

25 PROBABILITY

By learning this lesson, You will be able to . . .

- ❖ Describes the nature of a dependent event.
- ❖ Provides examples of dependent events.
- ❖ Distinguishes and identifies dependent and independent events.
- ❖ Represents the sample space of a random experiment involving dependent events on a grid.
- ❖ Writes down the probability of a given event using the representation on a grid of the sample space of a random experiment involving dependent events.
- ❖ Solves problems involving dependent events using a grid.
- ❖ Represents on a tree diagram, all the possible events of a random experiment involving 2 stages of dependent events.
- ❖ States that the sum of all the probabilities on the branches of a tree diagram is 1 (one) for each stage.
- ❖ Solves problems involving dependent events using a tree diagram.

Introduction

Using a **grid or a tree diagram**, to solve problems related to **probabilities of events in a random experiment**

Common characteristics of random experiments.

- ❖ All the possible outcomes of the experiment are known before the experiment is carried out.
- ❖ The outcome of the experiment cannot be stated with certainty before the experiment is carried out.
- ❖ The experiment can be repeated any number of times under the same condition.
- ❖ When the experiment is repeated, a pattern cannot be recognized in the outcomes.

For each of the following experiments, in the column to the right, mark “ ✓ “ if it is a random experiment and “ X “ if it is not a random experiment.

Experiment	Random / Not random
i. Tossing a coin and observing the side that turns up. ii. Rolling an unbiased die with its faces numbered from 1 to 6 and recording the number on the face that turns up. iii. Drawing a pen from a bag which contains pens of same type. iv. Choosing a leader of a group of students by voting v. Shooting at a target. vi. Choosing a leader of a group of students by drawing lots. vii. Tossing a ball in the air and observing whether it falls to the ground.	

Sample space – The set which consists of all the possible outcomes of a random experiment is called its sample space.

Events – An event is a any subset of the sample space.

Probability of a selected event $P(A) = \frac{\text{Number of elements in the event } n(A)}{\text{Number of elements in the sample space } n(S)}$

Consider the experiment of observing the side, that turns up when an unbiased coin is tossed once.

- (i) Write the sample space of this experiment and find $n(s)$.
- (ii) If the event A is “head” turns up, write the elements in A and find $n(A)$
- (iii) Find $p(A)$, the probability that heads turns up.

Random experiment	Sample space (S)	Some selected events	Probability of the selected event
01. Tossing an unbiased coin once	$S = \{\text{Head, tail}\}$	$A = \{\text{Head turns up}\}$ $B = \{\text{Tail turns up}\}$	$P() = \frac{()}{()} = -$
02. Rolling an unbiased die with its faces numbered from 1 to 6.	$S = \{1,2,3,4,5,6\}$	$A = \{\text{Getting 3}\}$ $B = \{\text{Getting an odd number}\}$ $C = \{\text{Getting a number greater than 1}\}$	$P() = \frac{()}{()} = -$
03. Drawing a pen from a bag which contains 3 red pens and 2 blue pens.	$S = \{R_1, R_2, R_3, B_1, B_2\}$	$A = \{\text{Drawing a red pen}\}$ $B = \{\text{drawing a blue pen}\}$	$P() = \frac{()}{()} = -$
04. When a coin is tossed twice	$S = \{(T,T), (T,H), (H,T), (H,H)\}$	$A = \{\text{Getting head in both occasions}\}$ $B = \{\text{Getting the same side in both occasions}\}$	$P() = \frac{()}{()} = -$
05. When an unbiased die is tossed twice	$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), \dots\}$	In this occasion a grid is used to represent the sample space.	

Solving problems using a grid (Cartesian plane)

A grid is used to find the probability of a random experiment with many stages. A grid is used for a random experiment with two stages.

It is necessary to have a clear understanding about the independent events and dependent events to solve the problem related to probabilities, using the grid.

Independent events:-

If the occurrence of an event does not depend on the occurrence of another event, then the two events are **independent**.

Dependent events :-

If the occurrence of event depends on the occurrence of another event, then they are called. **dependent** events.

When an unbiased die numbered from 1 to 6 is tossed twice.

