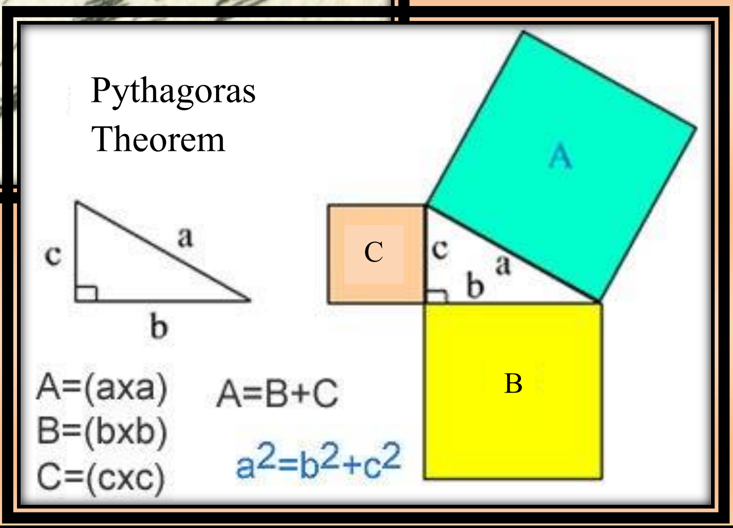
Grade 11

17 Pythagoras Theorem

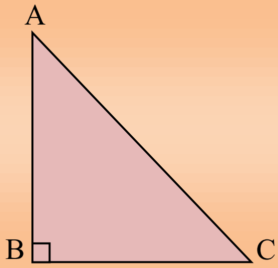
At the end of this lesson, you will be able to …

- Name the hypotenuse and the sides that include the right angle of a right angled triangle separately.
- Identify Pythagoras Theorem.
- Engage in calculations using the Pythagoras Theorem.
- Prove riders using Pythagoras Theorem.
- Concern about Pythagoras triples with whole numerals.



17.1 Introduction

In the century of 6 B.C a Greek mathematician named Pythagoras indroduced a wonderful relationship between the sides of a right angled triangle. Let us understand this relationship.



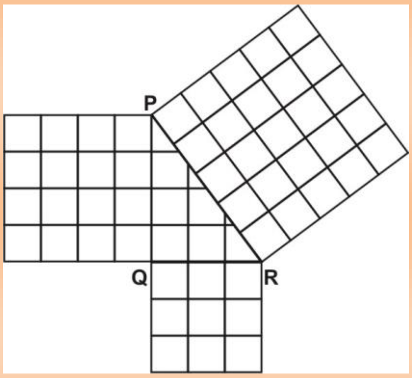
In the triangle given in the figure, the angle B is a right angle. AC is the hypotenuse of it. Hypotenuse is the side which is opposite the right angle and which is longest side of the triangle. AB and BC are the two sides which include the right angle.

After cutting three squares and a triangle using a paper with 1 cm^2 squares, the way of pasting them on a paper is shown in the figure.

The area of the square on PR = $25cm^2$,

The area of the square on QR = $9cm^2$,

The area of the square on PQ= $16cm^2$



Accordingly, the area of the square on PR is equal to the sum of the areas of the squares on QR and PQ. When taken as,

The area of the square on PR = PR^2

The area of the square on QR = QR^2

The area of the square on PQ = PQ^2

Then, $PR^2 = QR^2 + PQ^2$.

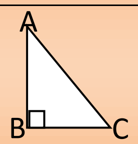
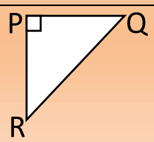
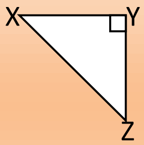
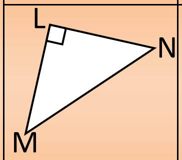
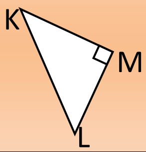
The above relationship is the wonderful relationship between the sides of a right angled triangle discovered by the mathematician named Pythagoras. That relationship is expressed as a theorem as follows.

