3 Indices and Logarithms II

By studying this lesson you will be able to

- use the table of logarithms to simplify expressions involving products and quotients of powers and roots of numbers between 0 and 1.
- identify the two keys $\boxed{\wedge}$ and $\boxed{\sqrt{\ }}$ on a scientific calculater, and simplify expressions involving decimal numbers, powers and roots using a scientific calculator.

Logarithms

$10^3 = 1000$ can be written using logarithms as $\log_{10} 1000 = 3$. As a convention we write "lg" instead of "$\log_{10}$". Now we can express the above expression as $\lg 1000 = 3$. It is important to mention the base if it is other than 10.

For example,

$\log_5 25 = 2$ because $5^2 = 25$,
$\lg 1 = 0$ because $10^0 = 1$, and
$\lg 10 = 1$ because $10^1 = 10$.

The logarithm of any positive number can be found using the table of logarithms. Do the following exercise to refresh the memory on using logarithms to simplify expressions involving multiplications and divisions of numbers.

Review Exercise

1. Complete the following tables.

(i)

Number	Scientific notation	Logarithm		Logarithm
		Characteristic	Mantissa	
73.45	7.345×10^1	1	0.8660	1.8660
8.7				
12.5				
725.3				
975				

(ii)

Logarithm	Logarithm		Scientific notation	Number
	Characteristic	Mantissa		
1.5492				
2.9059				
1.4036				
2.8798				
3.4909				

2. Use the table of logarithms to fill in the blanks.

a. lg 5.745 = 0.7593, therefore 5.745 = $10^{0.7593}$
b. lg 9.005 = ..., therefore 9.005 = $10^{\cdots\cdots}$
c. lg 82.8 = ..., therefore 82.8 = $10^{\cdots\cdots}$
d. lg 74.01 = ..., therefore 74.01 = $10^{\cdots\cdots}$
e. lg 853.1 = ..., therefore 853.1 = $10^{\cdots\cdots}$
f. antilog 0.7453 = 5.562, therefore 5.562 = $10^{0.7453}$
g. antilog 0.0014 = ..., therefore .... = $10^{0.0014}$
h. antilog 1.9251 = ..., therefore .... = $10^{1.9251}$
i. antilog 2.4374 = ..., therefore .... = $10^{2.4374}$
j. antilog 3.2001 = ..., therefore .... = $10^{3.2001}$

3. Fill in the blanks and find the value of P.

(i) In terms of logarithms

$$P = \frac{27.32 \times 9.8}{11.5}$$

lg P = lg ...... + lg ...... − lg ....
= .... + ... − ....
= ....
∴ P = antilog ....
= ....

(ii) Using indices

$$P = \frac{27.32 \times 9.8}{11.5}$$

$$= \frac{10^{\cdots\cdots} \times 10^{\cdots\cdots}}{10^{\cdots\cdots}}$$

$$= \frac{10^{\cdots\cdots}}{10^{\cdots\cdots}}$$

$= 10^{\cdots\cdots}$
$= \ldots\ldots \times 10^{\cdots\cdots}$
= ....

4. Simplify the expressions using logarithms.

a. 14.3×95.2

b. $2.575 \times 9.27 \times 12.54$

c. $\dfrac{9.87 \times 7.85}{4.321}$

3.1 Logarithms of decimal numbers less than one

Let us now consider how to use the table of logarithms to obtain the logarithms of numbers between 0 and 1, by paying close attention to how we obtained the logarithms of numbers greater than 1. For this purpose, carefully investigate the following table.

Number	Scientific Notation	Logarithm		Logarithm
		Characteristic	Mantissa	
5432	5.432×10^3	3	0.7350	3.7350
543.2	5.432×10^2	2	0.7350	2.7350
54.32	5.432×10^1	1	0.7350	1.7350
5.432	5.432×10^0	0	0.7350	0.7350
0.5432	5.432×10^{-1}	-1	0.7350	$\bar{1}.7350$
0.05432	5.432×10^{-2}	-2	0.7350	$\bar{2}.7350$
0.005432	5.432×10^{-3}	-3	0.7350	$\bar{3}.7350$
0.0005432	5.432×10^{-4}	-4	0.7350	$\bar{4}.7350$

According to the above table, the characteristic of the logarithm of numbers inbetween 0 and 1, coming after 5.432 in the first column, are negative. Even though the characteristic is negative, the mantissa of the logarithm, which is found using the table, is a positive number. The symbol "−" is used above the whole part to indicate that only the characteristic is negative. It is read as "bar".

For example, $\bar{2}.3725$ is read as "bar two point three, seven, two, five". Moreover, what is represented by $\bar{2}.3725$ is $-2 + 0.3725$.

The characteristic of the logarithm of a number between 0 and 1 is negative. The characteristic of the logarithm of such a number can be obtained either by writing it in scientific notation or by counting the number of zeros after the decimal point.

The characteristic of the logarithm can be obtained by adding one to the number of zeros after the decimal point (and before the next non-zero digit) and taking its negative value. Observe it in the above table too.

