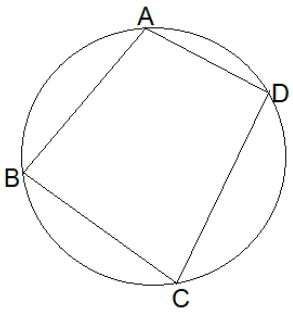
# 21 Cyclic Quadrilaterals

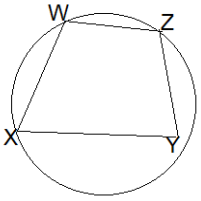
By learning this lesson, you will be able to;

- Identify cyclic quadrilaterals ,
- Identify the theorem “the opposite angles of a cyclic quadrilateral are supplementary” ,
- Identify the theorem “If the opposite angles of a quadrilateral are supplementary, then the vertices of the quadrilateral are on a circle” ,
- Identify the theorem “If one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle of the quadrilateral”.

Cyclic Quadrilateral ः- If all the four vertices of a quadrilateral are on a circle, it is a cyclic quadrilateral.

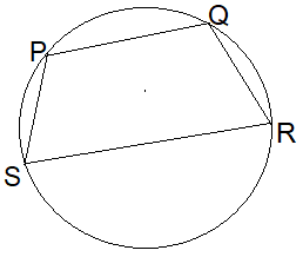



$ABCD$ is a cyclic quadrilateral

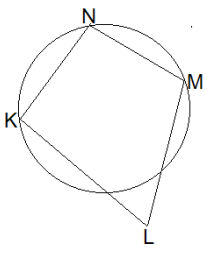



$Y$ vertex is inside the circle

$\therefore$ $WXYZ$ is not a cyclic quadrilateral




$PQRS$ is a cyclic quadrilateral




$L$ vertex is outside the circle

$\therefore$ $KLMN$ is not a cyclic quadrilateral

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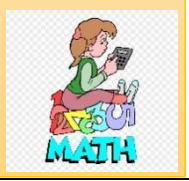

**Exercise 01** Write all the cyclic quadrilaterals in the figure given below.

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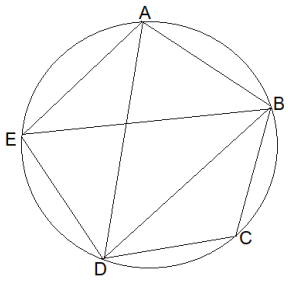
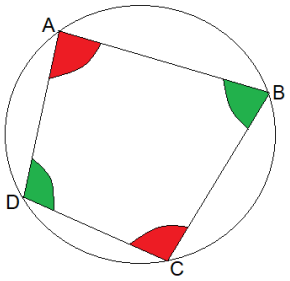
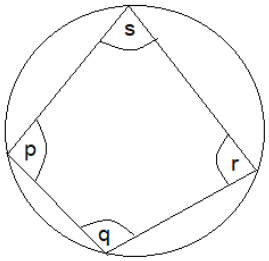



In the above figure, ABCD is a cyclic quadrilateral. The angle which is facing $\mathrm{B}\hat{A}\mathrm{D}$ is $\mathrm{B}\hat{C}\mathrm{D}$ and the angle which is facing of $\mathrm{A}\hat{D}\mathrm{C}$ is $A\hat{B}C$.

So, the opposite angle of $\mathrm{B}\hat{A}\mathrm{D}$ is $\mathrm{B}\hat{C}\mathrm{D}$ and the opposite angle of $\mathrm{A}\hat{D}\mathrm{C}$ is $A\hat{B}C$.

Activity




- Draw a cyclic quadrilateral according to the given figure.
- Mark the angles of the cyclic quadrilateral as $p, q, r, s$ and cut them.
- Paste the opposite angles $p$ and $r$ on a piece of paper such that they make a pair of adjacent angles and check whether they become supplementary $(180^0)$ using a protractor.
- Do the same step to $q$ and $s$ angles.
- What can you conclude about the opposite angles of a cyclic quadrilateral by this activity?
- **It can be concluded that the opposite angles of a cyclic quadrilateral are supplementary.**

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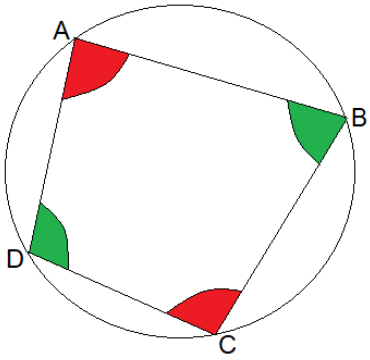
Theorem :- The opposite angles of a cyclic quadrilateral are supplementary.

