 36 cm^2. Find the volume of the pyramid.

3. The volume of a square pyramid of height 12 cm is 256 cm^3 . Find the length of a side of the base.

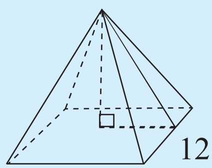
4. The height of a square pyramid is 5 cm. Its volume is 60 cm^3 . Find the area of its base.

5. The volume of a square pyramid is 216 cm^3 . The length of a side of its base is 9 cm. Find the height of the pyramid.

6. The area of the base of a square pyramid is 16 cm^2 and its volume is 80 cm^3. Find the height of the pyramid.

7. The side length of the base of a square pyramid is 12 cm and the slant height is 10 cm. Find

(i) the height, and
(ii) the volume
of the pyramid.



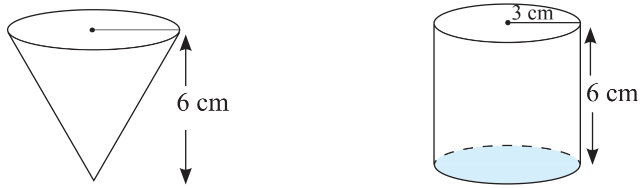
8. The side length of the base of a square pyramid is 10 cm and the slant height is 13 cm. Find
(i) the height, and
(ii) the volume
of the pyramid.

5.2 Volume of a right circular cone

Let us consider constructing a formula for the volume of a right circular cone. Do the following activity using a right circular cylinder and a right circular cylinder.

Activity

As shown in the figure, using a cardboard,construct a cone without a base, and a cylinder with a base but without a lid, of equal radii and equal height.



Fill the cone completely with sand and empty it into the cylinder. Find how many times you need to do this in order to completely fill the cylinder.

You will be able to observe that, pouring three times from the cone will completely fill the cylinder without any overflow. According to this observation,

$$\text{Volume of the cone} \times 3 = \text{Volume of the cylinder}$$

$$\text{Volume of the cone} = \frac{1}{3} \times \text{Volume of the cylinder}$$

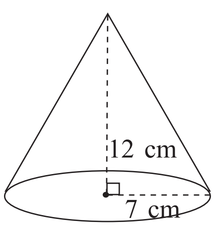
You have learned in a previous lesson that the volume of a cylinder, of radius r and height h, is given by $\pi r^2 h$. Therefore, the volume of a cone of radius r and height h is given by $\frac{1}{3}\pi r^2 h$.

$$\boxed{\text{Volume of the cone} = \frac{1}{3}\pi r^2 h}$$

The value of π is taken as $\frac{22}{7}$ in this lesson.

Example 1

Find the volume of a cone of radius 7 cm and height 12 cm.



$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12$$
$$= 616$$

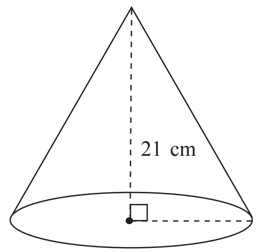
$\therefore$ volume of the cone is 616 cm^3.

Example 2

The circumference of the base of a cone is 44 cm. Its perpendicular height is 21 cm. Find the volume of the cone.

Circumference of the base = 44 cm
Let us take the radius as r centimetres

$$\therefore \quad 2\pi r = 44$$
$$2 \times \frac{22}{7} \times r = 44$$
$$\therefore \quad r = \frac{44 \times 7}{2 \times 22}$$
$$= 7$$



$\therefore$ radius of the cone is 7 cm.

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 21$$
$$= 1078$$

$\therefore$ volume of the cone is 1078 cm^3.

Example 3

Find,

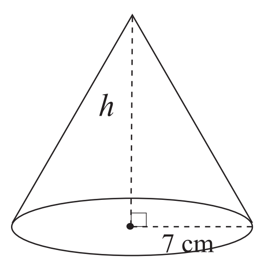
(i) the height

(ii) the volume

of a cone of radius 7 cm and slant height 25 cm.