Example

0.004302 Number of zeros after the decimal point and before the next non-zero digit is 2. Therefore characteristic of the logarithm is $\bar{3}$

0.04302	Number of zeros after the decimal point is 1; thus the characteristic of the logarithm is $\bar{2}$
0.4302	Number of zeros after the decimal point is 0; thus the characteristic of the logarithm is $\bar{1}$

Therefore, $\lg 0.004302 = \bar{3}.6337$.

When written using indices, it is;
$0.004302 = 10^{\bar{3}.6337}$. This can also be written as, $0.004302 = 10^{-3} \times 10^{0.6337}$.

Do the following exercise to practice taking logarithms of numbers between 0 and 1.

Exercise 3.1

1. For each of the following numbers, write the characteristic of its logarithm.

a. 0.9843 **b.** 0.05 **c.** 0.0725
d. 0.0019 **e.** 0.003141 **f.** 0.000783

2. Find the value.

a. lg 0.831 **b.** lg 0.01175 **c.** lg 0.0034
d. lg 0.009 **e.** lg 0.00005 **f.** lg 0.00098

3. Express each of the following numbers as a power of 10.

a. 0.831 **b.** 0.01175 **c.** 0.0034
d. 0.009 **e.** 0.00005 **f.** 0.00098

3.2 Number corresponding to a logarithm (antilog)

Let us recall how the antilog of a number greater than 1 is obtained.

$$\text{antilog } 2.7421 = 5.522 \times 10^2$$
$$= 552.2$$

When a number is written in scientific form, the index of the power of 10 is the characteristic of the logarithm of that number. The characteristic of the logarithm indicates the number of places that the decimal point needs to be shifted when taking the antilog.

Thus, we obtained 552.2 by shifting the decimal point of 5.522 two places to the right. However, when the characteristic is negative the decimal point is shifted to the left side.

antilog $\bar{2}.7421 = 5.522 \times 10^{-2}$ (Decimal point needs to be shifted 2 places to the left)

$= 0.05522$ (Because of bar 2, there is one 0 after the decimal point)

antilog $\bar{1}.7421 = 5.522 \times 10^{-1}$ (Decimal place needs to be shifted one place to the left)

$= 0.5522$ (Because of bar 1, there are no zeros after the decimal place)

Exercise 3.2

1. Express each of the following numbers, given in the scientific form, in decimal form.

a. 3.37×10^{-1} **b.** 5.99×10^{-3} **c.** 6.0×10^{-2}
d. 5.745×10^{0} **e.** 9.993×10^{-4} **f.** 8.777×10^{-3}

2. Find the value using the logarithmic table.

a. antilog $\bar{2}.5432$ **b.** antilog $\bar{1}.9321$ **c.** antilog 0.9972
d. antilog $\bar{4}.5330$ **e.** antilog $\bar{2}.0000$ **f.** antilog $\bar{3}.5555$

3.3 Addition and subtraction of logarithms with negative characteristics

(a) Addition

The mantissa of a logarithm is obtained from the table of logarithms and is always positive. But we now know that, the characteristic can be positive, negative or zero. In $\bar{2}.5143$, the mantissa, .5143, is positive and the characteristic, $\bar{2}$, is negative. When adding or subtracting such numbers, it is important to simplify the characteristic and the mantissa separately.

Example 1

Simplify and express the answer in log form.

(i) $\bar{2}.5143 + \bar{1}.2375$ (ii) $\bar{3}.9211 + 2.3142$ (iii) $\bar{3}.8753 + \bar{1}.3475$

(i) $\bar{2}.5143 + \bar{1}.2375$

$$= -2 + 0.5143 + (-1) + 0.2375$$
$$= (-2 - 1) + (0.5143 + 0.2375)$$
$$= -3 + 0.7518$$
$$= \underline{\underline{\bar{3}.7518}}$$

(ii) $\overline{3}.9211 + 2.3142$ $= -3 + 0.9211 + 2 + 0.3142$

$= (-3 + 2) + (0.9211 + 0.3142)$

$= -1 + 1.2353$

$= -1 + 1 + 0.2353$

$= \underline{\underline{0.2353}}$

(iii) $\overline{3}.8753 + 1.3475$ $= -3 + 0.8753 + 1 + 0.3475$

$= (-3 + 1) + (0.8753 + 0.3475)$

$= -2 + 1.2228$

$= -2 + 1 + 0.2228$

$= \underline{\underline{\overline{1}.2228}}$

(b) Subtraction

As in addition, logarithms should be subtracted from right to left, remembering that the mantissa is positive.

Example 2

Simplify and express the answer in log form.

