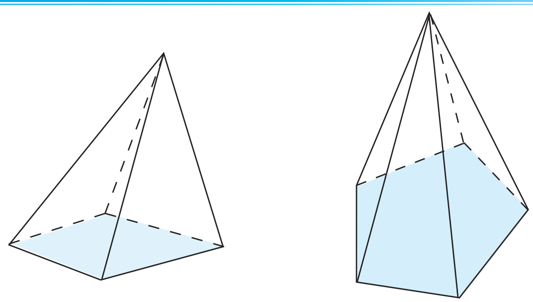
4 Surface area of solids

By studying this lesson you will be able to,

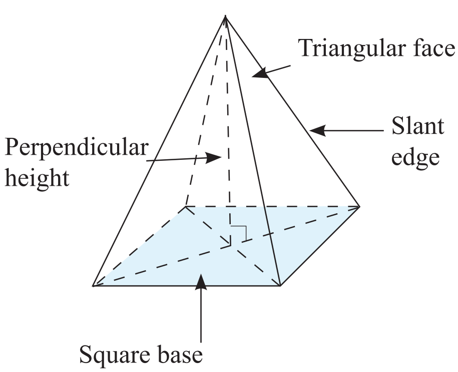
- find the surface area of a right pyramid with a square base,
- find the surface area of a right circular cone, and
- find the surface area of a sphere.

Pyramid



Carefully observe the solid objects in the above figure. Note that their faces are polygons. Of these faces, the horizontal face at the bottom is called the base. All the faces, except the base are of triangular shape. The common point of these triangular faces is called the "apex". A solid object with these properties is called a "pyramid". Note that the bases of the Pyramids shown above are respectively, the shape of a quadrilateral, a pentagon and a hexagon.

Right pyramid with a square base



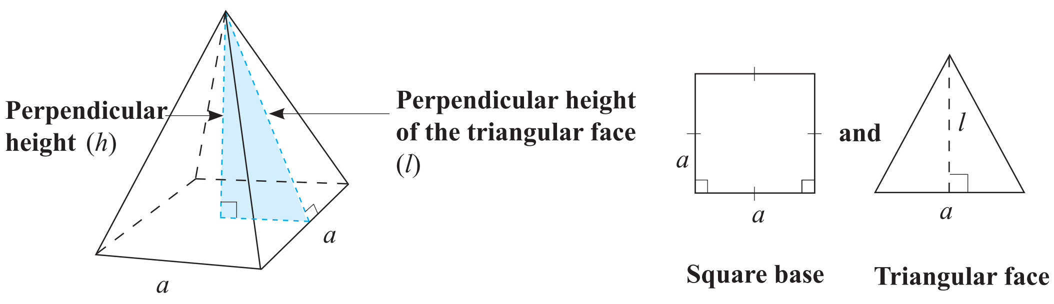
The base of the pyramid in the figure is a square. All the remaining faces are triangular in shape. If the line segment connecting the apex and the midpoint of the square base (that is the intersection point of the two diagonals) is perpendicular to the base, then such a pyramid is called a "square based right pyramid". The length of the line segment connecting the apex and the midpoint of the base is called the perpendicular height (or simply the height) of the pyramid. The edges of the triangular faces which are not common to the base are called slant edges. In this lesson, we will only consider finding the surface area of square based right pyramids.

Note: A tetrahedron can also be considered as a pyramid. All the faces of a tetrahedron are triangular in shape. Any one of the faces can be taken as the base. The concept of "right pyramid" can be defined even when the base is not a square. For example, we can define a right pyramid when the base of a pyramid is a regular polygon, as follows. First note that all the axes of symmetry of a regular polygon pass through a common point, which is called the centroid of the regular polygon. A pyramid, having a base which is a regular polygon, is called a right pyramid, if the line segment connecting the apex and the centroid of the base is perpendicular to the base.
If you study mathematics further, you will learn how to define the centroid, even when the base is not a regular polygon.

An important property of a square based right pyramid is that all its triangular faces are congruent to each other. Therefore, all the triangular faces have the same area. Moreover, note that each triangular face is an isosceles triangle, with one side a side of the square base and the other two sides equal in length.

4.1 Surface area of a square based right pyramid

To find the total surface area of a square based right pyramid we need to add the areas of the base and the four triangular faces.
Suppose the length of a side of the square base is "a" and the perpendicular height of a triangular face is "l".



