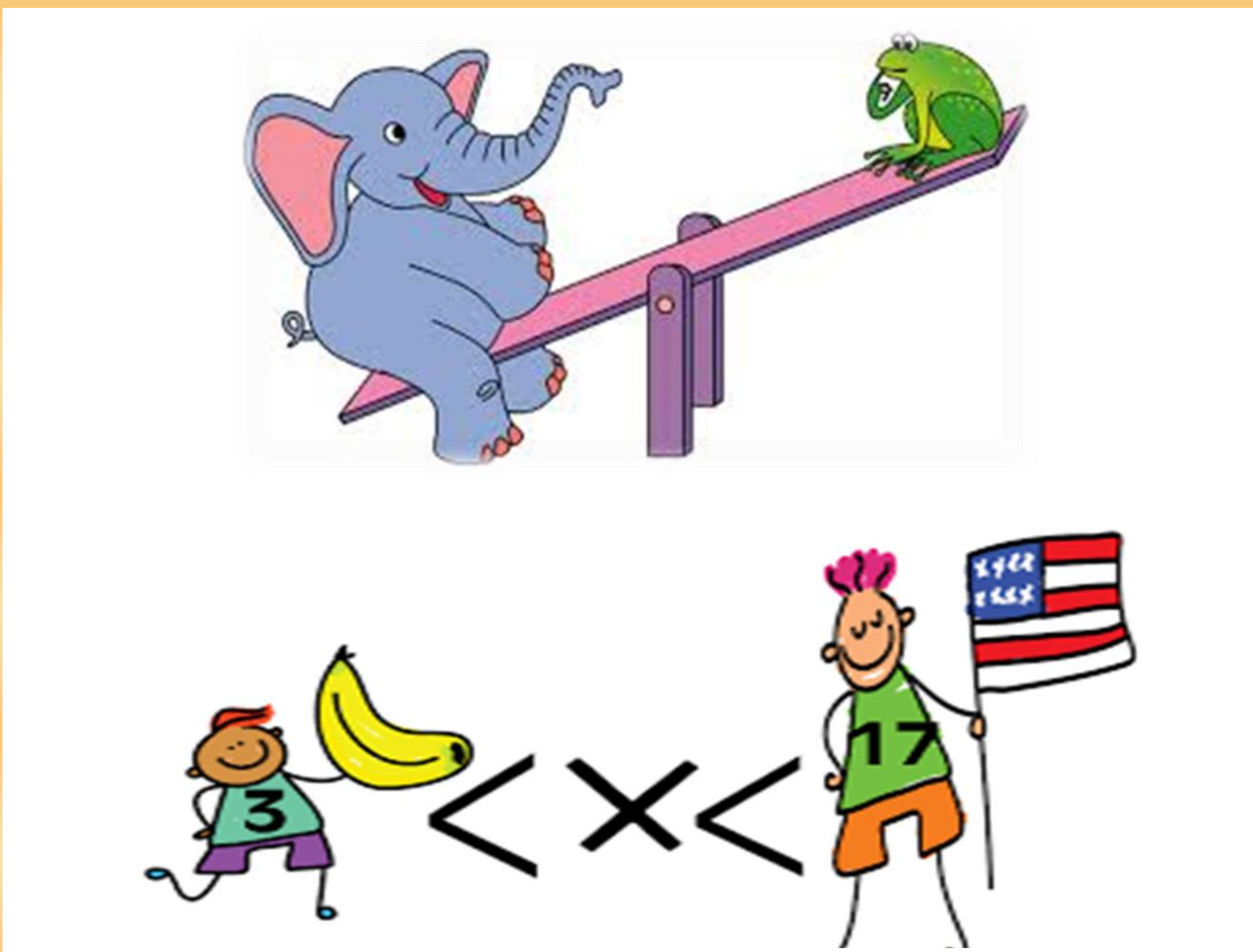
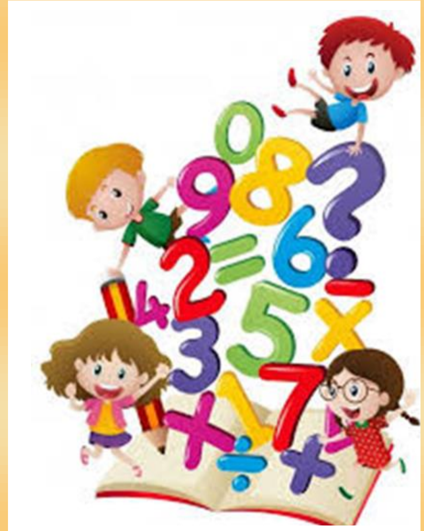
# Grade 11