Let us indicate the height of the cone by h centimetres. Let us apply Pythagoras' Theorem to the indicated triangle of the cone.

(i)
$$h^2 + 7^2 = 25^2$$
$$h^2 + 49 = 625$$
$$h^2 = 625 - 49$$
$$h = \sqrt{576}$$
$$h = 24$$



$\therefore$ height of the cone is 24 cm.

(ii) volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$

$= 1232$

$\therefore$ volume of the cone is 1232 cm^3.

Example 4

Find the perpendicular height of a cone of radius 3.5 cm and volume 154 cm^3.

Let us indicate the perpendicular height of the cone by h centimetres.

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$\therefore 154 = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h \qquad \left(\text{because } 3.5 = \frac{7}{2}\right)$$

$$\therefore h = \frac{154 \times 3 \times 7 \times 2 \times 2}{22 \times 7 \times 7}$$

$$= 12$$

$\therefore$ perpendicular height of the cone is 12 cm.

Exercise 5.2

1. Find the volume of a cone of radius 7 cm and height 12 cm.

2. Find the volume of a cone of diameter 21 cm and height 25 cm.

3. Find the volume of a cone of slant height 13 cm and base radius 5 cm.

4. Find the volume of a cone of diameter 12 cm and slant height 10 cm.

5. If the height of a cone, of volume 616 cm^3 is 12 cm, find the radius of the cone.

6. The volume of a cone is 6468 cm^3 and its height is 14 cm. Find the diameter of the cone.

7. The circumference of the base of a right cone is 44 cm and its slant height is 25 cm. Compute the following.

(i) Radius of the base

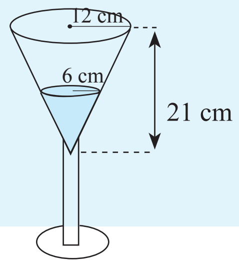
(ii) Height

(iii) Volume

8. The circumference of the base of a conical shaped container is 88 cm and its perpendicular height is 12 cm. Find the volume of the container.

9. How many cones of radius 7 cm and height 15 cm can be made by melting a solid metal cylinder of base radius 14 cm and height 30 cm?

10. A container is of the form of an inverted right circular cone. The radius of the cone is 12 cm and its height is 21 cm. If the container is filled with water to half its height, how much more water is required to fill it completely?

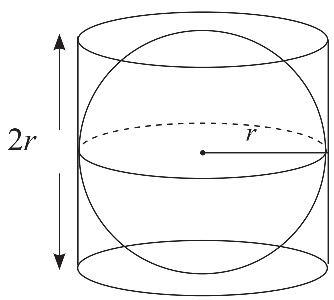


5.3 Volume of a sphere

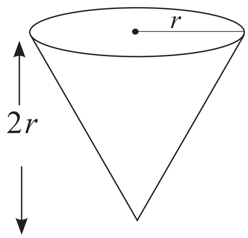
Recall the circumscribing cylinder of a sphere, that we leaned about in the lesson on the surface area of a sphere. Archimedes, the Greek mathematician who explained the surface area of a sphere in terms of the circumscribing cylinder, also explained the volume of a sphere in terms of the volume of the circumscribing cylinder. Let us do the following activity to understand his finding.

Activity

Find a small sphere of radius 3 cm. Using a piece of cardboard, create a cylinder with the same radius as that of the sphere and height equal to the diameter of the sphere. Do not close the two ends of the cylinder. Now carefully insert the sphere into the cylinder.



Take the cylinder, with the sphere inside it. Fill the top part of the cylinder, that is not ocupied by the sphere with sand. Cover top with a piece of cardboad and flip it over and fill the remaining part that is not occupied by the sphere also with sand. Put the total amount of sand that you used to fill the remaining volume of the cylinder after inserting the sphere into the cone that you made earlier. Note that the volume of sand fills the cone completely.



