

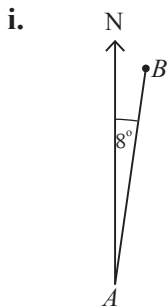
**By studying this lesson you will be able to;**

- identify bearings,
- draw a scale diagram of locations in a horizontal plane when bearings and distances are given, and find unknown quantities using the scale diagram.

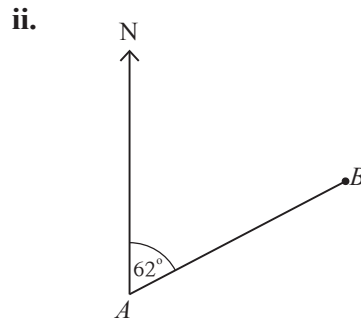
### 27.1 Bearing

Bearing is a measurement that is used to indicate a direction in a horizontal plane.

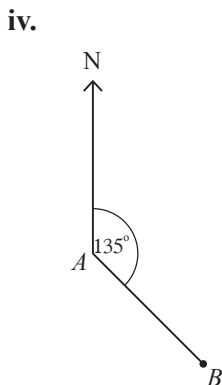
The bearing of the point  $B$  from the point  $A$  is the angle that the direction  $AB$  makes with the direction of North when measured from  $A$  in a clockwise direction. The following figures illustrate the bearing of  $B$  from  $A$  for different locations of  $A$  and  $B$ . Observe that the bearing is given in three digits.



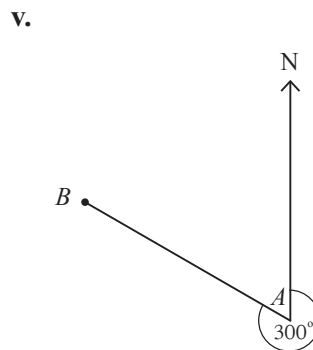
The bearing of  $B$  from  $A = 008^\circ$



The bearing of  $B$  from  $A = 062^\circ$



The bearing of  $B$  from  $A = 135^\circ$



The bearing of  $B$  from  $A = 300^\circ$

Since a bearing is always less than  $360^\circ$ , the maximum number of digits it can have is three. Therefore the norm is to always write bearings with three digits. If the angle is one of  $1^\circ, 2^\circ, \dots, 9^\circ$ , then the bearing is written as  $001^\circ, 002^\circ, \dots, 009^\circ$  and if the angle is one of  $10^\circ, 11^\circ, \dots, 99^\circ$ , then it is written as  $010^\circ, 011^\circ, \dots, 099^\circ$ .

Accordingly bearing is,

- i. measured starting from the North,
- ii. measured in a clockwise direction,
- iii. written with three digits.

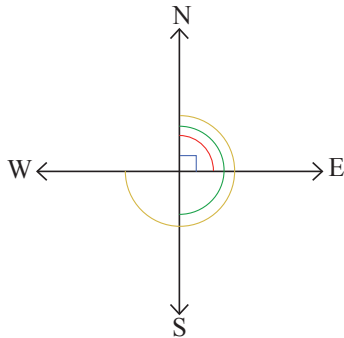
Since the North can be easily identified using a compass, bearings are used widely in sea and air travel.

Let us broaden our knowledge on bearings by studying the examples given below.

**Example 1**

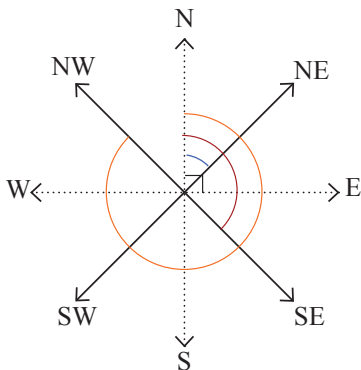
- i. Indicate the four main directions (cardinal directions) in terms of bearings.
- ii. Indicate the four sub-directions (intermediate directions) in terms of bearings.

i.



Direction	Bearing
North	$000^\circ$
East	$090^\circ$
South	$180^\circ$
West	$270^\circ$

ii.

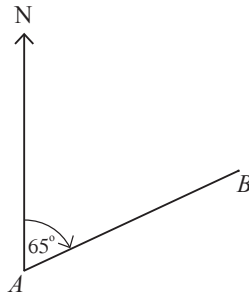


Direction	Bearing
Northeast	$045^\circ$
Southeast	$135^\circ$
Southwest	$225^\circ$
Northwest	$315^\circ$