# 20 Algebraic Inequalities

## At the end of this lesson you will be able to...

- Solve the inequalities of the form $ax \pm b \gtreqless c$ , $ax + b \gtreqless cx + d$ and represent the solutions on a number line.





Let us solve the equation **2x – 3 = 5**

2x – 3 = 5

- By adding 3 to both sides to remove the (-3) in the left side of the equation,

2x -3 + 3 = 5 + 3

2x = 8 (because -3 +3 = 0 and 5 + 3 = 8)

- By dividing both sides by 2 to remove the ×2 in the left side of the equation,

$\frac{2x}{2} = \frac{8}{2}$

x = 4

2x – 3 = 5 is an equation. When applying sign of “<” or “>” instead of the sign of “=” of the equation, it can be written as

2x – 3 < 5 or 2x – 3 > 5.

Then,

2x – 3 < 5 is an inequality.

2x – 3 > 5 is an inequality.

These inequalities also can be solved as the above equation.

**Let us solve the inequality 2x – 3 < 5.**

2x – 3 < 5

- By adding 3 to both sides to remove the (-3) in the left side of the inequality,

2x -3 + 3 < 5 + 3

2x < 8 (because -3 +3 = 0 and 5 + 3 = 8)

- By dividing both sides by 2 to remove the ×2 in the left side of the inequality,

$\frac{2x}{2} < \frac{8}{2}$

**x < 4**

**Let us solve the inequality 2x – 3 > 5.**

2x – 3 >5

- By adding 3 to both sides to remove the (-3) in the left side of the inequality,

2x -3 + 3 > 5 + 3

2x > 8 (because -3 +3 = 0 and 5 + 3 = 8)

- By dividing both sides by 2 to remove the ×2 in the left side of the inequality,

$\frac{2x}{2} > \frac{8}{2}$

**x > 4**

## 20.1 Solving the inequalities of the form $ax \pm b \gtreqless c$

**Example 01**

Consider the inequality 2x - 1 > 3.

i. Solving the inequality.

By adding 1 to both sides to remove the (-1) in the left side of the inequality

2x - 1 + 1 > 3 +1

2x > 4 (because -1 + 1 = 0 and 3 + 1 = 4)

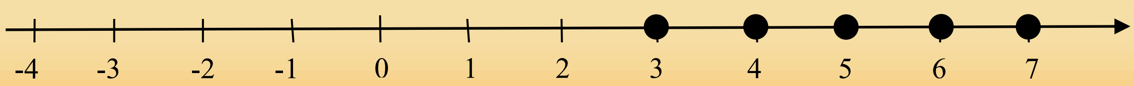
By dividing both sides by 2 to remove the ×2 in the left side of the inequality

$\frac{2x}{2} > \frac{4}{2}$

x > 2

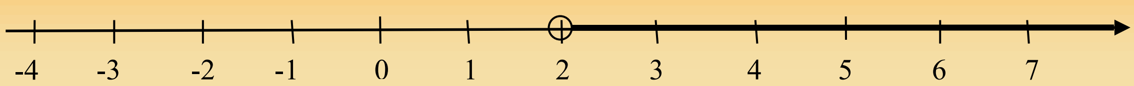
ii. Representing integer solutions on a number line.

Integers greater than 2 are relevant for the inequality $x > 2$, but 2 is not belongs to it. Therefore integers relevant for the inequality $x > 2$ are 3, 4, 5, 6, 7 …etc. The above integer values can be represented on a number line as follows.




iii. Representing solutions on a number line.