Let us formulate what you have observed in this activity.

Volume of the Circumscribing Cylinder = Volume of the Sphere + Volume of the Cone

Therefore, we can find the volume of the sphere by subtracting the volume of the cone from the volume of the circumscribing cylinder.

Volume of the Sphere = Volume of the Circumscribing Cylinder – Volume of the Cone

$$= \pi r^2 h - \frac{1}{3} \times \pi r^2 h$$

$$= \frac{2}{3} \pi r^2 h$$

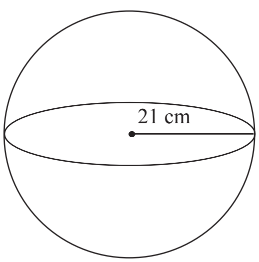
$$= \frac{2}{3} \pi r^2 \times 2r \quad (\text{Because } h = 2r)$$

$$= \frac{4}{3} \pi r^3$$

$$\boxed{\text{Volume of the Sphere} = \frac{4}{3} \pi r^3}$$

Example 1

Find the volume of a sphere of radius 21 cm.



Volume of the Sphere $= \frac{4}{3} \pi r^3$

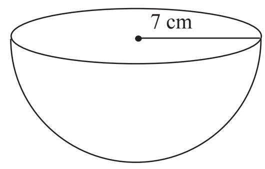
$$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 38\,808$$

$\therefore$ Volume of the sphere is 38 808 cm^3.

Example 2

Find the volume of a solid hemisphere of radius 7 cm.



$$\text{Volume of the hemisphere} = \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$= \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= 718.67$$

$\therefore$ Volume of the hemisphere is approximately 718.67 cm^3.

Example 3

Find the radius of a small spherical marble, of volume $113\frac{1}{7}$ cm^3.
Let us take the radius as r centimetres.

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$\therefore \quad \frac{4}{3}\pi r^3 = 113\frac{1}{7}$$

$$\therefore \quad r^3 = \frac{792}{7} \times \frac{3}{4} \times \frac{7}{22}$$

$$= 27$$

$$= 3^3$$

$$\therefore \quad r = 3$$

$\therefore$ Radius of the sphere is 3 cm.

Exercise 5.3

1. Find the volume of a sphere of radius 7 centimeters.

2. Show that the volume of a sphere of diameter 9 centimeters is $381\frac{6}{7}$ cm^3.

3. Find the volume of a spherical celestial body of radius 2.1 kilometers.

4. Find the volume of a hemisphere of radius 10.5 centimeters.

5. The volume of a sphere is $11492\frac{2}{3}$ cubic centimeters. Find the radius of the sphere.

6. Find the radius of the metal ball that is made by using all the metal obtained by melting 8 metal balls, each of radius 7 cm.

7. Show that 32 metal balls of radius 3 cm can be made by melting a solid metal hemisphere of radius 12 cm.

Summary

- The volume V of a square based right pyramid, of base length "a" and height "h" is

  $$V = \frac{1}{3}a^2h.$$

- The volume V, of a right circular cone of radius r and height h is

  $$V = \frac{1}{3}\pi r^2h.$$

- The volume V of a sphere of radius r is

  $$V = \frac{4}{3}\pi r^3.$$

Miscellaneous Exercise

1. Solid metal spheres of radius 3 cm were made by melting a metal block. The metal block had a square crossection of 12 cm sides. The length of the metal block was 22 cm. How many metal spheres were made?

2. A solid metal sphere of radius 3.5 cm was melted and casted into a circular cone of the same radius. Find the height of the cone, assuming there was no waste of the metal in the molding process.

3. A cone of slant height r and apex O was constructed using a metal lamina in the shape of the sector shown in the figure. The centre and the radius of the sector are O and r respectively. n pieces of ice in the shape of spheres of radius a are placed in this cone. (the cone is held inverted) If the cone is filled completely with water when all the ice melts show that $125na^3 = 9r^3$.