21 Equations

After studying this chapter you will be able to get a good knowledge of the following;

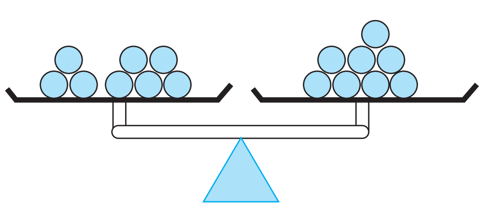
- ★ forming equations of the form $ax \pm b = c$.
- ★ solving equations by flow diagrams and algebraic methods.
- ★ building up simple formulae.

21.1 Forming simple equations

Before starting this chapter, it will be very useful to revise what you have learned in Grade 6 about algebraic expressions. In an algebraic expression, + and - signs, unknown terms, and numbers could be included, but the sign " = " is not included. When the sign " =" is introduced to an algebraic expression to balance it, the new algebraic relation you get is called an **equation**.

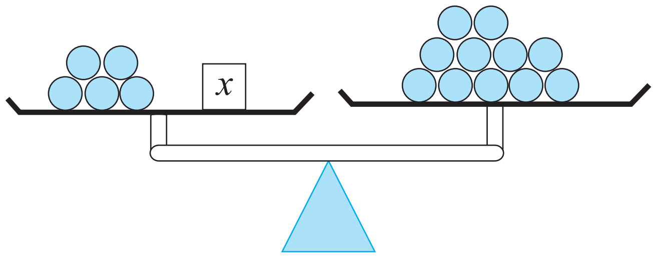
Now let us consider how a simple equation can be formed. What is an equation? A weighing balance or a scale is a good model to explain simple equations. There is a direct relationship between an equation and a balance, because, an equation expresses a balance between two sets of quantities.

Let us investigate and find the equation related to the balanced scale shown in the figure. If the weight of each marble is the same, the weight in the left pan of the balance should be equal to the weight in the pan on the right. Further the number of balls on the left side is equal to the number of balls on the right side.
Accordingly, this relationship can be written as $3+5=8$



$3 + 5 = 8$

Now look at the following figure. By the box in the left pan in the balanced scale, an unknown number of balls is indicated. Let us consider this unknown number of balls as 'x'.



$5 + x = 11$

Accordingly, the number of balls on the left side of the balance is $5 + x$ and the number of balls on the right side is 11. As mentioned above, when the algebraic expressions $5 + x$ and 11 are connected with the sign of equality we get the equation $5 + x = 11$.

Example 1

Ashani and Gayani are two students studying in Grade 7. One day, for the lesson on equations, they brought a balance, marbles, and a box containing 5g, 10g, 20g, and 50g weights to the class.

(i) When they put the weight 50g to one side of the balance and 5 marbles of equal weight to the other side, the two pans came to the same level. When the weight of a marble is taken as 'x', the equation shown by the balanced scale is,

$x + x + x + x + x = 50$

$\boxed{5x = 50}$

(ii) Then 20g was put to each pan of the above balance and again the two pans came to the same level. Now the equation shown by the balance is,

$5x + 20 = 50 + 20$

$\boxed{5x + 20 = 70}$

Activity 21.1

Continue the activity in the above example by putting marbles to the pan on the left of the balance and weights to the right side. Write the equation you get each time.

Exercise 21.1

(1) Form equations for each of the following situations.

(i) Rs. 60 was spent to buy 5 books at the rate of Rs. a a book.

(ii) Soma spent Rs. 30 to buy an apple and one orange. Take Rs. x as the price of an apple and Rs. 10 as the price of an orange.

(iii) Agasthi bought two toffees at the rate of Rs. 2 a toffee and an ice cream for Rs. p. The total amount he had to pay was Rs. 24.

(iv) The bus fare for a passenger from Negombo to Colombo is Rs. 30. There are 'x' passengers and the total amount paid is Rs. 300.

(v) The water board, distributes water at Rs. 5 per unit and has a monthly fixed charge of Rs. 50. In a certain month the number of units of water consumed was 'n' and the total bill that had to be paid was Rs. 120.