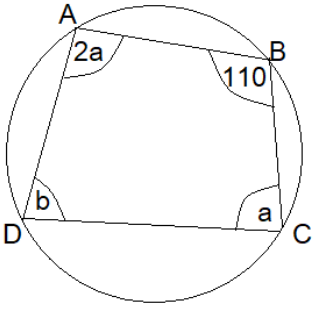



$$A\hat{D}C + A\hat{B}C = 180^0$$

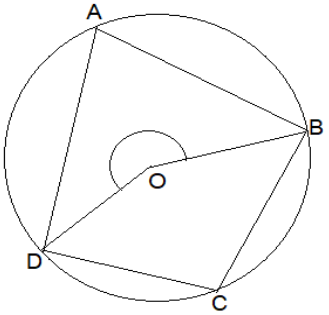
$$B\hat{A}D + B\hat{C}D = 180^0$$

**Exercise 02**

(01) $ABCD$ is a cyclic quadrilateral. Find the values of $a$ and $b$




(02) In the given figure, $ABCD$ is a cyclic quadrilateral. $B\hat{O}D\ (reflex) = 230^0$ Find the values of $B\hat{A}D$ and $B\hat{C}D$

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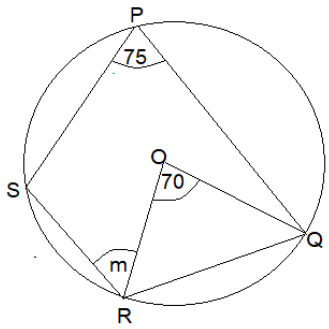
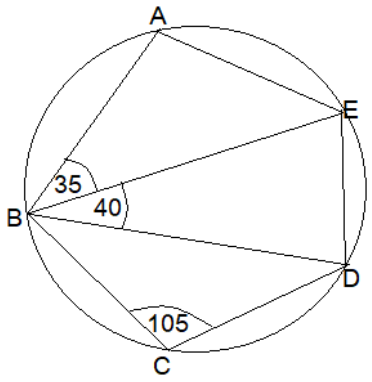
(03) $PQRS$ is a cyclic quadrilateral with centre $O$. Find the value of $m$ using the given information.

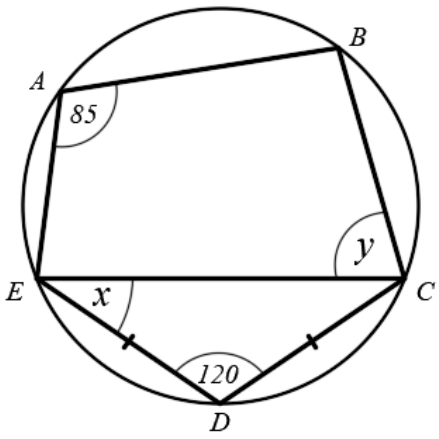



(04) Find the values of the following angles from the figure given below.

*i)* $B\hat{E}D$ *ii)* $A\hat{E}D$ *iii)* $B\hat{D}E$




(05) Find the values of $x$ and $y$





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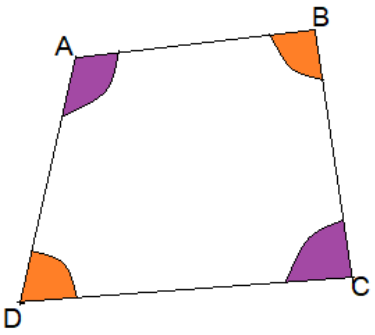
Theorem :- If the opposite angles of a quadrilateral are supplementary, then the vertices of the quadrilateral are on a circle

