22 Inequalities

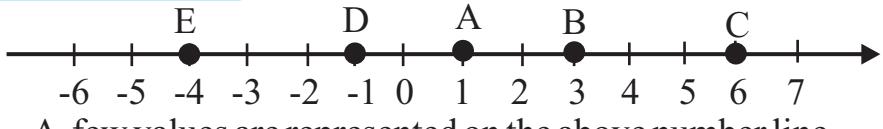
After studying this chapter you will be able to get a good understanding of,

- ★ solving inequalities of the form $ax > b$.
- ★ solving inequalities of the form $ax < b$.
- ★ solving inequalities of the form $(x \pm a) > b$.
- ★ solving inequalities of the form $(x \pm a) < b$.
- ★ representing the set of integral solutions relevant to each of them on a number line.

22.1 Can you remember what you have learned?

Do the following activity to recollect what you learned in Grade 6.

Activity 22.1



(i) A few values are represented on the above number line. What is the value each letter represents?

A = 1 B = ... C = ...
D = ... E = ...

(ii) Write as many relationships as possible by selecting two values at a time and using the sign > or <. (Example - 4 < 3)

(iii) Present the relationships you wrote to the whole class.

(iv) Write again the same relationship in (ii) above using the letters relevant to the given values. (Example B>A)

All the relationships that can be formed by connecting two quantities by the signs $<$ or $>$ are known as inequalities.

22.2 Algebraic inequalities

Balance scales are good models to explain inequalities.

(i) (ii)

← Lacee

Figure (i)

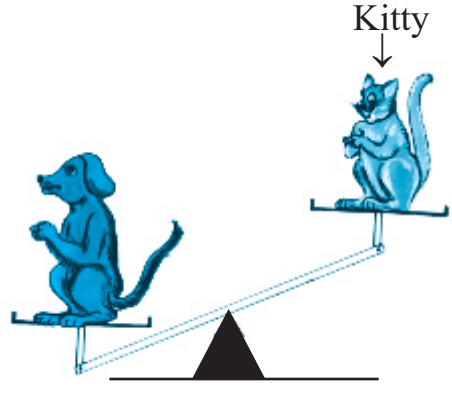


Figure (ii)

(i) According to figure (i) the weight of Lacee is 20 kg.
(ii) According to figure (ii) Weight of Lacee is greater than the weight of Kitty.

This can be written as an inequality,

Weight of Kitty $<$ Weight of Lacee

Since the weight of kitty is not known let us denote it by an unknown term 'x'.
Then the above inequality can be written as $x < 20$.

Now let us consider the following figure

Keep two similar balls each weigthing 5 kg on both sides of the scale. Accordingly, the weight in the left pan is 25 kg, and the weight in the right pan is $(x+5)$ kg.

Accordingly, the inequality illustrated by the above figure is,

$$x+5<25$$

Inequalities such as $ax < b$, $ax > b$, $x + a < b$, $x + a > b$ are said to be **algebraic inequalities**. Here 'a' , 'b' are whole numbers.

Example 1

(i) Express " x is greater than 6 " as an inequality.

$x > 6$

(ii) Express the inequality $x < 4$ in words.

"x is less than 4".

Exercise 22.1

(1) Write each of the following statements as an inequality.

(i) Value of 'a' is greater than 16

(ii) Value of 'y' is less than 8

(iii) Value of 'x' is greater than 10

(2) Write the following inequalities in words.

(i) $x < 5$ (ii) $x > 8$ (iii) $a < 2$ (iv) $y > 7$

(3) The age of Ravi is 13 years. Mohan is older than Ravi. If the age of Mohan is 'x' years, write this relation as an inequality.

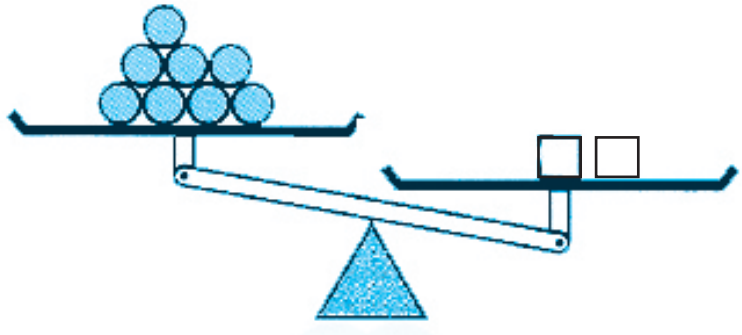
(4) Twice the weight of Mohan, is less than 80 kg. If Mohan's weight is 'y' kg, write this relation as an inequality.