(vi) The charges for the transportation of material by train is Rs. 5 for one kilogramme. One bag had "x" kilogrammes and another bag had 10 kilogrammes. Rs. 625 had to be paid as transportation fee.

(vii) The price of a pen is twice the price of a pencil. A pencil costs Rs. x. A student spent Rs. 40 for 2 pens and a pencil.

(2) Geetha is, a student studying in Grade 7. Her age is "x" years and her father's age is four times her age. Form equations for the following situations.

(i) When the ages of Geetha and her father are added the total is 60 years.

(ii) When the age of Geetha is subtracted from the age of her father the difference is 36 years.

(3) There are 100 bags of cement in a lorry. Form equations for the following situations.

(i) When 'x' bags of cement were unloaded from the lorry, 80 bags were remaining.

(ii) When '$2x$' more bags of cement were loaded to the above lorry the total number of bags was 140.

(4) A certain fish merchant sells "Hurulla" fish of the same weight. By taking the weight of a "Hurulla" as "h" grammes, form equations for the following situations.

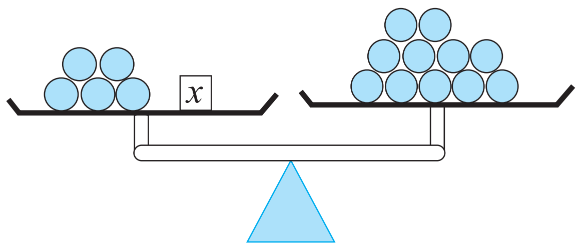
(i) When a weight of 1kg was placed on the right side of the balance, and 30 "Hurullas" of the same weight on the left side the two pans came to the same level.

(ii) When a weight of 2 kg was placed on the right side, and 30 "Hurullas" of the same weight with a 1 kg weight on the left side, the two pans balanced.

A mathematical statement obtained by connecting algebraic expressions with the equality symbol is called an **equation**.

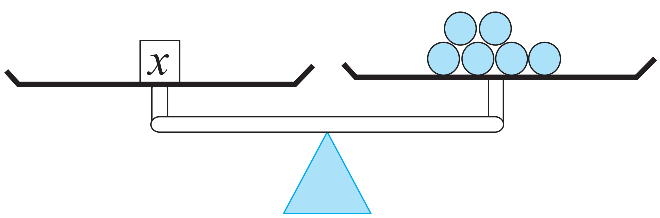
21.2 Solution of simple equations

Let us pay attention again to the balance we considered earlier.



$5 + x = 11$

Here the value of the unknown number of marbles denoted by 'x' can be found. When we remove 5 marbles from each side of the balance we again get a balanced scale.



$x = 6$

Hence the value of x is 6.

As explained in section 21.1, an equation consists of two algebraic expressions connected by the sign " = ". When numerical values are substituted for the unknown terms and simplified it can be decided whether the statement is true or false. When both sides or one side of the equation contains algebraic expressions the truth or false hood of that equation depends on the values of the unknown terms.

The value of the unknown term satisfying the equation is called its solution.

Accordingly $x = 6$ is the solution of the equation $5 + x = 11$. The reason for it is that when 6 is substituted for 'x' in the equation, the equality is satisfied. That is, 6 is the value of the unknown term 'x' for which $5 + x = 11$ is true.

> The value of the unknown term for which the equation is true is the solution of that equation.

