3   Factors and Multiples

After studying this chapter you will be able to get a good understanding,

- ✶ of finding the digital index of a number
- ✶ of checking whether a given number is divisible by 3, 4, 6 and 9 without actual dividing
- ✶ of finding the common factors of a few numbers
- ✶ of finding the H. C. F. of a few numbers
- ✶ of finding the L. C. M. of a few numbers
- ✶ of solving problems using the H.C.F and the L.C.M.

3. 1 Can you remember what you have learned...?

You have learnt in Grade 6, how to decide whether a number is divisible by 5 or 10 without a remainder. Let us revise those facts by doing the following activity.

Activity 3.1

Fill in the table given below.

Number	Digit in the units place	Is divisible by 5 (Put ✓ or ✗ mark)	Is divisible by 10 (Put ✓ or ✗ mark)
370			
1 115			
5 554			
1 300			
505			
1 005			
3 033			
111 110			
505 052			
120 035			

★ If a number has 0 in the units place then it is divisible by 10.

★ If a number has 0 or 5 in the units place then it is divisible by 5.

? Are all the numbers divisible by ten can be divided by 5?

3.2 Investigating whether a number is divisible by 3

Activity 3.2

63   23   46   36   69   79   133   156

★   Is the digit in the units place of each of the numbers above divisible by 3?

★   Is each of the above numbers divisible by 3 ?
Can it be decided whether a number is divisible by 3 or not divisible by 3, depending on the divisibility or non divisibility of the digit in the units place of the number by 3 ?

Activity 3.3

Now let us add the digits of a few numbers divisible by three until we get the sum as a single digit number.

(i)   21   → $2+1 = 3$ →

(iv) 87 ➡️ $8+7=15$ ➡️ $1+5=6$

(v) 348 ➡️ $3+4+8=15$ ➡️ $1+5=6$

(vi) 59 997 ➡️ $5+9+9+9+7=39$ ➡️ $3+9=12$ ➡️ $1+2=3$

Based on the result of the above activity, we can arrive at the following conclusions.

★ The sum of the digits of each of the above numbers is 3, 6, or 9. It means that the sum of the digits is divisible by 3.

★ When the sum of the digits of a number is taken as a single digit number it is called the **"Digital Index"**.

★ If the digital index of a number is divisible by three, that number is divisible by 3.

Example 1

Let us find whether each of the numbers 1137 and 4127 is divisible by 3.

(i) Let us find the digital index of 1137.

1 137 ➡️ $1+1+3+7=12$ ➡️ $1+2=3$

The digital index is 3 which is divisible by 3.
Hence 1137 is divisible by 3.

(ii) Let us find the digital index of 4127.

4 127 ➡️ $4+1+2+7=14$ ➡️ $1+4=5$

The digital index of 4127 is 5. It is not divisible by 3.
Hence the number 4127 is not divisible by 3.

(1)   Find the digital index of each of the following numbers and tabulate those numbers that are divisible by 3 and not divisible by 3.

(i)   714   (ii) 508   (iii)   1 005 (iv)   7 122 (v) 438

(vi)   59 983 (vii) 1 122 (viii) 313   (ix)   777   (x) 55 551

(xi)   7 117 (xii) 9 136

(2)   A two digit number, divisible by 3 has 1 in the units place. Write all such numbers.

(3)   If the number 3 - 47 is divisible by 3 write all the digits that are suitable for the blank.

(4)   Write three numbers with three digits divisible by three which start with the digit 5 and end with 2.

3.3 Investigating whether a number is divisible by 4

Activity 3.4

⟨34⟩ ⟨418⟩ ⟨116⟩ ⟨508⟩ ⟨700⟩ ⟨252⟩ ⟨314⟩

⟨1132⟩ ⟨3700⟩ ⟨1214⟩
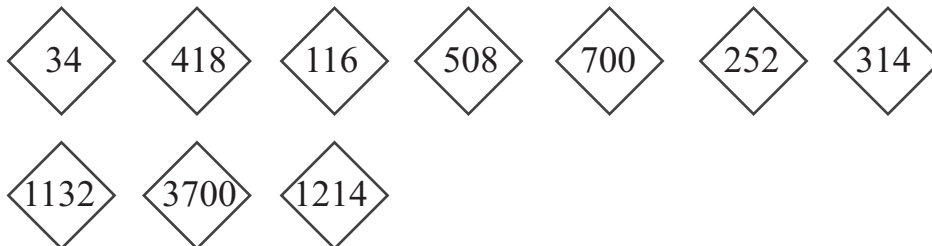
Number	Is the digit at the end divisible by 4?	Is the digital index divisible by 4?	Is the number formed with the last two digits divisible by 4?	Is the number divisible by 4? (Find by division)
34				
418				
116				
508				
700				
252				
314				
1132				
3700				
1214				

According to the table you have completed, which test should you apply out of the above to find whether a given number is divisible by 4?