(There are 4 such faces)

Now, we can find the total surface area as follows.

$$\left.\begin{array}{c}\text{Total surface area of}\\\text{the square based right}\\\text{pyramid}\end{array}\right\} = \left\{\begin{array}{c}\text{Area of the}\\\text{square base}\end{array}\right\} + 4 \times \left\{\begin{array}{c}\text{Area of a triangular}\\\text{face}\end{array}\right\}$$

$$= a \times a + 4 \times \frac{1}{2} \times a \times l$$

$$= a^2 + 2al$$

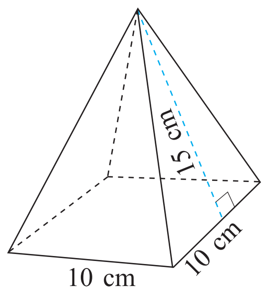
If the total surface area is A,

$$\boxed{A = a^2 + 2al}$$

Let us now consider some solved examples on the surface area of a square based right pyramid.

Example 1

The base length of a square based right pyramid is 10 cm and the perpendicular height of a triangular face is 15 cm. Find the total surface area of the pyramid.



Area of the base $= 10 \times 10$
$= 100$

Area of a triangular face $= \frac{1}{2} \times 10 \times 15$
$= 75$

Area of all four triangular faces $= 75 \times 4$
$= 300$

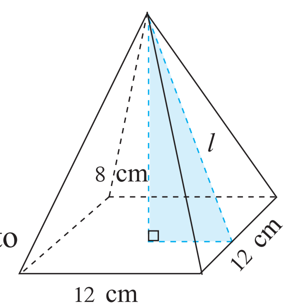
Total surface area of the pyramid $= 100 + 300$
$= 400$

$\therefore$ Total surface area is 400 cm^2.

Example 2

Shown in the figure is a square based right pyramid of perpendicular height 8 cm and base length 12 cm. Find

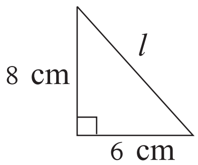
(i) the perpendicular height of a triangular face,
(ii) the area of a triangular face, and
(iii) the total surface area of the pyramid.

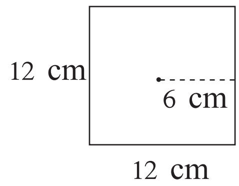


Let us take the perpendicular height of a triangular face to be l cm.
Consider the shaded triangle in the above figure.

By applying Pythagoras' theorem to this triangle,

(i) $l^2 = 8^2 + 6^2$
$= 64 + 36$
$= 100$
$\therefore\ l = \sqrt{100}$
$= 10$





$\therefore$ Perpendicular height of a triangular face is 10 cm.

(ii) Area of a triangular face $= \frac{1}{2} \times 12 \times 10$

$= 60$

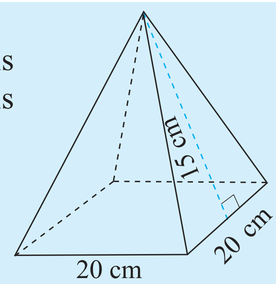
$\therefore$ Area of a triangular face is 60 cm^2.

(iii) Total surface area of the pyramid $= 12 \times 12 + 4 \times 60$
$= 144 + 240$
$= 384$

$\therefore$ Total surface area is 384 cm^2.

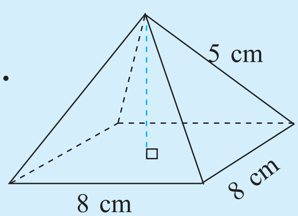
Exercise 4.1

1. The base length of a square based right pyramid is 20 cm and the perpendicular height of a triangular face is 15 cm. Find the total surface area of the pyramid.



2. In a square based right pyramid, the length of a side of the square base is 8 cm and the perpendicular height of a triangular face is 20 cm. What is the surface area of the pyramid?

3.

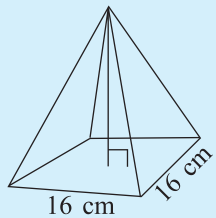


The slant height of a square based right pyramid is 5 cm, and the length of a side of the base is 8 cm. Find, the total surface area of the pyramid.

4. If the length of a side of the square base of a right pyramid is 20 cm and the perpendicular height is 12 cm, find the total surface area of the pyramid.

5. The base length of a square based right pyramid is 16 cm and the perpendicular height is 6 cm. Find the

(i) perpendicular height of a triangular face.
(ii) the total surface area of the pyramid.



6. Find the total surface area of a square based right pyramid of slant height is 13 cm, and the side length of the base equal to 20 cm.

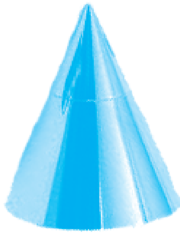
7. The surface area of a square based right pyramid is 2400 cm^2. If the length of a side of the base is 30cm, find
(i) the perpendicular distance from the apex to a side of the base, and
(ii) the height of the pyramid.