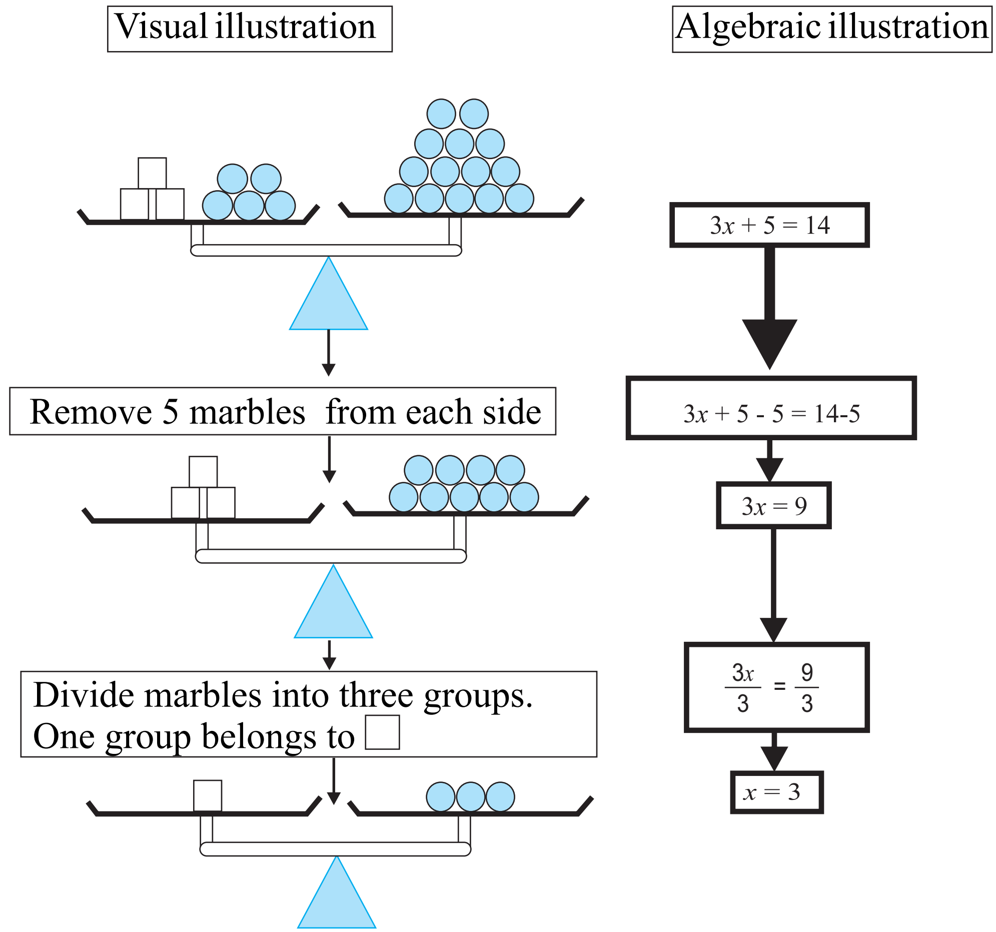
Now let us investigate different methods of solving an equation. There are two ways of solving an equation. They are,

(a) solving by algebraic method
(b) solving by flow diagrams

(a) Solution of simple equations by the algebraic method

Example 2

Let us solve the equation $3x + 5 = 14$. The unknown term in this equation is 'x'. Let us consider how this simple equation can be solved.