Example 2

Let us select the numbers which are divisible by 4 out of the following numbers.

3 700, 364, 1 554, 10 432, 7 500, 3 108, 5 014

Out of the above numbers, let us select the numbers having the last two digits divisible by 4 or having the last two digits as 00. They are 3700, 364, 10432, 7500, 3108.

Out of the numbers written in the beginning of this activity what are the numbers that are divisible by 4?

Accordingly we can arrive at the following conclusion.

A number is divisible by 4 if the number formed by the last two digits of that number is either 00 or divisible by 4.

Exercise 3.2

(1) Without dividing the whole number by 4, select out of the following numbers which are divisible by 4.

(i) 5 312　　(ii) 4 414　　(iii) 304　　(iv) 3 100　　(v) 5 392

(vi) 11 132　(vii) 7 438　(viii) 5 076　(ix) 31 328　(x) 40 468

(2) Write three numbers divisible by 4, each starting with 3 and ending with 2.

(3) Write all the counting numbers less than hundred having 6 as the digit in the units place which are divisible by 4.

(4) Write 10 numbers divisible by 2 and not by 4.

(5) Write 20 numbers greater than 1 which are divisible by 4 and each having the last digit as 0.

3.4 Investigating whether a number is divisible by 6

Activity 3.5

Complete the following table (use ✓ or X)

Number	Is it divisible by 2?	Is it divisible by 3?	Is it divisible by 6?
324	....	....	....
225	....	....	....
174	....	....	....
1 160	....	....	....
1 258	....	....	....
2 247	....	....	....
2 412	....	....	....
2 117	....	....	....

According to the facts in the table you have completed above, what are the two properties a number divisible by 6 should have?

If a number is divisible by both 2 and 3, it is divisible by 6.

3.5 Investigating whether a number is divisible by 9

A few multiples of 9 are given below.

9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117

Let us find the digital index of each number.

• 18 → 1 + 8 = •9
• 27 → 2 + 7 = •9
• 36 → 3 + 6 = •9

636 + 3 = •9
727 + 2 •=

→ 9   9 + 0 = 9
→ 99 9 + 9 →1+8=9
→ 9   108 1+0+8

You can see that of all these numbers which are divisible by 9 the digital index is 9.

> If the digital index of a number is 9 that number is divisible by 9.

Example 3

◆ Let us find whether 963 and 10 089 are divisible by 9.
  Let us find the digital index of 963.
  $9 + 6 + 3 = 18 \longrightarrow 1 + 8 = 9$
  Since the digital index of 963 is 9, it is divisible by 9.
◆ Let us find the digital index of 10 089.
  $1 + 0 + 0 + 8 + 9 = 18 \longrightarrow 1 + 8 = 9$
  The digital index of this is also 9.
  Hence 10 089 is also divisible by 9.

Exercise 3.3

(1) Out of the following numbers separate those which are divisible by 6 without remainder.
  (i) 72   (ii) 679   (iii) 618   (iv) 696   (v) 625
  (vi) 10 097   (vii) 1 044   (viii) 578   (ix) 916   (x) 3 275

(2) Out of the following numbers select those that are divisible by 9 without remainder.
  (i) 108   (ii) 589   (iii) 716   (iv) 1 539   (v) 981
  (vi) 1 827   (vii) 989   (viii) 1 782   (ix) 2 160   (x) 998

(3) Out of the following numbers find those which are divisible by both 6, and 9 without remainder.
  (i) 144   (ii) 54   (iii) 595   (iv) 162   (v) 1 062

3. 6 Factors

Let us assume that 36 buttons have to be packeted equally. Further they

With 1 button   →   36 packets
With 2 buttons   →   18 packets
With 3 buttons   →   12 packets
With 4 buttons   →   09 packets
With 6 buttons   →   06 packets
With 9 buttons   →   04 packets
With 12 buttons → 03 packets
With 18 buttons → 02 packets
With 36 buttons → 01 packet

Accordingly 36 can be divided without a remainder by   1, 2, 3, 4, 6, 9, 12, 18 and 36.

> It is defined that if a whole number can be divided by another whole number the second number is a factor of the first number.

