Grade 9

MATHEMATICS

Competency –
31. Analyzes the likelihood of an event occurring to predict future events.

Competency Level –
31.1 Investigates the likelihood of an event by considering the outcomes of the experiment.

Unit – 24. Probability

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Probability

Random Experiments

Pay your attention for the following conversation

Samath: Imashi, Kavin, Nasli, Rehan… as all of you came today, shall we play the Elle game ?

Kavin: wow… it is a good idea.

Imashi: Yes, I like that idea too, then shall we divide into two teams?

Samath: First, two of us should be the leaders and then the others can separate into the two teams. Nasli and Rehan will be the two leaders of the teams.

Imashi: Good, now who is batting first?

Nasli: We…

Rehan: No… We…

Kavin: Wait… do not fight for it. Let us toss a coin and see who will be going to bat first. Now I am going to toss the coin, who is requesting the side?

Nasli: I am requesting to have the tail.

Kavin: It is tail, so the team of Nasli can bat first.

Rehan: No… it is unfair. She requested before the coin is tossed.

Kavin: Ok, then I will toss the coin again, Nasli, you can request the side.

Nasli: Give me the tail.

Kavin: This time, the side is Head, so Rehan's team can bat first.

You might come across with such occasion in the life. When tossing a coin, we cannot predict which side that we will get from the Head and the Tail. We can repeat this for several times. We cannot see a pattern in the result that we gain.

We consider only the experiments or observations where we know the set of possible outcomes but cannot predict a particular outcome which can be repeated for several times under the same condition as a random experiment.

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common properties of a random experiment

- The experiment can be repeated any number of times under the same condition.
- All the possible outcomes of the experiment are known before the experiment is carried out.
- The outcome of the experiment cannot be stated with certainty before the experiment is carried out.
- When the experiment is repeated, a pattern cannot be recognized in the outcomes.

Now let us consider about another example.

You have to take out an ice-cream randomly, from a box which contain three ice-creams of three different colors where we cannot see the things inside. Does this occurrence include the properties of a random experiment?

Exercise 01

Select the random and not random experiments from the given occurrences.

Experiment	Random / Not random
Selecting a card from a pack of cards with 52 cards.	
Observing the side which turned upward when tossing an unbiased die.	
Taking out a ball randomly from a bag which contains glass balls with the same size and shape.	
Observing whether the sum of two odd numbers is an odd number.	
Expressing the number of goals obtained by the each team in a football match.	
Rising of the sun from east on every day of the week.	

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Sample space

Example 01 –
When we spin this wheel once and noticed the location of the indicator, what can be the color?

It can be any color from the five colors in this wheel.
{Yellow, Pink, Red, Green, Blue}

Example 02 –

When we spin this wheel once and noticed the location of the indicator, what can be the color?
It can be any color from the three colors in this wheel.
{Red, Yellow, Blue}

Example 03 –

What can be the value that we can see in the top of the die when we toss an unbiased die numbered its sides from 1 – 6
{1, 2, 3, 4, 5, 6}

All the possible outcomes of a random experiment can be written as a set. This set is called the sample space and it is usually written by S.
The number of elements of is denoted by n(S).

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- Sample space related to Example – 01
  S = {Yellow, Pink, Red, Green, Blue}
  n(S) = 5
- Sample space related to Example – 02
  S = {Red, Yellow, Blue}
  n(S) = 3
- Sample space related to Example – 03
  S = {1, 2, 3, 4, 5, 6}
  n(S) = 6

Let us consider another example

In a transparent container there are two beads in the colors of Black and White. Take one ball randomly from it,

- mark the color,
- put it again to the container and
- take another bead from the bag. The relevant sample space for this experiment is,
  B – getting a black ball
  W – getting a white ball

S = {(B, W), (B, B), (W, W), (W, B)}

Something to think …

When tossing 2 unbiased dice with the faces marked from 1 – 6, how many elements will be there in the relevant sample space?

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Exercise 02

1. Write the sample space and the number of elements of the random experiment of tossing an unbiased coin.
2. Write the sample space of tossing two coins.
3. In a bag there are three balls of a Blue, a Red and a Yellow with the same size and the shape. Write the sample space of ta king a ball out randomly from the bag. How many elements are there in it?
4. In a bag there are three balls of a Blue, a Red and a Yellow with the same size and the shape.
   - Take a ball out from the bag,
   - mark its color,
   - put it again to the bag,
   - then take another ball from the bag. Write the relevant sample space of the experiment. How many elements are there in it?

Equally likely outcomes

An object when the material it is made of is uniformly distributed throughout the object then we call it a fair object. When the sample space of a random experiment is considered, if each out-come is equally likely to occur,

then the experiment is called an experiment with the equally likely outcomes.

Let us consider about the following situations

- **Tossing an unbiased cubical die.**
  Since, all the faces are equal in shape and area, there is an equal possibility to obtain any of the side in the die. So the outcomes of this experiment are equally likely events.
- **Taking a card out randomly from a card pack.**
  As we can assume that all the cards in the card pack are identical, we can consider the outcomes of the experiment are equally likely.