(i) $\overline{2}.5143 - 1.3143 = -2 + 0.5143 - (1 + 0.3143)$

$= -2 + 0.5143 - 1 - 0.3143$

$= -2 - 1 + 0.5143 - 0.3143$

$= -3 + 0.2000$

$= \underline{\underline{\overline{3}.2000}}$

(ii) $2.5143 - \overline{1}.9143 = 2 + 0.5143 - (-1 + 0.9143)$

$= 2 + 0.5143 + 1 - 0.9143$

$= 3 - 0.4000$

$= \underline{\underline{2.6000}}$

(iii) $0.2143 - \overline{1}.8143 = 0.2143 - (-1 + 0.8143)$

$= 0.2143 + 1 - 0.8143$

$= 1 - 0.6000$

$= \underline{\underline{0.4}}$

(iv) $\overline{2}.5143 - \overline{1}.9143 = -2 + 0.5143 - (-1 + 0.9143)$

$= -2 + 0.5143 + 1 - 0.9143$

$= -2 + 1 + 0.5143 - 0.9143$

$= -1 - 0.4000$

In the above example, the decimal part is negative. Because we need the decimal part of a logarithm to be positive, we will use a trick as follows to make it positive.

$-1-0.4=-1-1+1-0.4$ (Because $-1+1=0$, we have not changed the value.)

$=-2+0.6$

$=\bar{2}.6$

Actually what we have done is add -1 to the characteristic and $+1$ to the mantissa.

Note: We could have avoided getting a negative decimal part by doing the simplification as follows.

$-2+0.5143+1-0.9143=-2+1.5143-0.9143=-2+0.6=\bar{2}.6$

Exercise 3.3

1. Simplify

a. $0.7512+\bar{1}.3142$ **b.** $\bar{1}.3072+\bar{2}.2111$ **c.** $\bar{2}.5432+\bar{1}.9513$

d $\bar{3}.9121+\bar{1}.5431$ **e.** $0.7532+\bar{3}.8542$ **f.** $\bar{1}.8311+\bar{2}.5431+1.3954$

g. $3.8760-\bar{2}.5431$ **h.** $\bar{2}.5132-\bar{1}.9332$ **i.** $\bar{3}.5114-\bar{2}.4312$

j $\bar{2}.9372-1.5449$ **k.** $0.7512+\bar{1}.9431$ **l.** $\bar{1}.9112-\bar{3}.9543$

2. Simplify and express in log form.

a. $\bar{1}.2513+0.9172-\bar{1}.514$ **b.** $\bar{3}.2112+2.5994-\bar{1}.5004$

c. $\bar{3}.2754+\bar{2}.8211-\bar{1}.4372$ **d.** $0.8514-\bar{1}.9111-\bar{2}.3112$

e. $\bar{3}.7512-(0.2511+\bar{1}.8112)$ **f.** $\bar{1}.2572+3.9140-\bar{1}.1111$

3.4 Simplification of numerical expressions using the table of logarithms

The following examples show how numerical computations are done using the given logarithmic rules.

1. $\log_a (P\times Q)=\log_a P+\log_a Q$

2. $\log_a \left(\frac{P}{Q}\right)=\log_a P-\log_a Q$

Example 1

Simplify using the table of logarithms and logarithmic rules.

a. 43.85×0.7532 **b.** 0.0034×0.8752

c. $0.0875 \div 18.75$ **d.** $0.3752 \div 0.9321$

Two methods of simplifying are shown below.

Method 1:

a. 43.85×0.7532

Take $P = 43.85 \times 0.7532$

Then $\lg P = \lg (43.85 \times 0.7532)$

$= \lg 43.85 + \lg 0.7532$

$= 1.6420 + \bar{1}.8769$

$= 1 + 0.6420 - 1 + 0.8769$

$= 1.5189$

$\therefore \quad P = \text{antilog } 1.5189$

$= \underline{\underline{33.03}}$

Method 2:

Simplifying using indices

43.85×0.7532

$= 10^{1.6420} \times 10^{\bar{1}.8769}$

$= 10^{1.5189}$

$= 3.303 \times 10^{1}$

$= \underline{\underline{33.03}}$

b. 0.0034×0.8752

Take $P = 0.0034 \times 0.8752$.