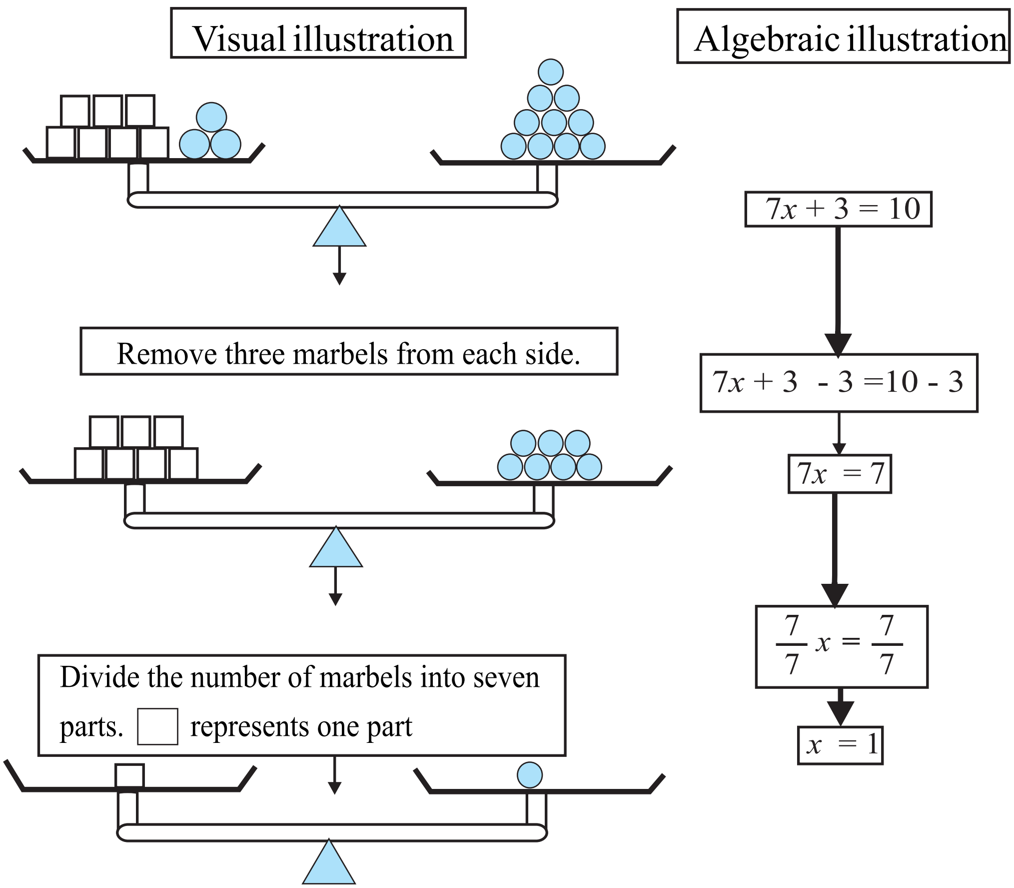


By the side of the visual illustration above are the steps that should be followed when solving an equation by the algebraic method.

You can see that if the three squares in the left side of the first balance above are removed by putting three balls for each, that balance will be balanced with 14 balls on each side. Similarly when 3, is substitued for 'x' in $3x + 5 = 14$, the equality becomes true. Therefore $x = 3$ is the solution of the equation $3x + 5 = 14$.

Example 3

Let us solve the equation $7x + 3 = 10$.



When all the squares in the first balance are removed by putting one marble for one square, the scale will balance with 10 balls on either side. Similarly in the algrebraic method when $x = 1$ is substituted in the equation $7x + 3 = 10$, the equation will be true. Hence $x = 1$ is the solution of the equation $7x + 3 = 10$.

Example 4

(i) Let us assume that the equation $2x - 1 = 5$ has to be solved. It can be solved by using the algebraic method you learned above as follows.

$$2x - 1 = 5$$
$$\downarrow$$
$$2x - 1 + 1 = 5 + 1$$

Add 1 to both sides

$$\downarrow$$
$$2x = 6$$
$$\downarrow$$
$$\frac{2x}{2} = \frac{6}{2}$$

Divide both sides by 2

$$\downarrow$$
$$x = 3$$

Accordingly, the solution of this is 3. Now substitute 3 for 'x' in the equation $2x - 1 = 5$ and examine whether both sides are equal and confirm that the solution of the equation $2x - 1 = 5$, is 3.

(ii) Solve the following equation $6x - 3 = 15$.

$$6x - 3 = 15$$

$$6x - 3 + 3 = 15 + 3$$
$$6x = 18$$

$$\frac{6x}{6} = \frac{18}{6}$$
$$x = 3$$

The solution of the equation, $6x - 3 = 15$ is $x = 3$.

(*b*) Solution of equations by using flow diagrams

Let us describe various activities and their inverses.

Activity	Inverse activity
Breathing in	Breathing out
A vehicle going forward	A vehicle reversing
Unfolding an aerial	Folding an aerial
Depositing money in a bank	Withdrawing money from a bank
Getting the result 5 by adding 2 to 3	Subtracting 2 from the result 5 and getting 3
Getting the result 6 by multiplying 3 by 2	Getting the first number 3 again by dividing 6 by 2.