In the inequality $x > 2$ , x cannot take the value 2. Therefore the point $x = 2$ is circled but not shaded (It is represented using the symbol "○") and the line is shades as follows. Then it represents all the values relevant to $x > 2$.




### Example 02

Considering the inequality $5x + 3 < 18$.

i. Solving the inequality $5x + 3 < 18$.

$5x + 3 - 3 < 18 - 3$ (By subtracting 3 to both sides to remove the 3 in the left side of the inequality)

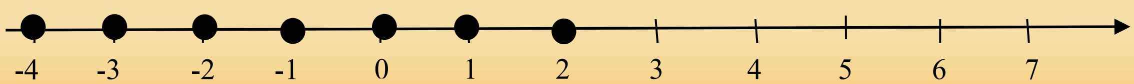
$5x < 15$ (because $+3 - 3 = 0$ and $18 - 3 = 15$)

$\frac{5x}{5} < \frac{15}{5}$ (By dividing both sides by 5 to remove the ×5 in the left side of the inequality)

$x < 3$

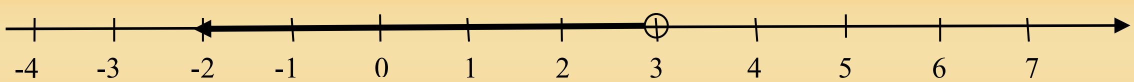
ii. Representing integer solutions on a number line.

Integers less than 3 are relevant for the inequality x < 3, but 3 is not belongs to it. Therefore integers relevant for the inequality x < 3 are 2, 1, 0, -1, -2, etc. The above integer values can be represented on a number line as follows.




iii. Representing solutions on a number line.

In the inequality x < 3, x cannot take the value 3. Therefore the point x = 3 is circled but not shaded (It is represented using the symbol "◯") and the line is shades as follows. Then it represents all the values relevant to x < 3.




### Example 03

Considering the inequality $2 - 3x \geq 5$.

i. Solving the inequality $2 - 3x \geq 5$.

$2 - 3x - 2 \geq 5 - 2$ (By subtracting 2 to both sides to remove the 2 in the left side of the inequality)

$-3x \geq 3$

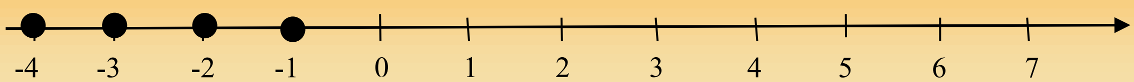
$\frac{-3x}{-3} \leq \frac{3}{-3}$ (By dividing both sides by (-3) to remove the ×(-3) in the left side of the inequality)

$x \leq -1$

❖ **When both sides of inequality is multiplied or divided by a same negative number, the inequality sign should be changed. ( It means the sign of “ >” is changed as “<” and the sign of “<” is changed as “ >”)**

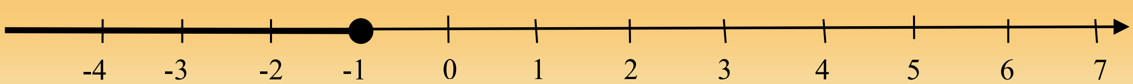
ii. Representing integer solutions on a number line.

In the inequality x ≤ -1, x can take -1 and the integers less than -1.the value of -1 is included for this inequality. Therefore integers relevant for the inequality x ≤ -1 are -1, -2,-3, etc. The above integer values can be represented on a number line as follows.




iii. Representing solutions on a number line.

In the inequality x ≤ -1, x can take -1and the integers less than -1. Therefore the point x = -1 is circled and shaded (It is represented using the symbol “●”) and the line is shaded as follows. Then it represents all the values relevant to x ≤ -1.




## Example 04

Considering the inequality $\frac{x}{2} - 3 \geq -4$ .

i. Solving $\frac{x}{2} - 3 \geq -4$.