Hence the factors of 36 are
1, 2, 3, 4, 6, 9, 12, 18 and 36.

Prime numbers

If a number has only two distinct factors that number is defined as a prime number.

Example 4

(i) $3 \rightarrow 1\underline{|3}\atop{3}$, $3\underline{|3}\atop{1}$

Accordingly 3 has only two distinct factors. They are 1 and 3.

(ii) $23 \rightarrow 1\underline{|23}\atop{23}$, $23\underline{|23}\atop{1}$

23 has only two distinct factors. They are 1 and 23.

Hence the numbers 3 and 23 are prime numbers.

The two distinct factors of a prime number are 1 and that number itself.

> If a number has more than two factors it is defined as a **composite number.**

Example 5

Let us find the factors of 6, 9 and 14.

$$\rightarrow \quad 1\overline{\smash{\big)}6} \quad 2\overline{\smash{\big)}6} \quad 3\overline{\smash{\big)}6} \quad 6\overline{\smash{\big)}6} \rightarrow$$
$$6 3 2 1$$

In 6

6 has four factors
(1, 2, 3, 6)

$$\rightarrow \quad 1\overline{\smash{\big)}9} \quad 3\overline{\smash{\big)}9} \quad 9\overline{\smash{\big)}9} \rightarrow$$
$$9 3 1$$

9 has three factors
(1, 3, 9)

In 9

$$\rightarrow \quad 1\overline{\smash{\big)}14} \quad 2\overline{\smash{\big)}14} \quad 7\overline{\smash{\big)}14} \quad 14\overline{\smash{\big)}14} \rightarrow$$
$$14 7 2 1$$

In 14

14 has 4 factors
(1, 2, 7, 14)

The numbers such as 6, 9, 14...etc, having more than two factors are called composite numbers.

Activity 3.6

(1) Complete the following table.

Number	Factors	Number of factors
1	1	1
2	1, 2	2
3	....	....
4	1, 2 , 4	3
5	....	....
6	....	....
7	....	....
8	....	....
9	....	....
10	....	....
13	....	....
17	....	....
29	....	....
25	....	....

What ... in column

Prime numbers	
Composite numbers	

3.7 Finding the factors of a number by dividing it by prime numbers

Example 6

Let us find the factors of 36.

$$36 = 2 \times 2 \times 3 \times 3 \times 1$$

```
2 | 36
2 | 18
3 | 9
3 | 3
    1
```

$2 \times 1 = 2$
$2 \times 3 = 6$
$2 \times 2 = 4$
$3 \times 1 = 3$
$3 \times 3 = 9$
$2 \times 3 \times 2 = 12$
$3 \times 3 \times 2 = 18$

By taking the multiplication bonds of these numbers, all the factors of these numbers can be obtained.

Accordingly the factors of 36 are, 1, 2, 3, 4, 6, 9, 12, 18, 36.

Example 7

Let us find all the factors of 63.

```
3 | 63
7 | 21
3 | 3
    1
```

$63 = 3 \times 7 \times 3 \times 1$
$3 \times 1 = 3$
$7 \times 1 = 7$
$3 \times 7 = 21$

Using the multiplication bonds of 3, 7, 3, 1, let us find the other factors
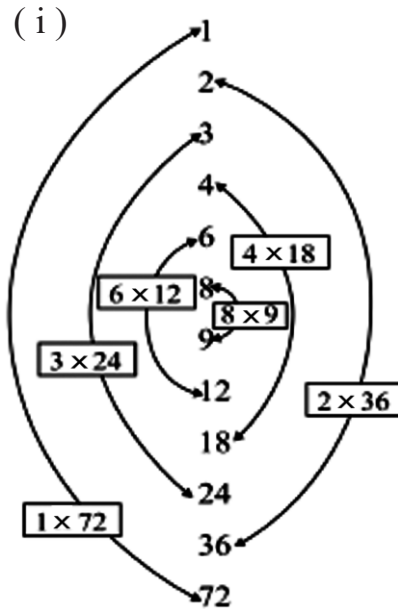$7 \times 3 \times 3 = 63$

Example 8

Accordingly all the factors of 63 are 1, 3, 7, 9, 21, 63.

```
2 | 72
2 | 36
2 | 18
3 | 9
3 | 3
    1
```

All the factors obtained are $1, 2, 2, 2, 3, 3$. Now using the above numbers and their products, the factors of 72 can be written in the ascending order as.