Let us consider the activity of multiplying 3 by 2 and subtracting 1 from the result.

This can be illustrated as

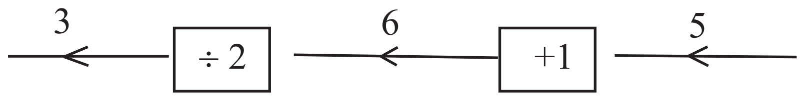
$$\xrightarrow{3} \boxed{\times 2} \xrightarrow{6} \boxed{-1} \xrightarrow{5}$$

This kind of a diagram is called a flow diagram. All the mathematical operations done here are written inside boxes.

The inverse action of the above action is as follows.

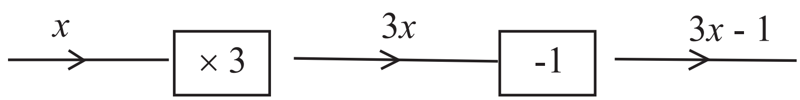
Add one to 5 and divide by 2

The flow diagram relevant to this inverse action is as follows.

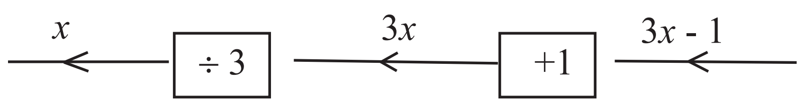


Example 5

Let us consider the flow diagram relevant to $3x - 1$ and its inverse.



The inverse can be given as,



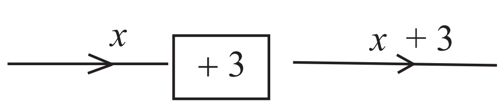
Simple equations can be solved easily by using inverse flow diagrams.

Example 6

Solve the equation $x + 3 = 5$ using a flow diagram.

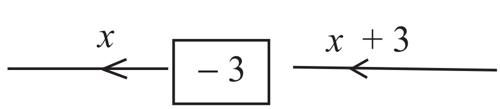
Step 1

Flow diagram



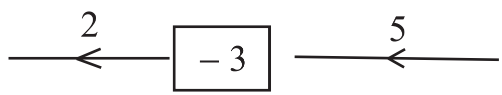
Step 2

Inverse flow diagram.



Step 3

The inverse flow diagram relevant to the eqution $x+3=5$

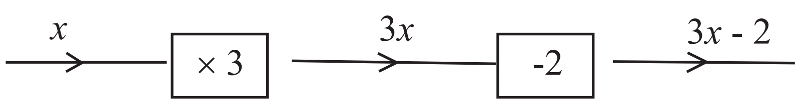


The solution of $x+3=5$ is $x=2$.

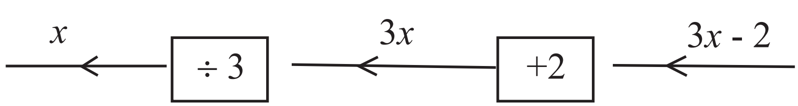
Example 7

Solve the equation $3x-2=5$ using a flow diagram.

(1) Flow diagram

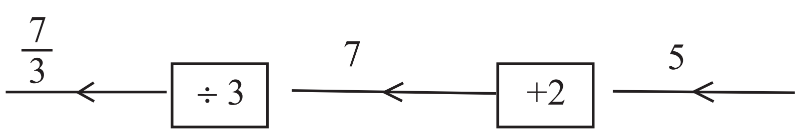


(2) Inverse flow diagram



To get the answer we use the inverse flow diagram.

(3) The inverse flow diagram relevant to $3x-2=5$.



$\therefore x = \frac{7}{3}$ is the solution of $3x-2=5$.

(I) If there are a few unknown terms on one side of an equation, simplify them first and then start solving the equation.

(ii) If any operation is performed on the left side of an equation the same operation should be performed on the right side of the equation too.