8. The area of the fabric that is used to make a tent in the shape of a square based right pyramid, is 80 m^2. Find the height of the tent, if the fabric is not used for the base of the tent and the length of a side of the base is 8cm.

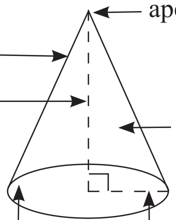
9. The height of a tent in the shape of a square based right pyramid is 4m and the perpendicular height of a triangular face is 5 m. If both the roof and the base of the tent is to be made from fabric, how much material is required?

10. It is required to construct a tent in the shape of a square based pyramid of base length 16 m and height 6 m. Find the fabric needed to construct the tent, also covering the base.

Cone



Shown above are some conical (cone shaped) objects. A cone has a circular plane surface and a curved surface. The circular plane surface is called the base of the cone. The point through which all the straight lines drawn on the curved surface pass through is called the "apex" of the cone.



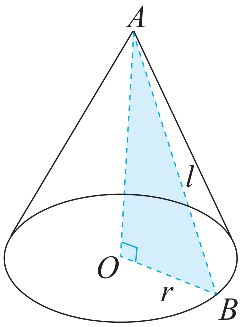
A cone is called a right circular cone if the line segment connecting the apex and the centre of the circular base is perpendicular to the circular base of a right circle cone. The radius of the base circle is called the radius of the cone. The length of the line segment connecting the centre of the base and the apex is the perpendicular height of a right circular cone. Moreover, any line segment connecting the apex and a point on the perimeter of the base circle is called a generator of the cone. The length of a generator is called the "slant height " of the cone.

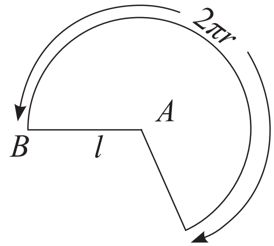
It is customary to use "r" for the radius, "h" for the height and "l" for the slant height of a cone.

4.2 Surface area of a right circular cone

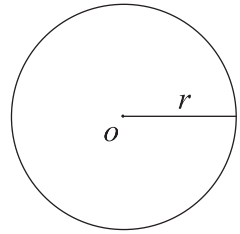
To explain a method to find the surface area of a right circular cone, consider a hollow right circular cone made from a thin sheet. Observe that the base of such a cone is a circular plane surface. Cutting open the curved surface along a generator, gives a lamina, the shape of a circular sector.

Given the radius and the slant height of a right circular cone, one can find surface area of the cone, by finding the area of the circular base and the area of the curved surface. We can use the formula πr^2 to find the area of the circular base. We can find the area of the circular sector as follows.





Curved surface



Circular base

The surface area of the curved surface is equal to the area of the sector that is obtained by cutting it open. Because the arc length of the sector is the circumference of the base circle, the arc length of the sector is equal to $2\pi r$. Also note that the radius of this sector is the slant height "l". Now, as you have learned in the lesson on the perimeter of a circular sector in Grade 10, if the angle of the sector is θ then $\frac{\theta}{360} \times 2\pi l = 2\pi r$. $\therefore\ \theta = \frac{2\pi r \times 360}{2\pi l}$ i . e, $\theta = \frac{360r}{l}$

The area of the sector with the above angle θ (as learnt in grade 10) is $\frac{\theta}{360} \times \pi l^2$.

By substituting θ from the above equation, we get $\frac{360r}{l} \times \frac{\pi l^2}{360}$. Accordingly the area of the curved surface of the cone is πrl.

Now we can add both areas, to get the total surface area of the cone.

$$\text{Total surface area of the cone} = \left\{\text{area of the curved surface of the cone}\right\} + \left\{\text{area of the circular base}\right\}$$

$$= \pi rl + \pi r^2$$

If the total surface area is A

$$\boxed{A = \pi rl + \pi r^2}$$

Let us now consider some solved examples on the surface area of a cone. In this lesson let us take the value of π as $\frac{22}{7}$.

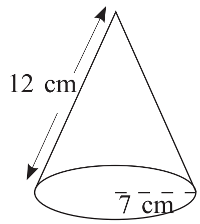
Example 1

Shown in the figure is a solid right circular cone. Its radius is 7 cm and slant height is 12 cm. Find the total surface area of the cone.

Area of the curved surface $= \pi rl$

$= \frac{22}{7} \times 7 \times 12$

$= 264$



Area of the circular base $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7$

$= 154$

$\therefore$ Total surface area of the cone $= 264 + 154$

$= 418$

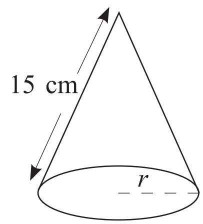
Total surface area of the cone is 418 cm^2.