(5) The number of floors of an upstair building 'A' is less than the number of floors of an upstair building 'B'. The number of floors in building 'B' is 20. Taking the number of floors in building 'A' as 'x' write this relation as an inequality.

22.3 Solution of algebraic inequalities

(a) Inequalities of the form ax > b

Example 2



$2x > 8$

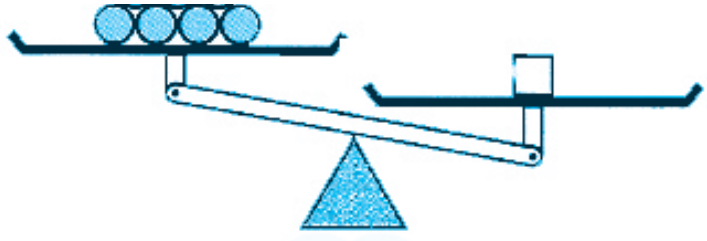
In the balance of the above figure, there are 8 equal iron balls each with a weight of 1 kg on one side and two equal unknown weights on the other side . If 'x' kg is taken as the weight of one of the two equal unknown weights, the inequality illustrated by the balance is,

$$2x > 8$$

How many iron balls must be removed to keep the balance as before if one of the two weights is removed?

To keep the balance unchanged, the number of balls that should be removed is 4.

Now look at the following figure.



$x > 4$

The inequality illustrated by the balance is $x > 4$
The values for 'x' which satisfy $x > 4$ are 5, 6, 7, ...
The values for 'x' which satisfy $2x > 8$ are 5, 6, 7, ...

Hence $2x > 8$ and $x > 4$ are two equivalent inequalities.

When both sides of an inequality are divided by the same positive number the inequality does not change.

(i)

$3x > 24$

$\frac{3x}{3} > \frac{24}{3}$

$x > 8$

(ii)

$5x > 30$

$\frac{5x}{5} > \frac{30}{5}$

$x > 6$

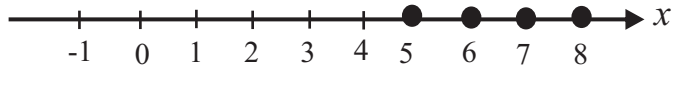
(iii)

$7x > 7$

$\frac{7x}{7} > \frac{7}{7}$

$x > 1$

You have learned in Grade 6, how whole numbers are represented on a number line. Accordingly the solutions of the inequality $2x > 8$, that is, $x > 4$, can be represented on a number line as follows.



The set of values that the unknown of an inequality can take which satisfy it, is known as its **solution**.

Example 3

(i) The price of 3 pens is greater than Rs. 27. If the price of one pen is Rs. 'x', write an inequality using 'x'.

(ii) Write the intergral values that for 'x' can take to satisfy the above inequality.

(i) The price of a pen = Rs. x

The price of three pens = Rs. $3x$

Since the price of three pens is greater than Rs. 27, the relevant inequality is $3x > 27$.

This can be solved as shown below.

$3x > 27$

$\downarrow$

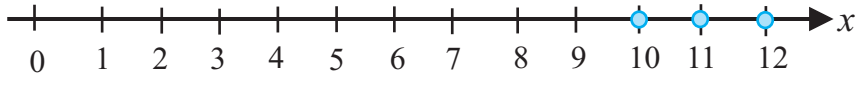
$\frac{3x}{3} > \frac{27}{3}$ (Divide both sides of the inequality by 3)

$\downarrow$

$x > 9$

Solutions are 10, 11, 12...

Accordingly, the price of a pencil can be a value such as Rs. 10, Rs. 11, Rs. 12,... When these solutions are represented on a number line, it is,



Example 4

Solve the inequality $3x > 15$ and write the integral solutions that 'x' can take.

$3x > 15$

$\downarrow$

$\frac{3x}{3} > \frac{15}{3}$ (Divide both sides of the inequality by 3)

$\downarrow$

$x > 5$

The whole number values 'x' can take are 6, 7, 8, 9, ... When represented on a number line, it is,

Exercise 22.2

(1) Solve each of the following inequalities and represent the solutions on a number line.