Exercise 21.2

(1) Solve each of the following equations by flow diagrams and inverse flow diagrams.

(i) $3x - 2 = 1$ (ii) $2x - 1 = 3$ (iii) $4x - 5 = 3$

(iv) $5x + 2 = 12$ (v) $5x - 3 = 12$ (vi) $18x - 1 = 17$

(vii) $2x + 2 = 8$ (viii) $x - 7 = 9$

(2) Solve the following equations.

(i) $3x - 7 = -1$ (ii) $3x - 7 = 5$

(iii) $3x + 17 = 53$ (iv) $3x = 15$

(v) $4x = 16$ (vi) $4x - 5 = 3$

(vii) $x - 6 = 2$ (viii) $3x - 1 = 5$

(ix) $5x - 2x + 1 = 4$ (x) $25x - 45 - 19x = 9$

(xi) $5x - 30 - 2x = -18$ (xii) $3x - 17 = -2$

(xiii) $9y - 16 + 3y = 20$

(3) The price of a tennis ball is 4 times the price of a rubber ball. A child bought a tennis ball and a rubber ball and he had to spend Rs. 100 for it. If the price of a rubber ball is Rs. 'x',

(i) write the price of the tennis ball in terms of 'x'.

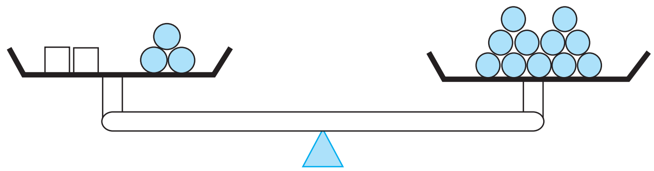
(ii) write the price of both types of balls in terms of 'x'.

(iii) using the algebraic expression obtained for (ii) above form an equation and by solving it, find the price of a rubber ball.

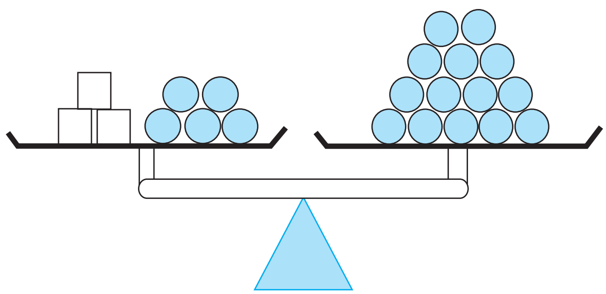
(iv) accordingly what is the price of a tennis ball?

(4) In the balances shown on the next page, $\boxed{}$ represents an unknown number of marbles. Let the number of marbles be 'x'.

(i)



(ii)



Write the equations presented by the balances (i) and (ii) above. Solve the equations and find the value of 'x' in each situation.

(5) If $2x - 3 = 5$, what is the value of $x + 4$?

(6) If $n - 246 = 762$, What is the value of $n - 247$?

(7) If $9c + 32 = 212$, do you agree that $5c = 100$?

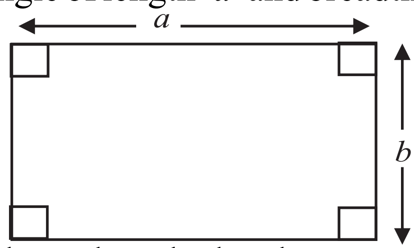
(8) Nimal had 350m of wire. He used it to build fence round a square piece of land. After completing four sides of the square once 110m were left. What is the length of one side of the square piece of land?

21.3 Building up simple formulae

The relationship between two or more quantities can be represented by a formula and those quantities can take various values. The difference between a formula and a normal equation is that a special quantity called the subject appears in it as an amount.

Example 8

Let us consider a rectangle of length 'a' and breadth 'b'.



The area of the rectangle can be calculated as,
(Area = length × breadth)
Accordingly, if the area is 'A', we get the formula

$$\boxed{A = a \times b} \text{ ——— } (1)$$

In this, the value of the special quantity 'A' is given by the product of the length and the breadth.