Then,

$\lg P = \lg (0.0034 \times 0.8752)$

$= \lg 0.0034 + \lg 0.8752$

$= \bar{3}.5315 + \bar{1}.9421$

$= -3 + 0.5315 - 1 + 0.9421$

$= -4 + 1.4736$

$= -4 + 1 + 0.4736$

$= -3 + 0.4736$

$= \bar{3}.4736$

$\therefore \quad P = \text{antilog } \bar{3}.4736$

$= \underline{\underline{0.002975}}$

Simplifying using indices

0.0034×0.8752

$= 10^{\bar{3}.5315} \times 10^{\bar{1}.9421}$

$= 10^{\bar{3}.4736}$

$= 2.975 \times 10^{-3}$

$= \underline{\underline{0.002975}}$

c. $0.0875 \div 18.75$

Take $P = 0.0875 \div 18.75$

Then, $\lg P = \lg (0.0875 \div 18.75)$

$= \lg 0.0875 - \lg 18.75$

$= \bar{2}.9420 - 1.2730$

$= -2 + 0.9420 - 1 - 0.2730$

$= -3 + 0.6690$

$= \bar{3}.6690$

$\therefore \quad P = \text{antilog } \bar{3}.6690$

$= \underline{\underline{0.004666}}$

Simplifying using indices

$0.0875 \div 18.75$

$= 10^{\bar{2}.9420} \div 10^{1.2730}$

$= 10^{\bar{2}.9420 - 1.2730}$

$= 10^{\bar{3}.6690}$

$= 4.666 \times 10^{-3}$

$= \underline{\underline{0.004666}}$

d. $0.3752 \div 0.9321$

Take $P = 0.3752 \div 0.9321$

Then, $\lg P = \lg (0.3752 \div 0.9321)$

$= \lg 0.3752 - \lg 0.9321$

$= \bar{1}.5742 - \bar{1}.9694$

$= -1 + 0.5742 - (-1 + 0.9694)$

$= -1 + 0.5742 + 1 - 0.9694$

$= -1 + 0.5742 + 0.0306$

$= -1 + 0.6048$

$= \bar{1}.6048$

$\therefore \quad P = \text{antilog } \bar{1}.6048$

$= \underline{\underline{0.4026}}$

Simplifying using indices

$0.3752 \div 0.9321$

$= 10^{\bar{1}.5742} \div 10^{\bar{1}.9694}$

$= 10^{\bar{1}.5742 - \bar{1}.9694}$

$= 10^{\bar{1}.6048}$

$= 4.026 \times 10^{-1}$

$= \underline{\underline{0.4026}}$

Example 2

Simplify using the table of logarithms.

$$\frac{8.753 \times 0.02203}{0.9321}$$

Take $P = \frac{8.753 \times 0.02203}{0.9321}$.

Then, $\lg P = \lg\left(\frac{8.753 \times 0.02203}{0.9321}\right)$

$= \lg 8.753 + \lg 0.02203 - \lg 0.9321$

$= 0.9421 + \bar{2}.3430 - \bar{1}.9694$

$= 0.9421 - 2 + 0.3430 - \bar{1}.9694$

$= \bar{1}.2851 - \bar{1}.9694$

$= -1 + 0.2851 - (-1 + 0.9694)$

$= -1 + 0.2851 + 1 - 0.9694$

$= \bar{1}.3157$

$\therefore\ P = \text{antilog } \bar{1}.3157$

$= \underline{\underline{0.2068}}$

Simplifying using indices

$$\frac{8.753 \times 0.02203}{0.9321}$$

$$= \frac{10^{0.9421} \times 10^{\bar{2}.3430}}{10^{\bar{1}.9694}}$$

$$= \frac{10^{\bar{1}.2851}}{10^{\bar{1}.9694}}$$

$$= 10^{\bar{1}.2851 - \bar{1}.9694}$$

$$= 10^{\bar{1}.3157}$$

$$= 2.067 \times 10^{-1}$$

$$= \underline{\underline{0.2068}}$$

Exercise 3.4

Find the value using the table of logarithms.

1. **a.** 5.945×0.782 **b.** 0.7453×0.05921 **c.** 0.0085×0.0943

d. $5.21 \times 0.752 \times 0.093$ **e.** $857 \times 0.008321 \times 0.457$ **f.** $0.123 \times 0.9857 \times 0.79$

2.

a. $7.543 \div 0.9524$ **b.** $0.0752 \div 0.8143$ **c.** $0.005273 \div 0.0078$

d. $0.9347 \div 8.75$ **e.** $0.0631 \div 0.003921$ **f.** $0.0752 \div 0.0008531$

3.

a. $\frac{8.247 \times 0.1973}{0.9875}$ **b.** $\frac{9.752 \times 0.0054}{0.09534}$ **c.** $\frac{79.25 \times 0.0043}{0.3725}$

d. $\frac{0.7135 \times 0.4391}{0.0059}$ **e.** $\frac{5.378 \times 0.9376}{0.0731 \times 0.471}$ **f.** $\frac{71.8 \times 0.7823}{23.19 \times 0.0932}$

3.5 Multiplication and division of a logarithm of a number by a whole number

We know that the characteristic of a number greater than one is positive. Multiplying or dividing such a logarithm by a number can be done in the usual way.

We know that the characteristic of the logarithm of a number between 0 and 1 is negative. $\overline{3}.8247$ is such a logarithm. When multiplying or dividing a logarithm with a negative characteristic by a number, we simplify the characteristic and the mantissa separately.

Multiplication of a logarithm by a whole number

Example 1

Simplify.

a. 2.8111×2 **b.** $\overline{2}.7512 \times 3$ **c.** $\overline{1}.9217 \times 3$

a.

$$2.8111 \times 2$$
$$= \underline{\underline{5.6222}}$$

b.

$$\overline{2}.7512 \times 3$$
$$= 3(-2 + 0.7512)$$
$$= -6 + 2.2536$$
$$= -6 + 2 + 0.2536$$
$$= -4 + 0.2536$$
$$= \underline{\underline{\overline{4}.2536}}$$

c.

$$\overline{1}.9217 \times 3$$
$$3(-1 + 0.9217)$$
$$= -3 + 2.7651$$
$$= -3 + 2 + 0.7651$$
$$= -1 + 0.7651$$
$$= \underline{\underline{\overline{1}.7651}}$$

Division of a logarithm by a whole number

Let us now consider how to divide a logarithm by a whole number. When the characteristic of a logarithm is negative the characteristic and the mantissa carry negative and positive values respectively. Therefore, it is important to divide the positive part and the negative part separately. Let us now consider some examples of this type.