(i)	$3x > 6$	(ii)	$2x > 4$	(iii)	$4x > 16$
(iv)	$8x > 24$	(v)	$12x > 24$	(vi)	$5x > 75$
(vii)	$4x > 20$	(viii)	$2x > 6$	(ix)	$6x > 60$

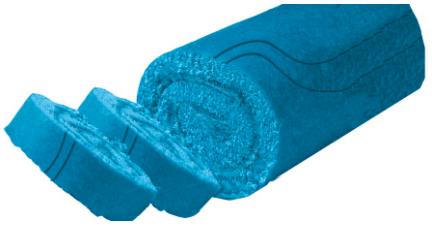
(2) Three times a number is greater than 9. Find the values of the number can take.

(b) Inequalities of the form $ax < b$

Activity 22.2

The picture of a cake of weight 6 kg is given below. This will be cut into equal sized pieces. Let us consider the weight of one piece as 'x' kg.

(i)

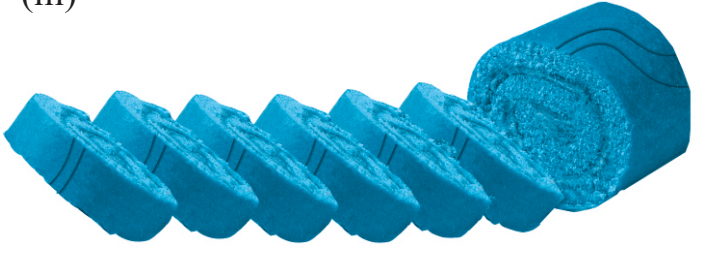


The weight of two pieces is '$2x$' and it has to be less than 6 kg.
Therefore $2x < 6$
Now fill in the blanks.

(ii)

$3x <$ ...

(iii)



..... < .....

Example 5

The price of three mangoes is less than Rs.12. If the price of one mango is Rs. 'x',

(i) write an inequality using 'x'

(ii) find the whole number values that can be assumed as the price of a mango, according to the above inequality.

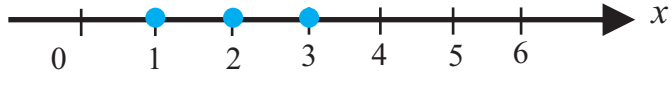
(i) The price of a mango = Rs. x
The price of three mangoes = Rs. $3x$
Then , $3x < 12$

(ii) $3x < 12$

$\frac{3x}{3} < \frac{12}{3}$ (Divide both sides of the inequality by 3)

$x < 4$

According to the above inequality,
the price of a mango is less than Rs. 4. Accordingly, the whole number values that can be taken as the price of a mango are Rs. 3, Rs. 2, and Rs.1.
These solutions can be represented on a number line as,



Example 6

Solve the inequality $5x < 25$ and represent the solutions on a number line.

$5x < 25$
When both sides are divided by 5 it will be,

$\frac{5x}{5} < \frac{25}{5}$

$x < 5$

Accordingly, 'x' can take values such as 4, 3, 2, 1, 0, -1, -2, -3, ...
When these numbers are represented on a number line it will be as follows.

Exercise 22.3

Solve each of the following inequalities and represent the integral solutions on a number line.

(i)	$2x < 12$	(ii)	$4x < 4$	(iii)	$6x < 12$
(iv)	$4x < 12$	(v)	$11x < 33$	(vi)	$4x < 16$
(vii)	$6x < 18$	(viii)	$10x < 30$		

(c)Inequalities of the form $x \pm a > b$ and $x \pm a < b$

Example 7

Let us solve the inequality $x + 2 > 6$.

$x + 2 > 6.$

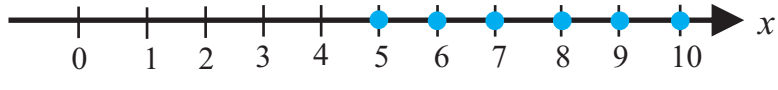
$\downarrow$

$x + 2 - 2 > 6 - 2$ (subtract 2 from both sides.)

$\downarrow$

$x > 4$

$\therefore$ The whole number solutions of the inequality are 5, 6, 7, 8 ...
When these solutions are represented on a number line, we get,



Example 8

Solve the following inequalities and represent the integral solutions on a number line.

(i) $x + 8 > 12$ (ii) $x - 2 > 3$ (iii) $x + 1 < 6$ (iv) $x - 3 < 1$

By adding the same number to both sides or subtracting the same number from both sides of an inequality, the inequality does not change.