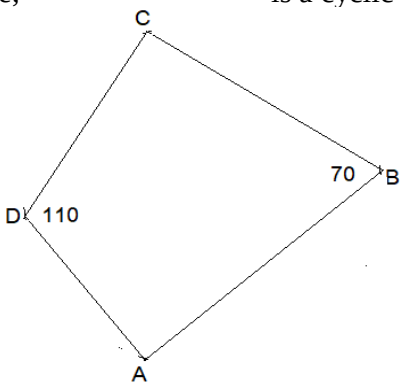


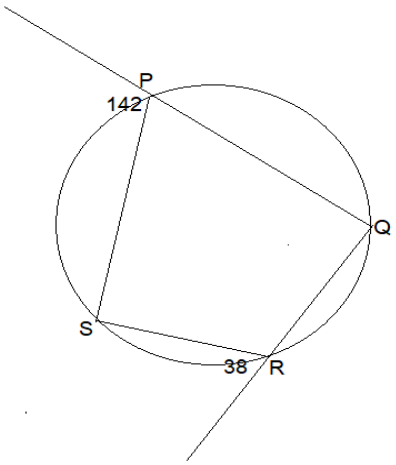

If, $B\hat{A}D + B\hat{C}D$ or $A\hat{D}C + A\hat{B}C = 180^0$.

Then A, B, C and D are on a circle. It's means, $ABCD$ is a cyclic quadrilateral.

**Exercise 03**

(01) According to the information in the figure, show that $ABCD$ is a cyclic quadrilateral.




(02) According to the information in the figure, show that

$PQRS$ is a cyclic quadrilateral.

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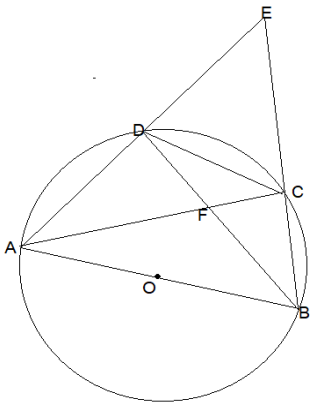
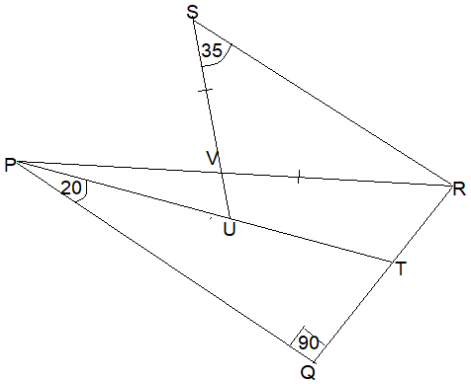
(03) In the circle with center $O$, $AB$ is a diameter and extended $AD$ and $BC$ lines meet at $E$. Prove that $DFCE$ is a cyclic quadrilateral.




(04) Show that $UTVR$ is a cyclic quadrilateral.

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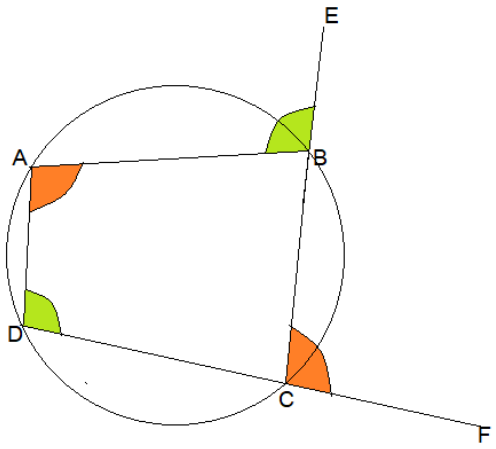



$ABCD$ is a cyclic quadrilateral . $CB$ is extended up to $E$.

Then, $A\hat{B}E$ is an external angle and the interior opposite angle of it is, $A\hat{D}C$

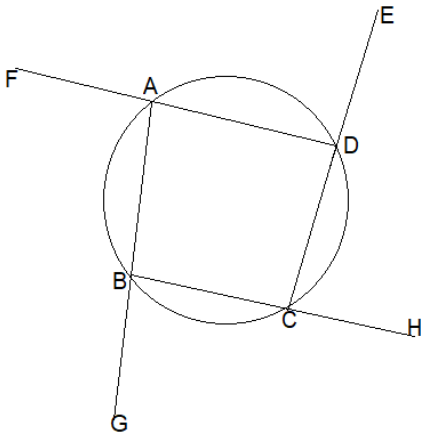
Also

$ABCD$ is a cyclic quadrilateral. $DC$ is extended up to $F$

Then, $B\hat{C}F$ is an external angle and the interior opposite angle of it is $B\hat{A}D$

**Exercise 04**

Complete the given table according to $ABCD$ cyclic quadrilateral




| Extended arm | External angle | Interior opposite angle |
|---|---|---|
| $AB$ | $G\hat{B}C$ | $A\hat{D}C$ |
| $DA$ | | |
| $BC$ | | |
| $CD$ | | |