Example 2

The circumference of the base of a right circular cone is 88 cm and its slant height is 15 cm. Find the area of the curved surface.

Circumference of the circular base = 88 cm

Let us take the radius as r cm.



Then, $2\pi r = 88$

$2 \times \frac{22}{7} \times r = 88$

$r = \frac{88 \times 7}{2 \times 22}$

$r = 14$

Surface area of the curved surface $= \pi rl$

$= \frac{22}{7} \times 14 \times 15$

$= 660$

$\therefore$ Surface area of the curved surface of the cone is 660 cm^2.

Example 3

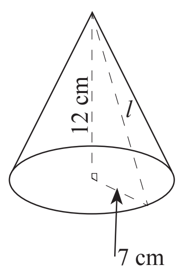
Find,

(i) the slant height,

(ii) the area of the curved surface, and

(iii) the total surface area of the cone,

accurate up to one decimal place, of a cone of radius 7 cm and perpendicular height 12 cm.



Take the slant height of the cone to be l cm.

According to Pythagoras' theorem,

$$\text{(i)}\quad l^2 = 7^2 + 12^2$$
$$= 49 + 144$$
$$= 193$$
$$l = \sqrt{193}$$
$$= 13.8 \quad \text{(Use the division method to find the square root)}$$

$\therefore$ The slant height of the cone is 13.8 cm.

(ii) The area of the curved surface $= \pi r l$

$$= \frac{22}{7} \times 7 \times 13.8$$
$$= 303.6$$

$\therefore$ The area of the curved surface is 303.6 cm^2.

(iii) Area of the circular base $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7$$
$$= 154$$

Total surface area of the cone $= 303.6 + 154$

$$= 457.6$$

$\therefore$ Total surface area of the cone is 457.6 cm^2.

Exercise 4.2

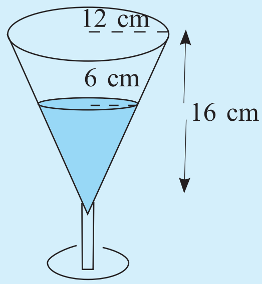
1. Find the area of the curved surface of a right circular cone of base radius 14 cm and slant height 20 cm.

2. Of a right circular solid cone, the radius of the base is 7 cm and the height is 24 cm. Find,
 (i) the slant height, and
 (ii) the area of the curved surface.

3. If the slant height, of a conical shaped sand pile with a base circumference 44 cm, is 20 cm, find
 (i) the radius of the base, and
 (ii) the area of the curved surface.

4. Find the total surface area of a right circular cone of base radius 10.5 cm and slant height 15 cm.

5. The slant height of a conical shaped solid object is 14 cm. If the area of the curved surface is 396 cm^2, find
(i) the radius of the cone, and
(ii) the perpendicular height of the cone.

6. 

Shown in the picture, is a thin glass container in the shape of a cone filled with a juice to half its height. The radius of the glass is 12 cm and its height is 16 cm. Find the area of the region on the glass surface that is in contact with the juice.

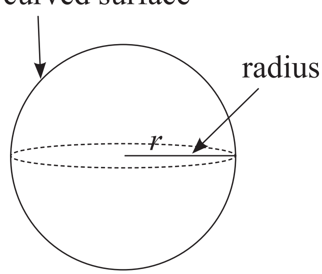
Sphere


shot put


tennis ball


Foot ball

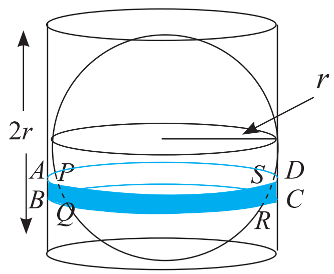
There is no doubt that you know what properties a sphere has. In mathematics, a sphere is defined as the set of points in three dimensional space that lies at a constant distance from a fixed point. The fixed point is called the centre of the sphere and the constant distance from the centre to a point on the sphere is called the radius of the sphere. A sphere has only one curved surface, and has no edges or vertices.



We will usually use "r" to indicate the radius of the sphere.

4.3 Surface area of a sphere

A cylinder with the same radius as the sphere and height equal to the diameter of the sphere is called the circumscribing cylinder of the sphere. The sphere will tightly fit in the circumscribing cylinder of the sphere.



The following fact regarding the surface area of a sphere and its circumscribing cylinder was observed by the Greek mathematician Archimedes, who lived around 225 B.C.