Example 2

Simplify.

a. $2\,.\,5142 \div 2$ **b.** $\overline{3}.\,5001 \div 3$ **c.** $\overline{4}.\,8322 \div 2$

because, $(-3 + 0.5001) \div 3$ because, $(-4 + 0.8322) \div 2$

$2\,.\,5142 \div 2$
$= \underline{\underline{1.\,2571}}$

$\overline{3} \div 3 = \overline{1}$
$0.5001 \div 3 = 0.1667$
$\therefore\ \overline{3}.\,5001 \div 3$
$= \underline{\underline{\overline{1}.\,1667}}$

$\overline{4} \div 2 = \overline{2}$
$0.8322 \div 2 = 0.4161$
$\therefore\ \overline{4}.\,8322 \div 2$
$= \underline{\underline{\overline{2}.\,4161}}$

In the above example, the characteristic of the logarithm was perfectly divisible. Let us consider in the following example, how division is done when the whole part is not perfectly divisible.

Example 3

Simplify.

a. $\overline{1}.\,5412 \div 2$ **b.** $\overline{2}.\,3713 \div 3$ **c.** $\overline{3}.\,5112 \div 2$

a. $\overline{1}.\,5412 \div 2$ can be written as $(-1 + 0.5412) \div 2.$

Because the whole part, $\overline{1}$, is not perfectly divisible by 2, let us write it as $\overline{2} + 1$. Now, we can perform the division as follows

a. $\overline{1}\,.\,5412 \div 2 = (-1 + 0.5412) \div 2$
$= (-2 + 1 + 0.5412) \div 2$
$= (-2 + 1\,.\,5412) \div 2$
$= \underline{\underline{\overline{1}\,.\,7706}}$

b. $\overline{2}.\,3713 \div 3$
$= (-2 + 0.3713) \div 3$
$= (-3 + 1 + 0.3713) \div 3$ beacause $(-2 = -3 + 1)$
$= (\overline{3} + 1.3713) \div 3$
$= \underline{\underline{\overline{1}.\,4571}}$

c. $\overline{3}.\,5112 \div 2$
$= (-3 + 0.5112) \div 2$
$= (-4 + 1 + 0.5112) \div 2$ because $(-3 = -4 + 1)$
$= \overline{4} + 1.5112 \div 2$
$= \underline{\underline{\overline{2}.\,7556}}$

These types of divisions and multiplications are important when simplifying using the table of logarithms. Do the following exercise to strengthen this knowledge.

Exercise 3.5

1. Find the value.

a. $\overline{1}.5413 \times 2$ **b.** $\overline{2}.7321 \times 3$ **c.** 1.7315×3

d. 0.4882×3 **e.** $\overline{3}.5111 \times 2$ **f.** $\overline{3}.8111 \times 4$

2. Find the value.

a. $1.9412 \div 2$ **b.** $0.5512 \div 2$ **c.** $\overline{2}.4312 \div 2$

d. $\overline{3}.5412 \div 3$ **e.** $\overline{2}.4712 \div 2$ **f.** $\overline{4}.5321 \div 2$

g. $\overline{1}.5432 \div 2$ **h.** $\overline{2}.9312 \div 3$ **i.** $\overline{3}.4112 \div 2$

j. $\overline{1}.7512 \div 3$ **k.** $\overline{4}.1012 \div 3$ **l.** $\overline{5}.1421 \div 3$

3.6 Finding powers and roots of numbers using the table of logarithms

Recall that $\log_2 5^3 = 3 \log_2 5$.

This follows from the logarithmic rule $\log_a m^r = r \log_a m$.

Similarly, the logarithm of a root can be written using this rule, as follows.

(i) $\log_a \sqrt{5} = \log_a 5^{\frac{1}{2}}$ (because $\sqrt{5} = 5^{\frac{1}{2}}$)

$= \frac{1}{2} \log_a 5$ (using the above logarithmic rule)

(ii) $\lg \sqrt{25} = \lg 25^{\frac{1}{2}}$

$= \frac{1}{2} \lg 25$

The following examples consider how to extract roots and powers of a number using the table of logarithms.

Example 1

Find the value.

a. 354^2 **b.** 0.0275^3 **c.** 0.9073^4

a. Take $P = 354^2$.

$\lg P = \lg 354^2$

$= 2 \lg 354$

$= 2 \lg 3.54 \times 10^2$

$= 2 \times 2.5490$

$= 5.0980$

$\therefore \quad P = \text{antilog } 5.0980$

$= 1.253 \times 10^5$

$= \underline{\underline{125\,300}}$

b. Take $P = 0.0275^3$.

$\lg P = \lg 0.0275^3$

$= 3 \lg 0.0275$

$= 3 \times \overline{2}.4393$

$= 3 \times (-2 + 0.4393)$

$= -6 + 1.3179$

$= -6 + 1 + 0.3179$

$= -5 + 0.3179$

$= \overline{5}.3179$

$\therefore \quad P = \text{antilog } \overline{5}.3179$

$= 2.079 \times 10^{-5}$

$= \underline{\underline{0.00002079}}$

c. Take $P = 0.9073^4$.