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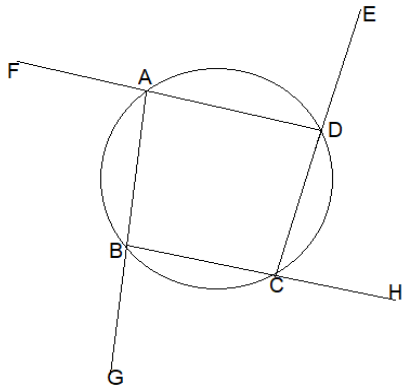
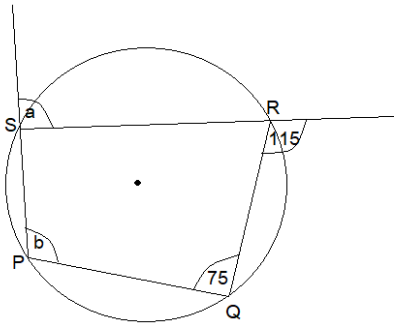
Theorem :- If one side of a cyclic quadrilateral is produced, the exterior angle so formed is equal to the interior opposite angle of the quadrilateral.



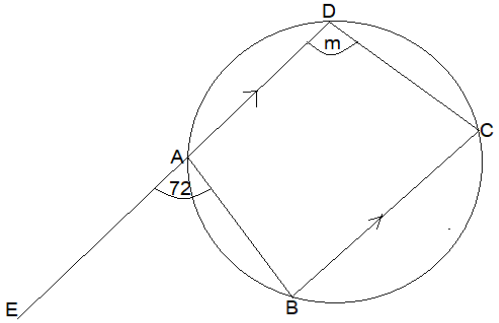

| |
|---|
| $G\hat{B}C = A\hat{D}C$ |
| $F\hat{A}B = B\hat{C}D$ |
| $D\hat{C}H = A\hat{B}D$ |
| $A\hat{D}E = A\hat{B}C$ |

**Exercise 05**

(01) $PQRS$ is a cyclic quadrilateral. Find the values of $a$ and $b$




(02) $ABCD$ is a cyclic quadrilateral. $BC // DE$ and $B\hat{A}E = 72^0$. Find the value of $m$

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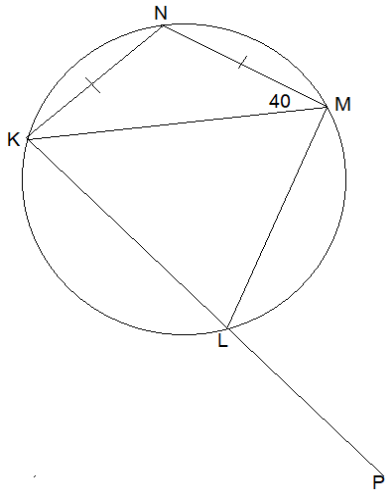
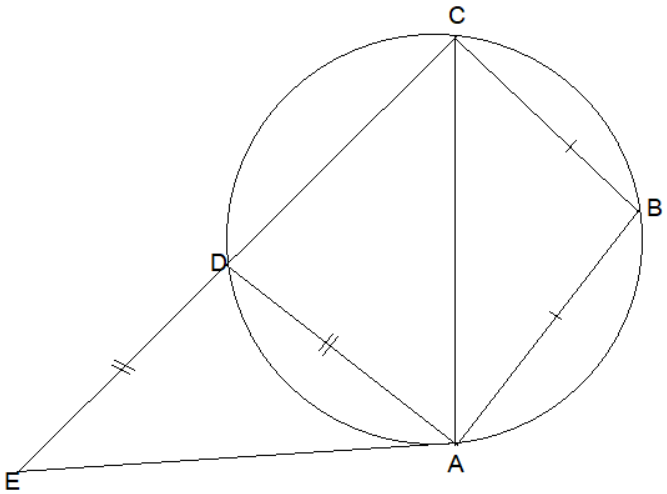
(03) $KLMN$ is a cyclic quadrilateral. Find the value of $M\hat{L}P$ .




04) $ABCD$ is a cyclic quadrilateral. $CD$ is extended up to $E$ such that $AB = BC$ and $AD = DE$ . Also $B\hat{A}C = 35^0$ .

i) Find the value of $D\hat{A}E$

ii) Show that $AE$ and $BD$ lines are parallel

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