4 Indices

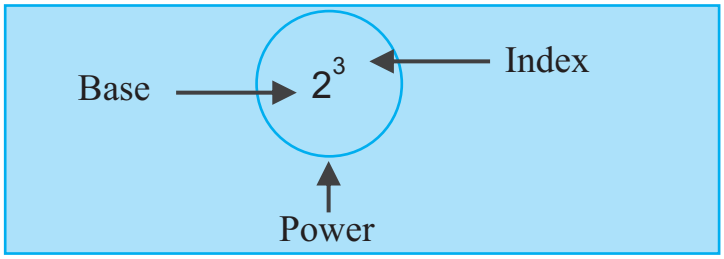
By studying this chapter you can get a good understanding of,

- ✶ introducing the powers of which the base is an algebraic term.
- ✶ expanding the powers of which the base is an algebraic term.
- ✶ finding by substitution the value of a power, the base of which is an algebraic term.

Generation	My family	Number
Me		2^0
Parents		2^1
Grand fathers and grand mothers		2^2
Great grand fathers and Great grand mothers		2^3
Great Great grand fathers and Great Great grand mothers	?	?

4.1 Do you remember what you have learned

You have studied indices in Grade six. Recollect how you wrote a number in the index form and how you expanded a power. Again let us revise how it should be when we write a number as a power of a given number and how the base should be when we write a given number as a power of a given index.



Let us now engage in the following activity.

Activity 4.1

Fill in the blanks of the table given below.

Power	Base	Index	Value
2^3	2	3	$2 \times 2 \times 2 = 8$
3^3	.....	.....	.....
4^3	.....	.....	.....
5^3	.....	.....	.....
6^2	.....	.....	.....
7^2	.....	.....	$7 \times 7 = 49$
10^3	.....	.....	.....
11^3	.....	.....	.....
12^2	.....	.....	.....

Activity 4. 2

Copy the following exercise in your exercise book and write suitable values in the circles and in the squares.

Example :- $2 \times 2 \times 2 \times 2 = \square^{\textcircled{4}}_{2}$

(i) $6 \times 6 = \square^{\bigcirc}$

(ii) $2 \times 2 \times 2 \times 2 = \square^{\bigcirc}$

(iii) $12 \times 12 \times 12 = \square^{\bigcirc}$

(iv) $4 \times 4 \times 4 \times 4 = \square^{\bigcirc}$

(v) $9 \times 9 = \square^{\bigcirc}$

(vi) $\square_{\bigcirc} \times \square = 10$

Exercise 4.1

(1) Write the numbers given below as products of prime factors and write down the relevant power.
   (i) 16, as a power of 2
   (ii) 27, as a power of 3
   (iii) 125, as a power of 5

(2) Express the following numbers as powers.
   (i) 16, as a power of 4
   (ii) 64, as a power of 8
   (iii) 216, as a power of 6
   (iv) 1000, as a power of 10

(3) What should be the base when 81 is written as a power of which the index is 4?

(4) What should be the base when 32 is written as a power of which the index is 5?

(5) (i) What should be the base, when 36 is written as a power of which the index is 2?
   (ii) Write down the power 7^{10} in the expanded form.
   (iii) Write down as a power the value of $4^7 \times 4^3$.
   (iv) Express 180 as a product of powers of prime numbers.
   (v) Find the value of $5 \times 6^2 \times 10^3$.
   (vi) Find the value of $5^2 \times 8^2 \times 3^3$.

(6) Agasthi is 3 years old and his elder brother is three times as old as he. Agasthi's father's age is 2 years greater than ten times the age of Agasthi. Express the ages of Agasthi's elder brother and his father, as powers .

4.2 Powers with bases in algebraic symbols

We have already studied powers with the base as numerical values. In this section we also study powers with base as algebraic symbols.

You will be able to get a good understanding of the above mentioned, when you observe the examples shown below.

Example 1

$2 \times 2 \times 2 = 2^3$

$3 \times 3 \times 3 \times 3 = 3^4$

Let 'a' be a non - zero algebraic variable ($a \neq 0$) (unknown value).
Then $a \times a \times a = a^3$
We can write $a \times a \times a \times a = a^4$
Here a^4 is a power with the base "a" and index 4.

Example 2

Consider the statement, "when the age of a son is multiplied by itself the age of his father can be obtained".
Here as the age of the son is not given, let us assume it as an unknown term 'y'.
Then the father's age $= y \times y$
and the answer is y^2.

From the above examples, it is clear that, when an algebraic symbol is multiplied repeatedly by itself, it can be expressed as a power.

A power can be expressed not only as a power with the base as a numerical term but also as a power with the base as an algebric symbol.

Example 3

Shown below are some expressions written in the expanded form. Study how these are expressed as products of powers.