$\lg P = \lg 0.9073^4$

$= 4 \lg 0.9073$

$= 4 \times \overline{1}.9577$

$= 4 \times (-1 + 0.9577)$

$= -4 + 3.8308$

$= -4 + 3 + 0.8308$

$= -1 + 0.8308$

$= \overline{1}.8308$

$\therefore P = \text{antilog } \overline{1}.8308$

$= 6.773 \times 10^{-1}$

$= \underline{\underline{0.6773}}$

Simplifying using indices

$0.9073^4 = \left(10^{\overline{1}.9577}\right)^4$

$= 10^{\overline{1}.9577 \times 4}$

$= 10^{\overline{1}.8308}$

$= 6.773 \times 10^{-1}$

$= \underline{\underline{0.6773}}$

Example 2

(i) $\sqrt{8.75}$ (ii) $\sqrt[3]{0.9371}$ (iii) $\sqrt[3]{0.0549}$

(i) Take $P = \sqrt{8.75}$.

$P = \sqrt{8.75}$ can be written as

$$
\begin{aligned}
P &= 8.75^{\frac{1}{2}} \\
\lg P &= \lg 8.75^{\frac{1}{2}} \\
&= \frac{1}{2} \lg 8.75 \\
&= \frac{1}{2} \times 0.9420 \\
&= 0.4710 \\
\therefore \quad P &= \text{antilog } 0.4710 \\
&= \underline{\underline{2.958}}
\end{aligned}
$$

(ii) Take $P = \sqrt[3]{0.9371}$.

$$
\begin{aligned}
P &= 0.9371^{\frac{1}{3}} \\
\lg P &= \lg 0.9371^{\frac{1}{3}} \\
&= \frac{1}{3} \lg 0.9371 \\
&= \frac{1}{3} \times \bar{1}.9717 \\
&= (\bar{1}.9717) \div 3 \\
&= (-1 + 0.9717) \div 3 \\
&= (-3 + 2 + 0.9717) \div 3 \\
&= (-3 + 2.9717) \div 3 \\
&= -1 + 0.9906 \\
&= \bar{1}.9906 \\
\therefore \quad P &= \text{antilog } \bar{1}.9906 \\
&= \underline{\underline{0.9786}}
\end{aligned}
$$

Simplifying using indices.

$$
\begin{aligned}
\sqrt[3]{0.9371} &= 0.9371^{\frac{1}{3}} \\
&= \left(10^{\bar{1}.9717}\right)^{\frac{1}{3}} \\
&= 10^{\bar{1}.9717 \times \frac{1}{3}} \\
&= 10^{\bar{1}.9906} \\
&= 9.786 \times 10^{-1} \\
&= \underline{\underline{0.9786}}
\end{aligned}
$$

(iii) Take $P = \sqrt[3]{0.0549}$.

$$\begin{aligned}
\lg P &= \lg 0.0549^{\frac{1}{3}} \\
&= \frac{1}{3} \lg 0.0549 \\
&= \frac{1}{3} \times \bar{2}.7396 \\
&= (\bar{2}.7396) \div 3 \\
&= (-2 + 0.7396) \div 3 \\
&= (-3 + 1 + 0.7396) \div 3 \\
&= (-3 + 1.7396) \div 3 \\
&= -1 + 0.5799 \\
&= \bar{1}.5799 \\
\therefore P &= \text{antilog } \bar{1}.5799 \\
&= \underline{\underline{0.3801}}
\end{aligned}$$

Simplifying using indices.

$$\begin{aligned}
\sqrt[3]{0.0549} &= 0.0549^{\frac{1}{3}} \\
&= \left(10^{\bar{2}.7396}\right)^{\frac{1}{3}} \\
&= 10^{\bar{2}.7396 \times \frac{1}{3}} \\
&= 10^{\bar{1}.5799} \\
&= 3.801 \times 10^{-1} \\
&= \underline{\underline{0.3801}}
\end{aligned}$$

Now do the following exercise.

Exercise 3.6

1. Find the value using the table of logarithms.

a. $(5.97)^2$ **b.** $(27.85)^3$ **c.** $(821)^3$

d. $(0.752)^2$ **e.** $(0.9812)^3$ **f.** $(0.0593)^2$

2. Find the value using the table of logarithms.

a. $\sqrt{25.1}$ **b.** $\sqrt{947.5}$ **c.** $\sqrt{0.0714}$

d. $\sqrt[3]{0.00913}$ **e.** $\sqrt[3]{0.7519}$ **f.** $\sqrt{0.999}$

3.7 Simplification of expressions involving powers and roots using the table of logarithms

The following example demonstrates how to simplify an expression involving roots, powers, products and divisions (or some of these) using the table of logarithms.