By adding 3 to both sides to remove the (-3) in the left side of the inequality,

$\frac{x}{2} - 3 + 3 \geq -4 + 3$

$\frac{x}{2} \geq -1$ (because -3 +3 = 0 and -4 +3 = -1)

*Written by – BML Balasooriya, ISA, Kotawehera Education Division / Translated by – Nirosha Herath, Nk/ Sri Sumangala National School, Wariyapola.*
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$\frac{x \times 2}{2} \geq -1 \times 2$ (By multiplying both sides by 2 to remove ÷2 in the left side of the inequality)

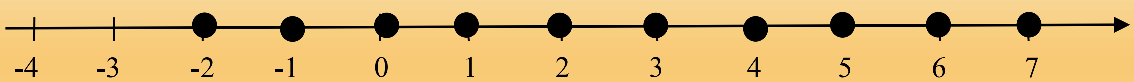
$x \geq -2$

ii. Representing integer solutions on a number line.

In the inequality $x \geq -2$ , x can take the value -2 and the integers greater than -2.

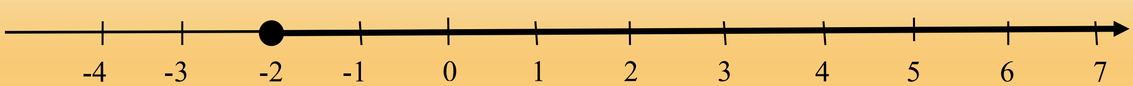
∴ The integer values relevant for the inequality $x \geq -2$ are -2, -1, 0, 1, 2, 3 ...etc.

The above integer values can be represented on a number line as follows.




iii. Representing solutions on a number line.

The value of -2 is included for the inequality $x \geq -2$.Therefore x = -2 is circled and shaded (it is represented using the symbol "●") and the line is shaded as follows.Then it represents all the values related to $x \geq -2$.



## Example 05

Considering the inequality $2 - \frac{x}{5} < -4$.

i. Solving $2 - \frac{x}{5} < -4$.

$2 - \frac{x}{5} - 2 < -4 - 2$ (By subtracting 2 to both sides to remove the 2 in the left side of the inequality)

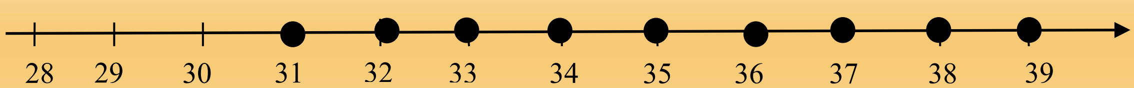
$-\frac{x}{5} < -6$ (because 2 -2 = 0 and -4 -2 =-6 )

By multiplying both sides by (-5) to remove ÷ (-5) in the left side of the inequality,

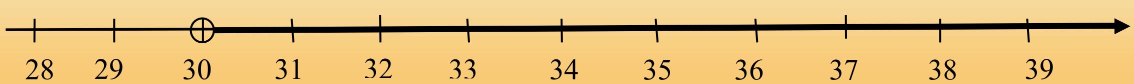
$\frac{-x}{5} \times (-5) > -6 \times (-5)$

x > 30 ( The inequality sign is changed because multiplied by a negative number.)

ii. Representing integer solutions on a number line.




iii. Representing solutions on a number line.




## Exercise 01

- Solve each of the following inequalities.

i. $5x - 1 > 9$

ii. $3 - 2x > 1$

iii. $\frac{3x}{4} + 5 \leq 11$

- Do the revision exercise in the text book.

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## 20.1 Solving the inequalities of the form $ax + b \gtrless cx + d$

### Example 01

Considering the inequality $5x – 4 < 4x$.

i. Solving.

$5x - 4 < 4x$

By subtracting 4x from both sides to remove 4x in the right side of the inequality,

$5x - 4 -4x < 4x – 4x$

$x – 4 < 0$ (because $4x – 4x = 0$ and $5x – 4x = x$)

By adding 4 to both sides to remove -4 in the left side of the inequality,

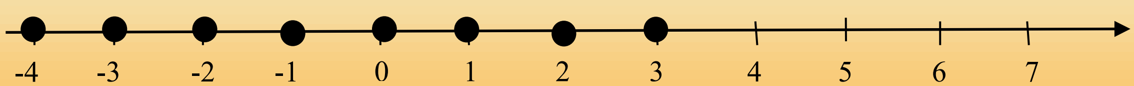
$x – 4 + 4 < 0 + 4$

$x < 4$ (because $- 4 + 4 = 0$ and $0 + 4 = 4$ )

ii. Representing integer solutions on a number line.