All the factors

(i)

1	$\longrightarrow$	1
2	$\longrightarrow$	2
3	$\longrightarrow$	3
2×2	$\longrightarrow$	4
2×3	$\longrightarrow$	6
$2 \times 2 \times 2$	$\longrightarrow$	8
3×3	$\longrightarrow$	9
$2 \times 2 \times 3$	$\longrightarrow$	12
$2 \times 3 \times 3$	$\longrightarrow$	18
$2 \times 2 \times 2 \times 3$	$\longrightarrow$	24
$2 \times 2 \times 3 \times 3$	$\longrightarrow$	36
$2 \times 2 \times 2 \times 3 \times 3$	$\longrightarrow$	72

(b)

(i)   Write down all the possible ways of expressing 72 as a product of two numbers.

(ii) Write down all the factors of 72.

(iii) Write down all the prime factors of 72.

(iv) Write down 72 as a product of prime factors.

(i) $1 \times 72, \ 2 \times 36, \ 3 \times 24, \ 4 \times 18, \ 6 \times 12, \ 8 \times 9$

(ii) $1, \ 2, \ 3, \ 4, \ 6, \ 8, \ 9, \ 12, 18, \ 24, \ 36, \ 72$

(iii) 2, 3

(iv) $2 \times 2 \times 2 \times 3 \times 3$

Out of the factors of a number, the factors which are prime numbers are called **prime factors.**

(1)   Find the factors by the method of division.

(i) 28   (ii) 64   (iii) 93   (iv) 124   (v) 225

(2)   Find all the factors of each of the following numbers.

(i) 32   (ii) 48   (iii) 76   (iv) 108   (v) 132
(vi) 184   (vii) 196   (viii) 216   (ix) 121   (x)   420

(3)   Write each of the following numbers as a product of prime
      numbers.

(i) 20   (ii) 36   (iii) 50   (iv) 280   (v) 560

(4)   (i)   Find the factors of 54.
      (ii) Find the number of factors of 54.
      (iii) Write the prime factors of 54.

(5)   (i)   Find the factors of 294.
      (ii) Write the number of factors of 294.
      (iii) Write the prime factors of 294.
      (iv) Write 294 as a product of prime numbers.

(6)   Write separately the prime numbers and the composite
      numbers between 1 and 50.

3.8 Highest Common Factor (H. C. F.)

Example 9

Let us find the Highest Common Factor of 16 and 40.
First find the factors of 16.

```
2 | 16
2 | 8
2 | 4
2 | 2
    1
```

Factors of 16 are 1, 2, 4, 8, 16.

Then the factors of 40.

```
2 | 40
2 | 20
2 | 10
5 | 5
    1
```

Factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40.

Now take the common factors of 16 and 40.

1 ,	2 ,	4 ,	8 ,	16					
1 ,	2 ,	4 ,	5 ,	8 ,	10 ,	20 ,	40		

Factors of 16

Factors of 40

The highest common factor out of all the common factors of a few numbers is called the highest common factor of those numbers.

Example 10

- Another method of finding the H. C. F. of 16 and 40 is given below.
  First write 16 and 40 as product of prime factors.

  16 $\longrightarrow$ $2 \times 2 \times 2 \times 2$
  40 $\longrightarrow$ $2 \times 2 \times 2 \times 5$

  $\downarrow$ $\downarrow$ $\downarrow$
  2   2   2

  $\longrightarrow$

  The common factors of 16 and 40 are   2, 2, 2.
  Hence the product of the common factors of 16 and 40 is $2 \times 2 \times 2$

- Accordingly the H. C. F. of 16 and 40 is 8.
  Find the H. C. F. of 28, 42, 56.
  $28 = 2 \times 2 \times 7$

  $42 = 2 \times 3 \times 7$

  $56 = 2 \times 2 \times 2 \times 7$

The common factors of 28, 42, and 56 are 2 and 7.
The product of the common factors of 28, 42 and 56 is 14.

The product of the common factors of a few numbers is their highest common factor.

Activity 3.7

Number	As a product of prime factors
18	
36	
54	

Com

(1)   Find the H. C. F. of 36, 45, 72
      (i) using the factors.
      (ii) using the prime factors.

(2)   Find the H. C. F. of each of the following sets of numbers.
      (i) 14 , 21   (iv) 6 , 8 , 12   (vii) 6 , 12 , 15
      (ii) 36 , 60   (v) 16 , 36 , 48   (viii) 96 , 64 , 160
      (iii) 72 , 114   (vi) 12 , 24, 64   (ix) 12, 28, 44
      (x) 24, 36 , 48 , 60

(3) There are three pieces of wire each of length 24cm, 36cm, 48cm respectively. Rings of equal circumference have to be constructed, using those wires without cutting off excess lengths from any wire. What is the circumference of the largest possible ring?

      length around the ring = circumference

3.9 Finding the Least Common Multiple (L. C. M.)

Out of the common multiples of a few numbers the smallest value is **Example 11** ommon multiple of those numbers.