Example 1

Simplify. Give your answer to the nearest first decimal place.

a. $\dfrac{7.543 \times 0.987^2}{\sqrt{0.875}}$ **b.** $\dfrac{\sqrt{0.4537} \times 75.4}{0.987^2}$

a. Take $P = \dfrac{7.543 \times 0.987^2}{\sqrt{0.875}}$

$$
\begin{aligned}
\text{Then } \lg P &= \lg\left(\frac{7.543 \times 0.987^2}{\sqrt{0.875}}\right) \\
&= \lg 7.543 + \lg 0.987^2 - \lg 0.875^{\frac{1}{2}} \\
&= \lg 7.543 + 2 \lg 0.987 - \frac{1}{2} \lg 0.875 \\
&= 0.8776 + 2 \times \bar{1}.9943 - \frac{1}{2} \times \bar{1}.9420 \\
&= 0.8776 + 2 \times \bar{1}.9943 - \frac{\bar{2} + 1.9420}{2} \\
&= 0.8776 + \bar{1}.9886 - (\bar{1} + 0.9710) \\
&= 0.8776 + \bar{1}.9886 - \bar{1}.9710 \\
&= 0.8662 - \bar{1}.9710 \\
&= 0.8952 \\
\therefore \quad P &= \text{antilog } 0.8952 \\
&= 7.855
\end{aligned}
$$

$$\therefore \frac{7.543 \times 0.987^2}{\sqrt{0.875}} \approx \underline{\underline{7.9}} \quad \text{(to the nearest first decimal place)}$$

This simplification can be done by using indices too as follows.

Simplifying using indices.

$$\frac{7.543 \times 0.987^2}{\sqrt{0.875}} = \frac{7.543 \times 0.987^2}{0.875^{\frac{1}{2}}}$$

$$= \frac{10^{0.8776} \times \left(10^{\bar{1}.9943}\right)^2}{\left(10^{\bar{1}.9420}\right)^{\frac{1}{2}}}$$

$$= \frac{10^{0.8776} \times 10^{\bar{1}.9886}}{10^{\bar{1}.9710}}$$

$$= \frac{10^{0.8662}}{10^{\bar{1}.9710}}$$

$$= 10^{0.8662 - \bar{1}.9710}$$

$$= 10^{0.8952}$$

$$= 7.855 \times 10^0$$

$$= 7.855$$

$$\therefore \frac{7.543 \times 0.987^2}{\sqrt{0.875}} \approx 7.9 \quad \text{(to the nearest first decimal place)}$$

b. Take $P = \dfrac{\sqrt{0.4537} \times 75.4}{0.987^2}$

$$\lg P = \lg\left(\frac{0.4537^{\frac{1}{2}} \times 75.4}{0.987^2}\right)$$

$$= \lg 0.4537^{\frac{1}{2}} + \lg 75.4 - \lg 0.987^2$$

$$= \frac{1}{2}\lg 0.4537 + \lg 75.4 - 2\lg 0.987$$

$$= \frac{1}{2} \times \bar{1}.6568 + 1.8774 - 2 \times \bar{1}.9943$$

$$= \bar{1}.8284 + 1.8774 - \bar{1}.9886$$

$$= 1.7058 - \bar{1}.9886$$

$$= 1.7172$$

$$P = \text{antilog } 1.7172$$

$$= \underline{\underline{52.15}}$$

$$\frac{\sqrt{0.4537} \times 75.4}{0.987^2} \approx \underline{\underline{52.2}} \quad \text{(to the nearest first decimal place)}$$

Simplifying using indices is given below.

$$\frac{\sqrt{0.4537} \times 75.4}{0.987^2} = \left(\frac{0.4537^{\frac{1}{2}} \times 75.4}{0.987^2}\right)$$

$$= \frac{\left(10^{\bar{1}.6568}\right)^{\frac{1}{2}} \times 10^{1.8774}}{\left(10^{\bar{1}.9943}\right)^2}$$

$$= \frac{10^{\bar{1}.8284} \times 10^{1.8774}}{10^{\bar{1}.9886}}$$

$$= 10^{1.7058 - \bar{1}.9886}$$
$$= 10^{1.7172}$$
$$= 52.15$$

$\approx \underline{\underline{52.2}}$ (to the nearest first decimal place)

Exercise 3.7

Use the table of logarithms to compute the value.

a. $\dfrac{8.765 \times \sqrt[3]{27.03}}{24.51}$ **b.** $\dfrac{\sqrt{9.18} \times 8.02^2}{9.83}$ **c.** $\dfrac{\sqrt{0.0945} \times 4.821^2}{48.15}$

d. $\dfrac{3 \times 0.752^2}{\sqrt{17.96}}$ **e.** $\dfrac{6.591 \times \sqrt[3]{0.0782}}{0.9821^2}$ **f.** $\dfrac{3.251 \times \sqrt[3]{0.0234}}{0.8915}$

3.8 Applications of logarithms

The table of logarithms can be used to do computations efficiently in many problems that involve products and divisions of numbers. Such an example is given below.