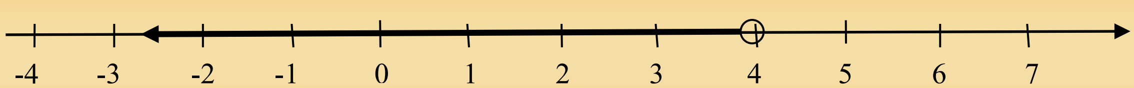
Integers less than 4 are relevant for the inequality $x < 4$, but 4 is not belongs to it.

Therefore the integers relevant for the inequality $x < 4$ are 3, 2, 1, 0, -1 ...etc. The above integer values can be represented on a number line as follows.




iii. Representing solutions on a number line.

4 is not belongs to the set of solutions of the inequality $x < 4$. Therefore the point $x = 4$ is circled but not shaded ( It is represented using the symbol "○") and the line is shaded as follows.





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## Example 02

Considering the inequality $3x - 6 \geq 5x$.

i. Solving $3x - 6 \geq 5x$.

$$3x - 6 \geq 5x$$

By subtracting 5x from both sides to remove 5x in the right side of the inequality,

$$3x - 6 - 5x \geq 5x - 5x$$

$$-2x - 6 \geq 0 \text{ (because } 5x - 5x = 0 \text{ and } 3x - 5x = -2x)$$

By adding 6 to both sides to remove -6 in the left side of the inequality,

$$-2x - 6 + 6 \geq 0 + 6$$

$$-2x \geq 6 \text{ (because } -6 + 6 = 0 \text{ and } 0 + 6 = 6)$$

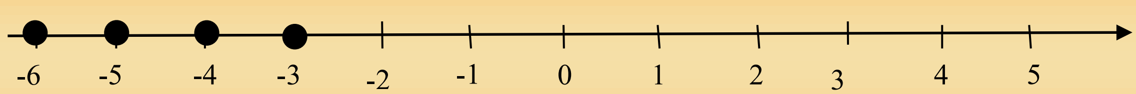
By dividing both sides by (-2) to remove the × (-2) in the left side of the inequality,

$$\frac{-2x}{-2} \leq \frac{6}{-2}$$

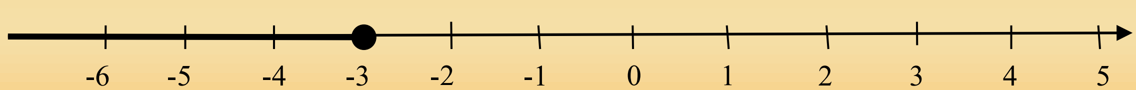
When dividing by a negative number the inequality sign is changed.

$$x \leq -3 \text{(because } \frac{-2}{-2} = 1 \text{ and } \frac{6}{-2} = -3)$$

ii. Representing integer solutions on a number line.




iii. Representing solutions on a number line.

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**Example 03**

Considering the inequality $7 - x \leq 6x$.

i. Solving.

$7 - x \leq 6x$

By subtracting 6x from both sides to remove 6x in the right side of the inequality,

$7 - x - 6x \leq 6x - 6x$

$7 - 7x \leq 0$ (because 6x – 6x = 0 and -x – 6x = -7x)

By subtracting 7 to both sides to remove the 7 in the left side of the inequality,

$7 - 7x - 7 \leq 0 - 7$

$-7x \leq -7$ (because 7 - 7 = 0 and 0 - 7 = -7)