Pythagoras Theorem:-

In a right angled triangle, the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on the other two sides which contain the right angle.

Exercise 1

Complete the table given below.

Triangle	Hypotenuse	Sides that include the right angle	Pythagoras relationship
	AC	AB, BC	$AC^2 = AB^2 + BC^2$
			
			
			
			

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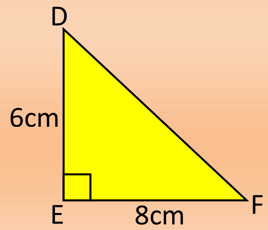
17.2 Calculations using Pythagoras Theorem

Example 1

Using the measurements given in the figure, find the length of the side DF.



$DF^2 = DE^2+EF^2$

$DF^2 = 6^2+8^2$

$DF^2 = 36+64$

$DF^2 = 100$

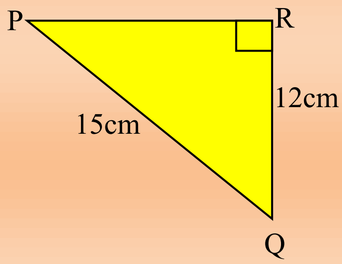
$DF = \sqrt{100}$

$DF = 10$

$\therefore$ The length of the side DF = 10cm

Example 2

Find the length of the side PR.



$PQ^2 = PR^2+RQ^2$

$15^2 = PR^2 + 12^2$

$15^2-12^2 = PR^2$

$225-144 = PR^2$

$81 = PR^2$

$\sqrt{81} = PR$

$9 = PR$

$\therefore$ The length of the side PR = 9cm

It is possible to find the value of 15^2- 12^2, by taking as the difference of two squares.

$(15 - 12)(15+12) = 3\times27 = 81$

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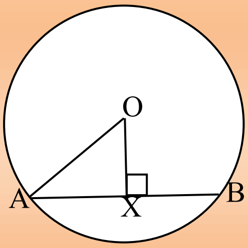
Example 3

O is the centre of the circle given in the figure. AB = 16 cm and the radius of the circle is 10 cm. Find the length of OX.



Because OX is perpendicular to the chord AB, AX=XB.

Therefore AX = 8cm.

By applying the Pythagoras Theorem to the triangle AOX,

$OA^2 = OX^2+AX^2$

$10^2 = OX^2+8^2$

$10^2-8^2 = OX^2$

$100-64 = OX^2$

$36 = OX^2$

$\sqrt{36} = OX^2$

$6 = OX$

$\therefore OX = 6cm$

Example 4

Find the length of the side RS.

$RT^2 = RS^2+ST^2$

$(\sqrt{11})^2 = RS^2+(\sqrt{7})^2$

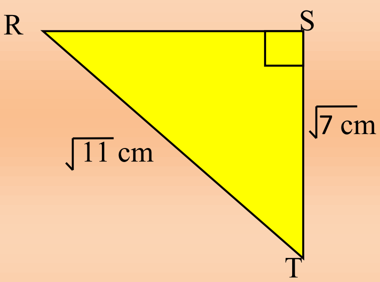
$11 = RS^2+7$

$11-7 = RS^2$

$4 = RS^2$

$\sqrt{4} = RS$

$RS = 2$



$\sqrt{11}^2$ can be taken as,

$\sqrt{11}^2 = \sqrt{11}\times\sqrt{11} = \sqrt{121} = 11$

Or

$\mathbf{11^{(1/2)^2} = 11}$

$\therefore$ The length of the side RS = 2cm

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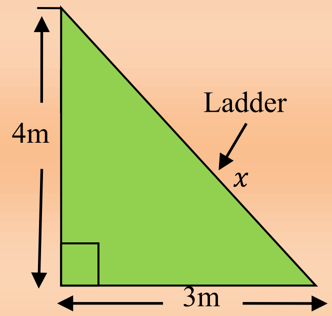
Example 5

A ladder is kept against a vertical wall as shown in the figure. The ladder is touching the wall from 4m away the bottom of the wall. Find the length of the ladder.