Find the L. C. M. of the numbers 3, 4, 6. Let us first write the multiples of 3, 4 and 6.

3 →	3 ,	6 , 9 ,	12 ,	15 , 18 , 21,	24 ,	27 , 30 , 33,	36
4 →	4 , 8 ,		12 ,	16 , 20 ,	24 ,	28 , 32 ,	36
6 →	6 ,		12 ,	18 ,	24 ,	30 ,	36

The common multiples of 3, 4 and 6 are 12, 24, 36, etc.

Out of these multiples the smallest is 12.
In otherwards, the smallest number which is divisible by 3, 4 and 6 is 12. Therefore the L. C. M. of 3, 4 and 6 is 12.

Example 12

Find the L. C. M. of 8 and 14.

Method of writing as the product of prime numbers

2	8
2	4
2	2
	1

2	14
7	7
	1

● 8 ➡ $2 \times 2 \times 2 = 2^3$

● 14 ➡ 2×7

Different digits among the factors   2 , 7

The highest powers of them $2^3 , 7^1$

The product of those highest powers $= \cancel{2^3} \times 7$

Example 13   d 14 $= 56$

Method of division by prime numbers

The L. C. M. of 8 and 14 can be found by dividing 8 and 14 by prime numbers.

2	8 . 14
2	4 . 7
2	7 . 7

(Here, since 7 cannot be divided by 2, it has been written as it is)

Accordingly,
the L.C.M. of 8 and 14 = $2 \times 2 \times 2 \times 7$

$= \underline{56}$

Example 14

Find the L. C. M. of 6, 12, 32 by the method of division.

2	6 , 12 , 32
2	3 , 6 , 16
2	3 , 3 , 8
2	3 , 3 , 4
2	3 , 3 , 2
3	3 , 3 , 1

$2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$

The L. C. M. of 6, 12 and 32 $= \underline{\underline{96}}$

Activity 3.8

Fill in the blanks of the following table. Hence find the L. C. M. and the H.

Number	As a product of prime factors
12	$2 \times 2 \times 3$
18	....
24	....

(1)   Find the L. C. M. of each of the following sets of numbers.

(i) 12 , 30   (iv) 3 , 6 , 8   (vii) 15 , 7 , 12

(ii) 84 , 132   (v) 60 , 80 , 90   (viii) 96 , 72 , 108

(iii) 18 , 21   (vi) 12 , 18 , 27   (ix) 64 , 96 , 128

(x) 70 , 80 , 90

(2)   Find the L. C. M. of the following numbers by finding their multiples.
        (i) 20 , 30   (ii) 8 , 14   (iii) 3 , 6 , 8   (iv) 15 , 7 , 12

(3)   Find the L. C. M. using the method of division.
        42 , 54 , 60

(4)   Find the L. C. M. of 10, 12, 16, 20.

(5)   A bell 'A' rings once in every 30 minutes and a bell 'B' rings once in every 45 minutes. Both bells rang at 6.00 a. m. Find the time the two bells will ring at the same time next before 12.00 noon.

(6) (i)   What is the smallest number divisible by both 50 and 60?
    (ii)   What is the highest number which divides both numbers 50 and 60.

(7)   Find the smallest number which can be divided by 8, 10 and 16 without a remainder.

Summary

* Numbers obtained by multiplying a certain number with various whole numbers are known as the multiples of that number.

* When the digits of a number are added until a single digit number is obtained it is called the digital index of the number.

* If the digital index of a number is 3, 6 or 9 that number is divisible by 3.

* If the number formed by the last two digits of a number is divisible by 4 or the last two digits are 00 that number is divisible by 4.

* If the digital index of a number is 3, 6 or 9 and the digit in the units place is 0 or an even number, that number is divisible by 6 .

* If the digital index of any number is 9, that number is divisible by 9.

* If a number can be divided by another number without a remainder, the second number is known as a factor of the first.

* Numbers which have only two distinct factors are called prime numbers.

* Numbers which have more than two factors are called composite numbers.

* The largest number, by which two or a few numbers can be divided without a remainder is called the highest common factor of those numbers.

* The smallest number divisible by two or more numbers is the least common multiple of those numbers.