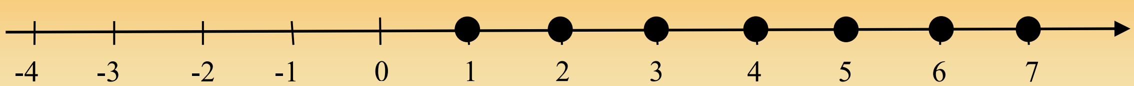
By dividing both sides by (-7),

$\frac{-7x}{-7} \geq \frac{-7}{-7}$

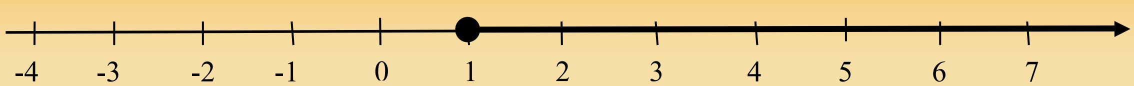
When dividing by a negative number the inequality sign is changed.

$x \geq 1$(because $\frac{-7}{-7} = 1$)

ii. Representing integer solutions on a number line.




iii. Representing solutions on a number line.



## Example 04

Considering the inequality $3x - 2 > 2x + 5$.

i. Solving.

$3x - 2 > 2x + 5$

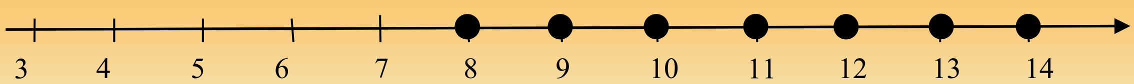
$3x - 2 - 2x > 2x + 5 - 2x$ (By subtracting 2x from both sides of the inequality)

$x - 2 > 5$ (because $2x - 2x = 0$ and $3x - 2x = x$)

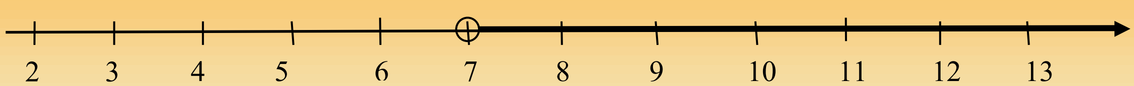
$x - 2 + 2 > 5 + 2$ (adding 2 to both sides of the inequality)

$x > 7$ (because $-2 + 2 = 0$ and $5 + 2 = 7$)

ii. Representing integer solutions on a number line.




iii. Representing solutions on a number line.




## Example 05

Considering the inequality $4x + 1 \leq 7x - 8$.

i. Solving.

$4x + 1 \leq 7x - 8$

$4x + 1 - 1 \leq 7x - 8 - 1$ (By subtracting (-1) from both sides of the inequality)

$4x \leq 7x - 9$ (because $+1 - 1 = 0$ and $-8 - 1 = -9$)

$4x - 7x \leq 7x - 9 - 7x$ (By subtracting 7x from both sides of the inequality)

$-3x \leq -9$ (because $4x - 7x = -3x$ and $7x - 7x = 0$)

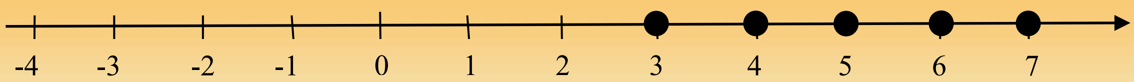
$-3x \leq -9$ (By dividing both sides of the inequality by (-3)),

$\frac{-3x}{-3} \geq \frac{-9}{-3}$

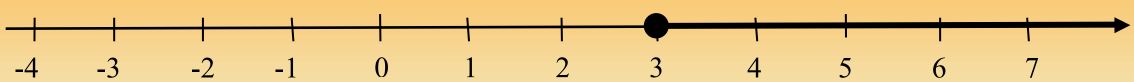
When dividing by a negative number the inequality sign is changed.

$x \geq 3$(because $\frac{-9}{-3} = 3$)

ii. Representing integer solutions on a number line.




iii. Representing solutions on a number line.




## Exercise 02

- By solving each of the following inequalities represent the,
  - Integer solutions on a number line.
  - Solutions on a number line.

I. $2x + 9 \leq 11x$

II. $7x > -12 + x$

III. $5x + 8 \geq 20 - x$

IV. $x - 2 < 3x$

V. $12 - 3x < 9$

- Do the exercise 20.1 in the text book.