$\therefore x^2 = 4^2 + 3^2$

$x^2 = 16+9$

$x^2 = 25$

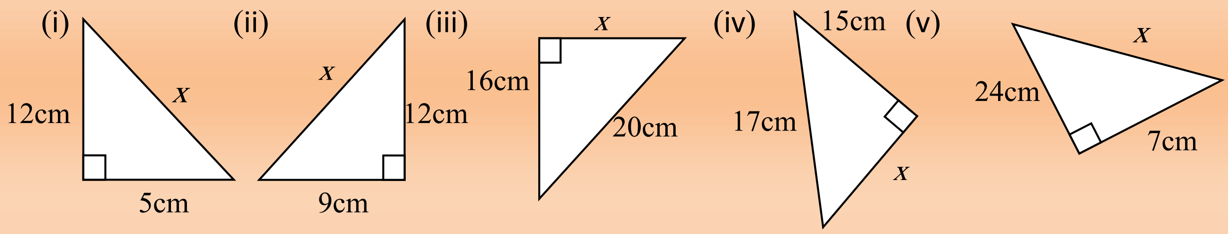
$x = \sqrt{25}$

$x = 5$

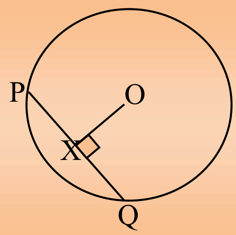
$\therefore$ The length of the ladder = 5m

Exercise 2

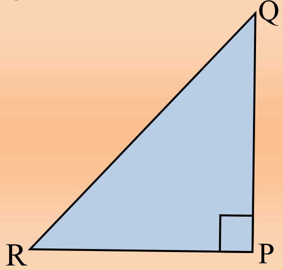
1. In each of the triangles given below, find the length of the sides indicated by x



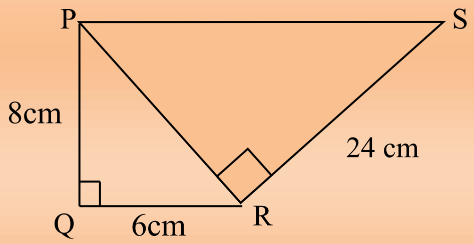
2. If the length of the chord PQ = 8cm and OX = 3cm of the circle with centre O. Find the radius of the circle.



3. The height of the vertical post PQ which is fixed on a horizontal ground is 8m. A supporting cable is tightly tied from the top of the post to the wedge R which is situated 6m away from the bottom P of the post. Find the length of the cable QR.



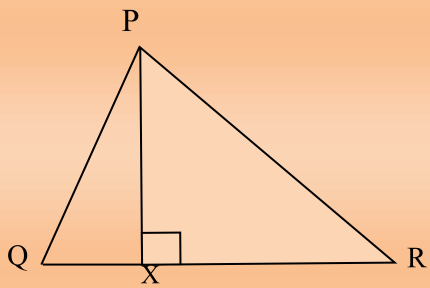
4. Find the length of PS.



- Do the Exercise 17.1 in the text book.

17.3 Proving riders using Pythagoras Theorem

Example 1



In the triangle PQR the perpendicular drawn from P to the side QR is PX.

Show that $PQ^2 + XR^2 = PR^2 + QX^2$.

Firstly let us write an equation in terms of PQ^2 by applying the Pythagoras Theorem to the triangle PQX.

$$PQ^2 = PX^2 + QX^2 \quad \text{—— ①}$$

Then let us write an equation in terms of XR^2

$$XR^2 = PR^2 - PX^2 \quad \text{—— ②}$$

Now let us add ① & ②

$$PQ^2 + XR^2 = PX^2 + QX^2 + PR^2 - PX^2$$

Because 08 $PX^2 - PX^2 = 0$(zero)

$$PQ^2 + XR^2 = QX^2 + PR^2$$

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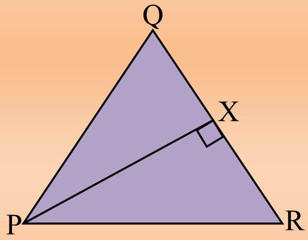
Example 2

In the equilateral triangle PQR, PX is the perpendicular drawn from P to QR. Show that $PX^2 = \frac{3}{4}QR^2$.