- **Tossing a match box.**
  Match box is in the shape of a cuboid. All the sides of it are not equal in size. When tossing a match box, the possibility of getting any side is not equal. Therefore, the outcomes are not considered as equally likely events.
- **Taking a chocolate biscuit from the following bottle without looking to that bottle.**
  We can see that the number of chocolate biscuits is less than the number of vanilla biscuits. Therefore, there is a less possibility to get a chocolate biscuit from the bottle than that of a vanilla biscuit. So the outcomes of this experiment are not equally likely.

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Exercise 03

State whether the following experiments are with equally likely outcomes or not.

1. Tossing an unbiased die
2. Taking out a ball randomly from a bag with 5 balls which are equal in shape and size.
3. The indicator will stop at Blue when spinning the wheel shown in the diagram once.
4. Taking a ball out randomly from the container shown.
5. The marshmallow taken out from the container is being in the color of pink.

Probability of an event when the outcomes are equally likely

Let us consider the experiment of tossing an unbiased die
Number of times that we can obtain the side 2 = 1
Total number of outcomes in the sample space = 6
Probability of getting 2 = $\frac{1}{6}$

$$= \frac{\text{Probability of a selected outcome 1}}{\text{total number of outcomes in the sample space of the random experiment}}$$

Now let us consider about obtaining an odd number when tossing an unbiased die.

There are three odd numbers in the die. The probability of obtaining an odd number is $\frac{3}{6}$

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The probability of an event in the sample space of a random experiment with equally likely outcomes depends on two different factors.

Number of elements in the event

Number of elements in the sample space

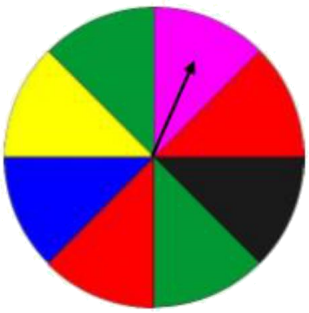
Probability of the event = $\frac{Number\ of\ elements\ in\ the\ event}{Total\ number\ of\ elements\ in\ the\ event}$

We can write it by using symbols as follows.

When the number of elements in the sample space S is n(S), and the number of elements of the event a is n(A), then the probability of occurring the event A is given by p(A).

$$p(A) = \frac{n(A)}{n(S)}$$

Example 01



Calculate the probability of stopping the indicator in the color of Red when spinning the wheel once.

The wheel has been divided into 8 equal parts and there are 2 red parts in the wheel

If the probability of indicating the Red is $p(A)$

$$p(A) = \frac{2}{8}$$

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Exercise 04

1. Isira takes a card randomly from a set of cards with the same size and the shape numbered as 1,4, 5, 6 and 9. Find the probability of that card having an even number.
2. When tossing an unbiased die numbered from 1 – 6, find the probability of getting 7.
3. The letter selected from the set of letters of the word "MATHEMATICS" is being "A"
4. There are six grape flavored toffees, four orange flavored toffees and three cherry flavored toffees in the following container. Answer the questions given below related to that container of toffees.

   I. Find the following probabilities
      II. selecting a cherry flavored toffee
      III. selecting an orange flavored toffee
      IV. selecting a grape flavored toffee
   II. Which flavored toffees do have the highest probability of taking out?
   III. Obtaining what flavor of toffee is having the least probability, of taking out
   IV. Calculate the probability of getting a cherry or grape flavored toffee.
   V. One of your friends needed to have toffee with the flavor of cherry or orange. When considering the probability of obtaining a toffee, which flavor have the most probability to gain out of those two flavors.
   VI. After having three orange flavored toffees, two cherry flavored toffees and 4 grape flavored toffees, which flavor of the remaining toffees have the highest probability to gain.

Remember,

In an experiment with equally likely outcomes,

$$\text{Probability of a selected outcome} = \frac{1}{\text{total number of outcomes in the sample space of the random experiment}}$$

$$\text{Probability of the event} = \frac{\text{Number of elemnts in the event}}{\text{Total number of elements in the event}}$$

$$p(A) = \frac{n(A)}{n(S)}$$

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Answers

Exercise 01

Experiment	Random / Not random
Selecting a card from a pack of cards with 52 cards.	Random
Observing the side which turned upward when tossing an unbiased die.	Random
Taking out a ball randomly from a bag which contains glass balls with the same size and shape.	Random
Observing whether the sum of two odd numbers is an odd number.	Not random
Expressing the number of goals obtained by the each team in a football match.	Random
Rising of the sun from east on every day of the week.	Not random

Exercise 02

1. S = {Head, Tail}, n(S) = 2
2. H – getting Head
   T – getting Tail
   S = {(H, H), (H, T), (T, H), (T, T)}

3. S = {Blue, Red, Yellow}, n(S) = 3
4. B – getting the Blue ball
   R – getting the Red ball
   Y – getting the Yellow ball
   S = {(B, B), (B, R), (B, Y), (R, B), (R, R), (R, Y), (Y, B), (Y, R), (Y, Y)}, n(S) = 9

Exercise 03

1. equally likely events
2. equally likely events
3. not equally likely events
4. not equally likely events
5. not equally likely events

Exercise 04

(1) $\frac{2}{5}$ (2) 0 (3) $\frac{1}{4}$

(4) (i) a. $\frac{3}{13}$ b. $\frac{4}{13}$ c. $\frac{6}{13}$ (ii) Grape toffee (iii) Cherry toffee (iv) $\frac{9}{13}$

(v) Orange toffee (vi) Grape toffee

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