Example 1

The volume V, of a sphere of radius r is given by, $V = \frac{4}{3}\pi r^3$. By taking $\pi = 3.142$ and given that $r = 0.64$ cm, use the table of logarithms to find the volume of the sphere to the nearest first decimal place.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times 3.142 \times 0.64^3$$
$$\lg V = \lg\left(\frac{4}{3} \times 3.142 \times 0.64^3\right)$$
$$= \lg 4 + \lg 3.142 + 3\lg 0.64 - \lg 3$$
$$= 0.6021 + 0.4972 + 3 \times \bar{1}.8062 - 0.4771$$
$$= 0.6021 + 0.4972 + \bar{1}.4186 - 0.4771$$
$$= 0.5179 - 0.4771$$
$$= 0.0408$$
$$\therefore \quad V = \text{antilog } 0.0408$$
$$= 1.098$$
$$\approx 1.1 \quad \text{(to the nearest first decimal place)}$$

$\therefore$ The volume of the sphere is 1.1 cm^3.

Exercise 3.8

1. The mass of one cubic centimeter of iron is 7.86 g. Find the mass, to the nearest kilogram, of a cuboidal shaped iron beam, of length, width and depth respectively 5.4 m, 0.36 m and 0.22 m.

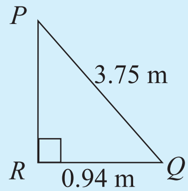
2. Find the value of g, if g is given by $g = \frac{4\pi^2 l}{T^2}$ where, $\pi = 3.142$, $l = 1.75$ and $T = 2.7$

3. A circular shaped portion of radius 0.07 m was removed from a thin circular metal sheet of radius 0.75 m.

(i) Show that the area of the remaining part is $\pi \times 0.82 \times 0.68$.

(ii) Taking π as 3.142, find the area of the remaining part using the table of logarithms.

4. The figure shows a right triangular block of land. If the dimensions of two sides are 3.75 m and 0.94 m, show that the length of PR is $\sqrt{4.69 \times 2.81}$ and find the length of PR in metres to the nearest second decimal place.



3.9 Using a calculator

Logarithms have been used for a long time to do complex numerical computations. However, its use has now been replaced to a great extent by calculators. Computations that can be done using an ordinary calculator is limited. For complex computations one needs to use a scientific calculator. The keyboard of a scientific calculator is much more complex than that of an ordinary calculator.

Evaluating powers using a calculator:

521^3 can be computed by entering $521 \times 521 \times 521$ into an ordinary calculator. However, this can be computed easily using a scientific calculator, by either using the key indicating x^n or by [^].

Example 1

Find the value of 275^3 using a calculator.
Show the sequence of keys that need to be activated to find 275^3.

[2] [7] [5] [x^n] [3] [=] or [2] [7] [5] [^] [3] [=]

20 796 875

Evaluating roots using a calculator:

You need to use the [shift] key, when finding roots. In addition to that, you also need to activate the keys denoted by [$\sqrt[x]{\ }$].

Example 1

Show the sequence of keys that need to be activated to find $\sqrt[4]{2313\,441}$ using a calculator.

[2] [3] [1] [3] [4] [4] [1] [shift] [x^n] [4] [=]
or
[2] [3] [1] [3] [4] [4] [1] [$x^{1/n}$] [4] [=]
or
[2] [3] [1] [3] [4] [4] [1] [$\sqrt[n]{x}$] [4] [=]

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Simplifying expressions involving powers and roots using a calculator:

Show the sequence of keys that need to be activated to find the value of

$$\frac{5.21^3 \times \sqrt[3]{4.3}}{3275}$$

5 . 2 1 x^n 3 × 4 . 3 $x^{1/n}$ 3 ÷ 3 2 7 5 = 0.070219546

Exercise 3.9

1. Show the sequence of keys that need to be activated to find each of the following values.

a. 952^2

b. $\sqrt{475}$

c. 5.85^3

d. $\sqrt[3]{275.1}$

e. $375^2 \times \sqrt{52}$

f. $\sqrt{4229} \times 352^2$

g. $\frac{37^2 \times 853}{\sqrt{50}}$

h. $\frac{\sqrt{751} \times 85^2}{\sqrt[3]{36}}$

i. $\frac{\sqrt{1452} \times 38.75}{98.2}$

j. $\frac{\sqrt[3]{827.3} \times 5.41^2}{9.74}$

Miscellaneous Exercise

1. Simplify using the table of logarithms. Verify your answer using a calculator.

(i) $\frac{1}{275.2}$ (ii) $\frac{1}{\sqrt{982.1}}$ (iii) $\frac{1}{\sqrt{0.954}}$

(iv) $0.5678^{\frac{1}{3}}$ (v) $0.785^2 - 0.0072^2$ (vi) $9.84^2 + 51.2^2$

2. Find the value of

(i) $\sqrt{\frac{a}{b}}$

(ii) $(ab)^2$

when $a = 0.8732$ and $b = 3.168$.

3. In $A = p\left(1 + \frac{r}{100}\right)^n$, find the value of A, when $P = 675$, $r = 3.5$ and $n = 3$.

4. A sector with an angle of 73° subtended at the center, was removed from a thin circular sheet.

(i) What fraction of the area of the circle is the area of the sector?

(ii) If the radius of the circle is 17.8 cm, find the area of the sector.