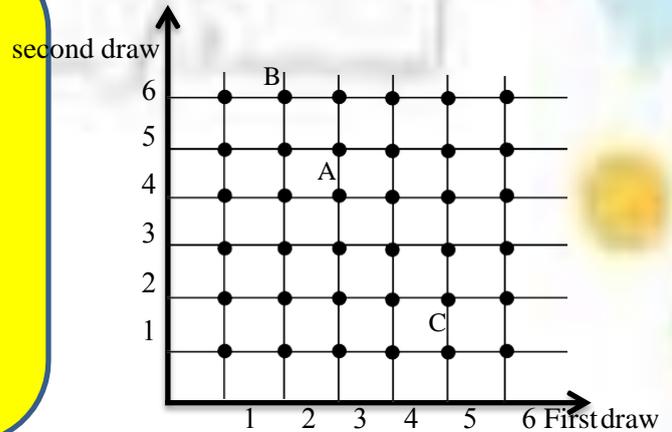
A grid is used to represent the sample space of two random experiments.

In this grid,

Point A represents :- Getting 3 in the first draw & getting 4 in the second draw.

Point B represents :- Getting 2 in the first draw & getting 6 in the second draw.

Point C represents :- Getting 5 in the first draw & getting 1 in the second draw.



Example 1

A box contains 2 red pens and 3 blue pens, all of which are otherwise identical. A pen is drawn out randomly and its colour is recorded.

After **replacing** it, a pen is randomly drawn out again and its colour is recorded.

Find the probability of the following events.

- The first pen being red
- The second pen being red.
- Both of the pens being blue
- Both of the pens being the same colour
- At least one pen being red

Answers

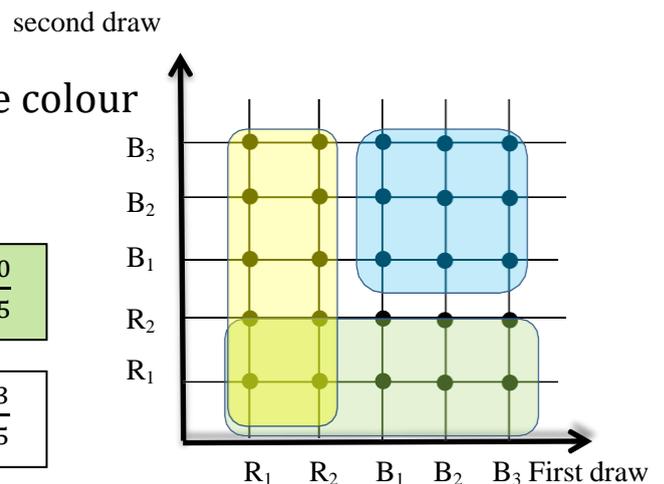
$$(i) \frac{10}{25}$$

$$(ii) \frac{10}{25}$$

$$(iii) \frac{9}{25}$$

$$(iv) \frac{13}{25}$$

$$(v) \frac{13}{25}$$



Example 2

A box contains 2 red pens and 3 blue pens, all of which are otherwise identical. A pen is drawn out randomly and its colour is recorded.

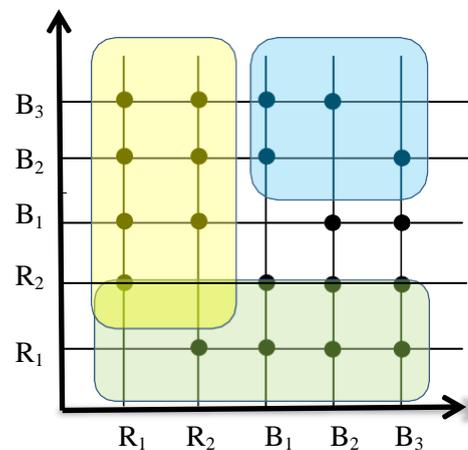
Without replacing the first pen, a second pen is randomly drawn out and its colour is recorded.

Find the probability of the following events.

The number of pens in the second draw is reduced by one because of the without replacement of the first draw.

- The first pen being red.
- The second pen being red
- Both of the pens being blue.
- Both of the pens being the same colour
- At least one pen being red.