(i) $2 \times 2 \times x = 2^2 \times x = 2^2 x$

(ii) $3 \times 3 \times a \times a = 3^2 \times a^2 = 3^2 a^2$

(iii) $x \times x \times x \times y \times y = x^3 \times y^2 = x^3 y^2$

(iv) $p \times p \times 2 \times 2 \times q = p^2 \times 2^2 \times q = 2^2 p^2 q$

(v) $4 \times m \times 2 \times m \times n = 2 \times 2 \times 2 \times m \times m \times n = 2^3 m^2 n$

Activity 4.3

Fill in the blanks in this table.

(i) $2 \times 2 \times x$	$2^2 \times x$	.....
(ii) $3 \times x \times x$	.....	$3x^2$
(iii)	$2^3 \times t^2$	.....
(iv) $x \times x \times x \times y$	.....	.....
(v) $5 \times 5 \times t \times x \times t$	.....	$5^2 t^2 x$
(vi)	.....	$6m^2n$

Exercise 4.2

(1) Write as a power:

$a \times a \times a \times a$.

(2) Write in the expanded form.

(i) a^5 (ii) $a^2 \times b^3$ (iii) $x^2 \times y^4 \times 3$ (iv) $x^2 y^3$

(v) $2^3 a^4$ (vi) $x^2 g^2$ (vii) $a^2 b^3 c^4$ (viii) $5^2 c d^3$

(3) Write as product of powers.

(i) $4 \times a^2 \times b^2$ (ii) $a \times a \times b \times b \times b \times c \times c \times c$

(iii) $x \times x \times 5 \times 5$

4.3 Finding the value of a power by substitution

By substituting different values to the variables of algebraic expressions with powers, the relevant answers can be obtained.

Example 4

In the algebraic expression $4a^2$; if $a = 3$, its value can be obtained as follows.

$$\begin{aligned} 4 \times a^2 &= 4 \times a \times a \\ &= 4 \times 3 \times 3 \\ &= 36 \\ 4a^2 &= 36 \end{aligned}$$

Example 5

Let us find the value of the expression x^3y^2, if $x = 2$ and $y = 3$.

$$\begin{aligned} x^3 y^2 &= x^3 \times y^2 \\ &= x \times x \times x \times y \times y \\ &= 2 \times 2 \times 2 \times 3 \times 3 \\ &= 8 \times 9 \\ &= 72 \end{aligned}$$

Example 6

Let us find the value of the expression $5x^3y^2$, if $x = 3$ and $y = 4$

$$\begin{aligned} 5x^3y^2 &= 5 \times x^3 \times y^2 \\ &= 5 \times x \times x \times x \times y \times y \\ &= 5 \times 3 \times 3 \times 3 \times 4 \times 4 \\ &= 5 \times 27 \times 16 \\ &= 2160 \end{aligned}$$

Activity 4.4

Complete the given table by substituting the values.
$x = 2$ and $y = 3$

Expression	Substitution	Expression in the expanded form	Value
x^3y^2	$2^3 \times 3^2$	$2 \times 2 \times 2 \times 3 \times 3$	.....
$3x^2y^4$	$3 \times 2^2 \times 3^4$	.....	.....
$10\,xy^3$	.....	.....	.....
$2\,(xy)^2$	.....	....	.....
$4\,x^2y^2$	.....	.....	.....
$5(xy)^2y$	.....	.....	.....

Exercise 4. 3

(1) Find the values of the terms given below by substituting $a = 3$.

(i) $3a$ (ii) a^3 (iii) $5a^2$ (iv) 2^2a^2 (v) 3^3a^2

(2) Find the values of the terms given below by taking $x = 1$ and $y = 2$.

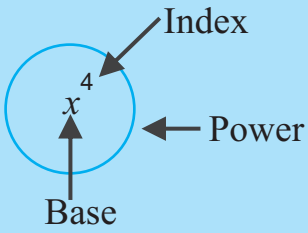
(i) $3x^2y$ (ii) $2xy^3$ (iii) $4x^2y^3$ (iv) $3x^3y^3$ (v) $6x^4y^2$

(3) Find the values of the terms given below by taking
$a = 2,\ b = 3,\ c = 4$.

(i) $2a^2bc$ (ii) $3a^3bc$ (iii) 3^3ab^2c (iv) abc^2 (v) $3a^2bc^2$

Summary

★ The procedure of repeated multiplication by the same number can be written in index form.



★ It is posible to write a power with the base as an algebraic symbol.

★ The use of the multiplication symbol is not required when we write as a product, numbers in the index form of algebraic terms in the index form.

Example:- $3^2 \times a^5 = 3^2 a^5$

★ We can find the values of powers with bases as algebraic symbols by substituting values to variables.