Now let us apply this formula to find the area of a rectangle of length 5 cm and breadth 3 cm.
Since $A = a \times b$,
Let us substitute $a = 5$ cm and $b = 3$ cm to the formula.
Then $A = 5 \text{ cm} \times 3 \text{ cm}$
$\quad = 15 \text{ cm}^2$
Hence the area of the rectangle is 15 cm^2.
Similarly, if the perimeter of the rectangle is taken as 'S',

$$S = a + b + a + b = 2a + 2b = 2\,(a + b)$$

$$\boxed{S = 2\,(a + b)} \text{ ——— } (2)$$

If the length of a square is 'x', then since the length and the breadth of a square are equal, the formulae (1) and (2) above relevant to it can be given as,

$$A = x \times x \longrightarrow \boxed{A = x^2}$$

$$S = 2\,(x + x) \longrightarrow \boxed{S = 4\,x}$$

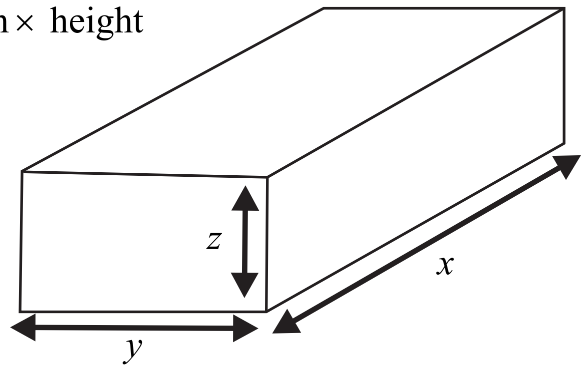
Example 9

Construct a formula for the volume 'V' of a cuboid of length 'x', breadth 'y' and height 'z'. Using that formula find the volume of a cuboid of length 7 cm, breadth 4 cm and height 3 cm.

V $= \text{length} \times \text{breadth} \times \text{height}$

$= x \times y \times z$

$$V = x\,y\,z$$



Let us substitute the values $x = 7$, $y = 4$, and $z = 3$ in the above formula.

$V = 7\text{cm} \times 4\text{ cm} \times 3\text{ cm}$

$V = 84\text{ cm}^3$

The volume of the cuboid is 84 cm^3.

Exercise 21.3

(1) The length of a certain rectangle is 'y' and its breadth is 'x'. If its perimeter is 'P', construct a formula for 'P' using 'x' and 'y'. Using the formula, find the perimeter of a rectangular piece of land of length 60 m and breadth 20 m.

(2) A certain factory produces 'n' cups per day. If the number of cups produced in 'x' days is 'm', construct a formula for 'm' in terms of 'n' and 'x'. Accordingly, find the number of cups produced in 25 days by a factory which produces 20 000 cups per day.

(3) A lorry transporting goods charges Rs. n for one kilogramme. If the total quantity of goods transported is 'm' kilogrammes and if 'P' is the amount of money needed for transportation, construct a formula for 'P'. Accordingly find the amount of money spent on a quantity of goods of weight 2500 kg transported at the rate of Rs. 10 per kilogramme.

(4) If the distance travelled by a motor cycle at a speed of 'v' for a time 't' is 'D', build up a formula for 'D' in terms of 'v' and 't'.

(5) When filling a tank with water, if 'n' litres of water flows into the tank every minute, and if 'V' is the quantity of water that is collected in the tank in a time of 't' minutes, construct a formula for 'V' in terms of 't' and 'n'.

(6) Construct formulae for the following
  (i) Subtracting three times "b" from "a" gives "c" .
  (ii) Adding twice 'q' to 'p' gives 'r'.

Summary

★ The mathematical relationship obtained by connecting algebraic expressions using the equality sign is called an equation.

★ The value of the unknown satisfying a simple equation is the solution of the equation.

★ An equation can be solved using flow diagrams and also by the algebraic method.

★ The relationship between quantities can be given by a formula.