Since PQR is an equilateral triangle PQ = PR = QR.

Because the perpendicular drawn from P, QX = XR.

Since one side of the equation that should be proved is PX^2, let us obtain an equation in term of PX^2 by applying the Pythagoras Theorem to the triangle PXQ.

$$PQ^2 = PX^2 + XQ^2$$

Since QX = 1/2 QR , let us substitute 1/2 QR instead of QX in the above equation.

$$PQ^2 = PX^2 + (1/2\ QR)^2$$

$$PQ^2 = PX^2 + 1/4\ QR^2$$

Since PQ = QR

$$QR^2 = PX^2 + 1/4\ QR^2$$

$$QR^2 - 1/4\ QR^2 = PX^2$$

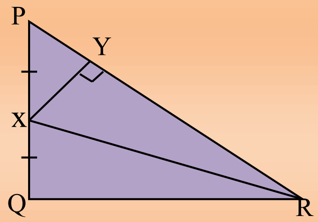
Since $QR^2 - 1/4\ QR^2 = 3/4\ QR^2$

$$PX^2 = 3/4\ QR^2$$

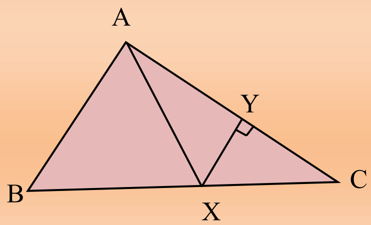
The above examples are written in details to make it easier for you to understand, but understand the steps of proofing properly and practice to doing it in the best way you can.

Exercise 3

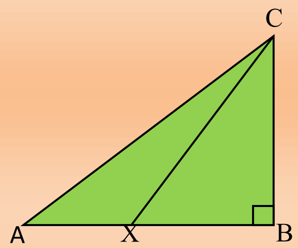
1) X is the midpoint of PQ. Show that $RY^2 - PY^2 = QR^2$.



2) In the triangle ABC, the point X is situated on the side BC such that AX = BX. XY is the perpendicular drown from X to AC. Show that ,
$AY^2 + XC^2 = BX^2 + YC^2$.



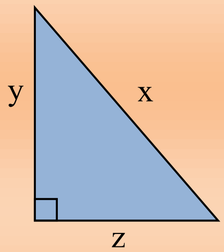
3) In the right angled triangle ABC, X is situated on AB such that
AX = 1/2 XB. Show that $AC^2 - XC^2 = 5AX^2$.



- Do the exercise 17.2 in the text book.

17.4 Pythagorean Triples

3 numbers that are satisfied the Pythagoras theorem are called as Pythagorean triples.



The lengths of the three sides of the triangle given in the figure are represented as x, y, z. According to the Pythagoras theorem, $x^2 = y^2 + z^2$.

Values of x, y and z which satisfy the above equation are known as Pythagorean triples.

Since $3^2 + 4^2 = 5^2$, (3,4,5) is a Pythagorean triple. Any multiple of the triple (3, 4, 5) is also a Pythagorean triple. Accordingly, (6, 8, 10), (9,12,15)..... are also Pythagorean triples. There are Pythagorean triples apart from it. 5,12,13 is such a triple. Any multiple of it is also a Pythagorean triple.

A method used to find Pythagorean triples introduced by a mathematician named Euclid is given in the following table. Study well how to obtained the first triple and obtain the rest triples by filling in the blanks in the table.

x	y	x^2	y^2	a	b	c	Pythagorean triple
				$x^2 - y^2$	$2xy$	$x^2 + y^2$	
2	1	4	1	3	4	5	3, 4, 5
3	2						
4	2						
5	3						
6	2						
7	3						

- Do the exercise 17.3 in the text book.