Eg :- (i) $x + 3 > 10$

$x + 3 - 3 > 10 - 3$

$x > 7$

(ii) $x - 2 > 1$

$x - 2 + 2 > 1 + 2$

$x > 3$

(i) $x + 8 > 12$

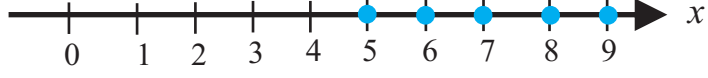
$\downarrow$

$x + 8 - 8 > 12 - 8$ (Subtract 8 from both sides of the inequality)

$\downarrow$

$x > 4$

Solutions are 5, 6, 7, 8 ... When these solutions are represented on a number line, we obtain,



(ii) $x - 2 > 3$

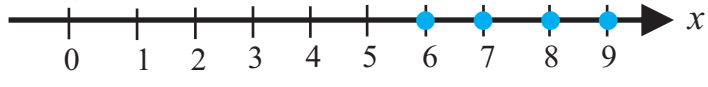
$\downarrow$

$x - 2 + 2 > 3 + 2$ (Add 2 to both sides of the inequality)

$\downarrow$

$x > 5$

Solutions are 6, 7, 8, 9, ... When these are represented on a number line, we obtain,



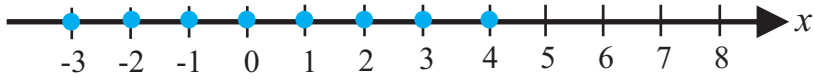
(iii) $x+1<6$

When 1 is subtracted from both sides.

$x+1-1<6-1$

$x<5$

Solutions are 4, 3, 2, 1, 0, -1, -2 ... When they are represented on a number line, we obtain,



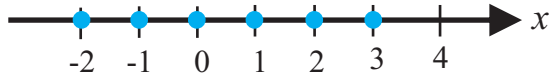
(iv) $x-3<1$

When 3 is added to both sides.

$x-3+3<1+3$

$x<4$

The solutions are 3, 2, 1, 0, -1, ... When represented on a number line, we obtain,



Exercise 22.4

(1) Solve the following inequalities.

(i) $x+2>4$ (ii) $x+4<8$ (iii) $x+1<2$

(iv) $x-3<4$ (v) $x-2>6$ (vi) $x+5<5$

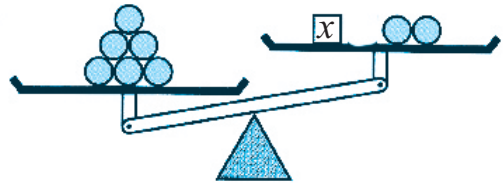
(2) Solve each of the following inequalities and represent the integral solutions on a number line.

(i) $x+3>5$ (ii) $x+7<12$ (iii) $2+x>6$

(iv) $x-2<8$ (v) $x-1>3$ (vi) $x-4>1$

(3) Write the inequalities shown by the balances on the next page. Represent their solutions on a number line. (Take $\square$ as a cube of weight 'x')

(*a*)



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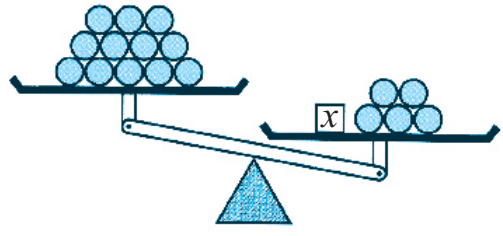

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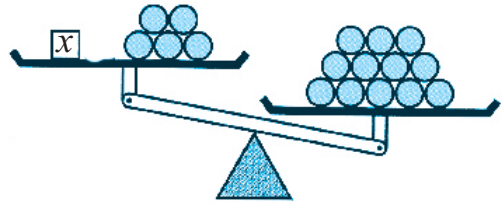

Summary

★ The solution of the inequality $ax > b$ is $x > \frac{b}{a}$. (here $a > 0$)

★ The solution of the inequality $ax < b$ is $x < \frac{b}{a}$. (here $a > 0$)

★ The solution of the inequality $x + a > b$ is $x > b - a$.

★ The solution of the inequality $x - a > b$ is $x > b + a$.

★ The solution of the inequality $x + a < b$ is $x < b - a$.

★ The solution of the inequality $x - a < b$ is $x < b + a$.

★ The numerical solution of each of the above inequalities can be represented on a number line.