Second draw



Answers

$$(i) \frac{8}{20}$$

$$(ii) \frac{8}{20}$$

$$(iii) \frac{6}{20}$$

$$(iv) \frac{8}{20}$$

$$(v) \frac{14}{20}$$

Refer exercise 25.1 in the Grade 11 Text book.

Solving problems using tree diagrams.

A tree diagram can be used to find the probabilities of events related to a random experiment having many stages. Let's consider only random experiments having two stages.

Example 1

A box contains 2 red pens and 3 blue pens, all of which are otherwise identical. A pen is drawn out randomly and its colour is recorded. After **replacing** it, a pen is randomly drawn out again and its colour is recorded.

Find the probability of the following events.

- The first pen being red
- The second pen being red.
- Both of the pens being blue
- Both of the pens being the same colour
- At least one pen being red.

Answers

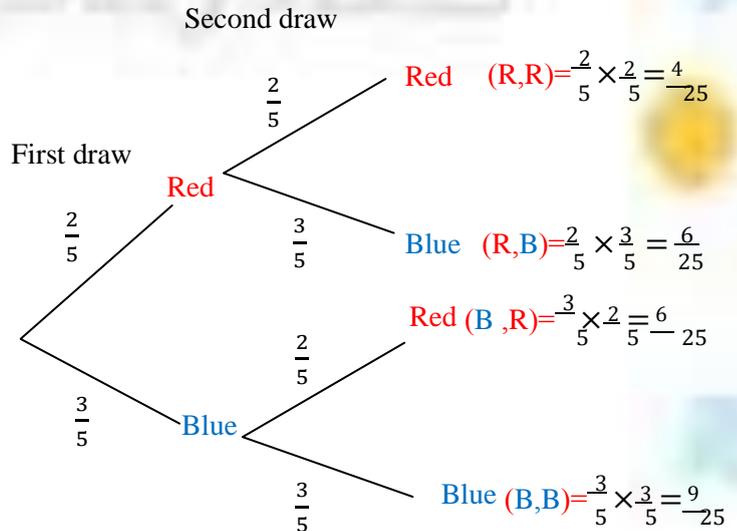
(i) $\frac{10}{25}$

(ii) $\frac{10}{25}$

(iii) $\frac{9}{25}$

(iv) $\frac{13}{25}$

(v) $\frac{16}{25}$



Example 2

A box contains 2 red pens and 3 blue pens, all of which are otherwise identical. A pen is drawn out randomly and its colour is recorded. **Without replacing** the first pen, a second pen is randomly drawn out and its colour is recorded.

Find the probability of the following events.

- The first pen being red.
- The second pen being red
- Both of the pens being blue.
- Both of the pens being the same colour
- At least one pen being red.

Answers

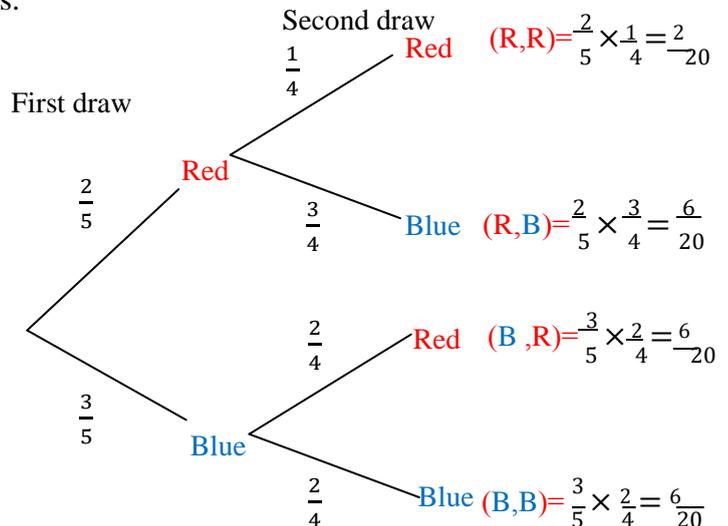
(i) $\frac{8}{20}$

(ii) $\frac{8}{20}$

(iii) $\frac{6}{20}$

(iv) $\frac{8}{20}$

(v) $\frac{14}{20}$



Refer exercise 25.2 in the Grade 11 Text book.