When the sphere is inside the circumscribing cylinder, any two planes parallel to the flat circular surfaces of the cylinder will bound equal surface areas on the curved surfaces of the sphere and cylinder.

For example, in the above figure, the area of the curved surface *PQRS* on the sphere is the same as the area of the curved surface *ABCD* on the cylinder.

Now, if we apply this fact to the entire cylinder, we see that the surface area of the sphere is equal to the surface area of the curved surface of the cylinder. We can use the formulae $2\pi rh$ to find the surface area of the curved surface of the circumscribing cylinder.

Area of the curved surface of the circumscribing cylinder $= 2\pi r \times 2r$

$= 4\pi r^2$

Therefore, the surface area of the sphere $= 4\pi r^2$

If the surface area is *A*,

$$A = 4\pi r^2$$

Example 1

Find the surface area of a sphere of radius 7 cm.

$$\text{Surface area of the sphere} = 4\pi r^2$$
$$= 4 \times \frac{22}{7} \times 7 \times 7$$
$$= 616$$

$\therefore$ surface area of the sphere is 616 cm^2.

Example 2

The surface area of a sphere is 1386 cm^2. Find the radius of the sphere.

Let r be the radius.

Then, $4\pi r^2 = 1386$

$$4 \times \frac{22}{7} \times r^2 = 1386$$

$$r^2 = \frac{1386 \times 7}{4 \times 22}$$

$$= \frac{441}{4}$$

$$r = \sqrt{\frac{441}{4}}$$

$$= \frac{21}{2}$$

$$= 10.5$$

$\therefore$ radius of the sphere is 10.5 cm.

Exercise 4.3

1. Find the surface area of a sphere of radius 3.5 cm.

2. Find the surface area of a sphere of radius 14 cm.

3. Find the radius of a sphere of surface area 5544 cm^2.

4. Find the (external) surface area of a hollow hemisphere of radius 7 cm.

5. Find the surface area of a solid sphere of diameter 0.5 cm.

6. Find the radius of a solid hemisphere with a surface area of 1386 cm^2.

Summary

- The surface area A of a square based right pyramid, of base length "a" and height "l" is

  $A = a^2 + 2al$

- The surface area A, of a right circular cone of radius r and slant height l is

  $A = \pi rl + \pi r^2$

- The surface area A of a sphere of radius r is

  $A = 4\pi r^2$.

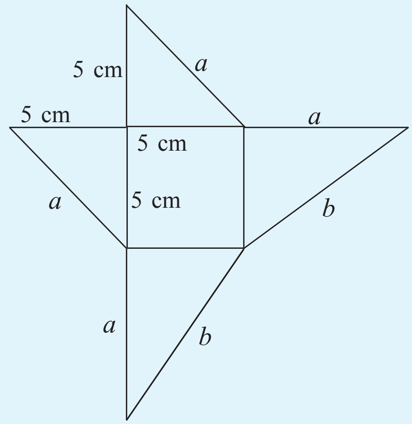
Mixed Exercise

1. Shown below is a net used to make a pyramid.

(i) Find the lengths indicated by a and b.

(ii) Give reasons as to why the resulting pyramid is not a right pyramid.

(iii) Find the total surface area of the pyramid.

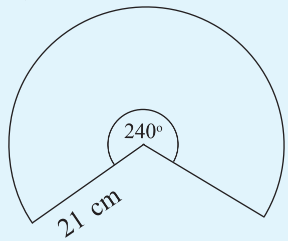


2. A right circular cone was made using a lamina in the shape of the sector shown in the figure.

A circular lamina of the same radius is fixed to the base of the cone.

(i) Find the radius of the cone.

(ii) Find the total surface area of the cone.

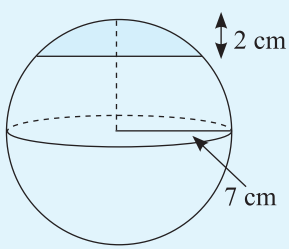


3. The ratio between the slant height and the perpendicular height of a cone is 5 : 4. The radius of the base of the cone is 6 cm.

(i) Find the slant height of the cone.

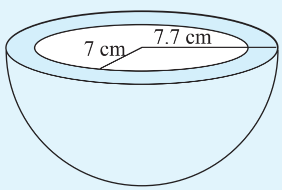
(ii) Find the surface area of the curved surface of the cone.

4.



On a sphere of radius 7 cm, paint was applied from the top downwards, a perpendicular distance of 2 cm. Find the area of the painted region. (Hint: make use of knowledge on the circumscribing cylinder)

5.



The internal radius of a hemispherical clay pot is 7 cm and the external radius is 7.7 cm. Find the total surface area of the pot.