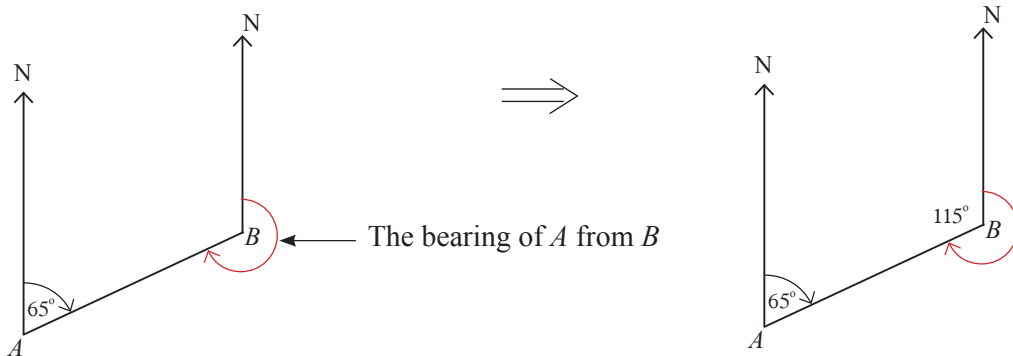
### Example 2

The bearing of  $B$  from  $A$  is  $065^\circ$ . Illustrate this information in a rough sketch and find the bearing of  $A$  from  $B$ .

Since the bearing of  $B$  from  $A$  is  $065^\circ$ , the angle drawn from the direction of North at  $A$  to the direction of  $AB$  in the clockwise direction is  $65^\circ$ .



Now, to find the bearing of  $A$  from  $B$ , a line needs to be drawn in the direction of North from  $B$ , and the angle that is formed when this line is rotated in a clockwise direction about  $B$  from the direction of North to the direction of  $BA$  needs to be found.



The lines drawn at  $A$  and  $B$  in the direction of North are parallel. The pair of allied angles formed by the transversal  $AB$  intersecting these lines are supplementary. Using this fact, the value  $115^\circ$  has been found. The required bearing is indicated in the figure given above.

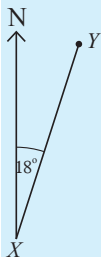
As the sum of the angles around a point is  $360^\circ$ ,

$$\begin{aligned} \text{the bearing of } A \text{ from } B &= 360^\circ - 115^\circ \\ &= \underline{\underline{245^\circ}} \end{aligned}$$

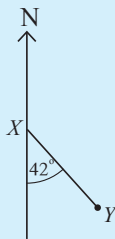
**Exercise 27.1**

1. In each of the following situations, find the bearing of  $Y$  from  $X$ .

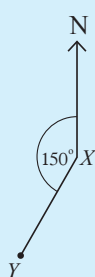
**i.**



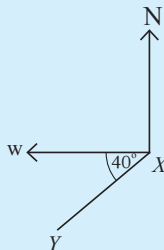
**ii.**



**iii.**



**iv.**



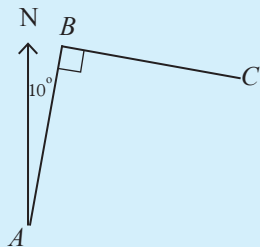
**v.**



2. Using a protractor to measure the angles, illustrate each of the following bearings by drawing a figure.

- i.** The bearing of  $F$  from  $E$  is  $005^\circ$ .
- ii.** The bearing of  $Q$  from  $P$  is  $075^\circ$ .
- iii.** The bearing of  $N$  from  $M$  is  $105^\circ$ .
- iv.** The bearing of  $H$  from  $J$  is  $270^\circ$ .
- v.** The bearing of  $D$  from  $C$  is  $310^\circ$ .

3.



Based on the information given in the figure,

- i.** determine the bearing of  $B$  from  $A$ ,
- ii.** determine the bearing of  $A$  from  $B$ ,
- iii.** determine the bearing of  $B$  from  $C$ .

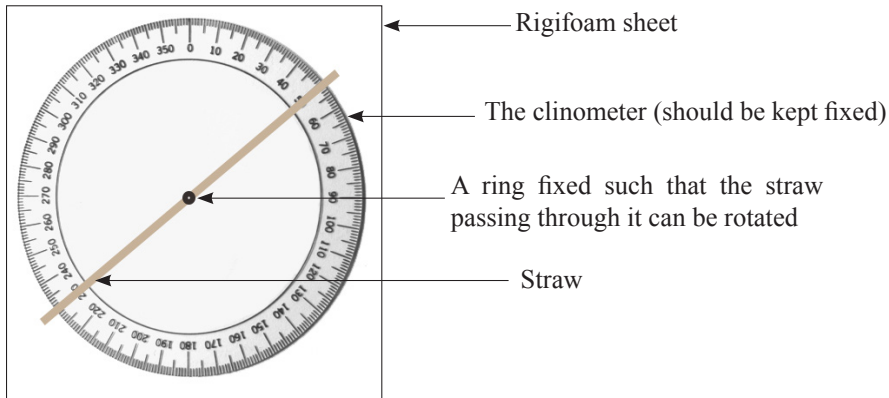
4.  $ABC$  is an equilateral triangle.  $B$  is situated to the north of  $A$ .

- i.** Illustrate this information by a rough sketch.
- ii.** By considering the sketch determine the following.
  - a.** Bearing of  $B$  from  $A$
  - b.** Bearing of  $C$  from  $A$
  - c.** Bearing of  $C$  from  $B$
  - d.** Bearing of  $B$  from  $C$
  - e.** Bearing of  $A$  from  $C$
  - f.** Bearing of  $A$  from  $B$

## 27.2 Clinometer

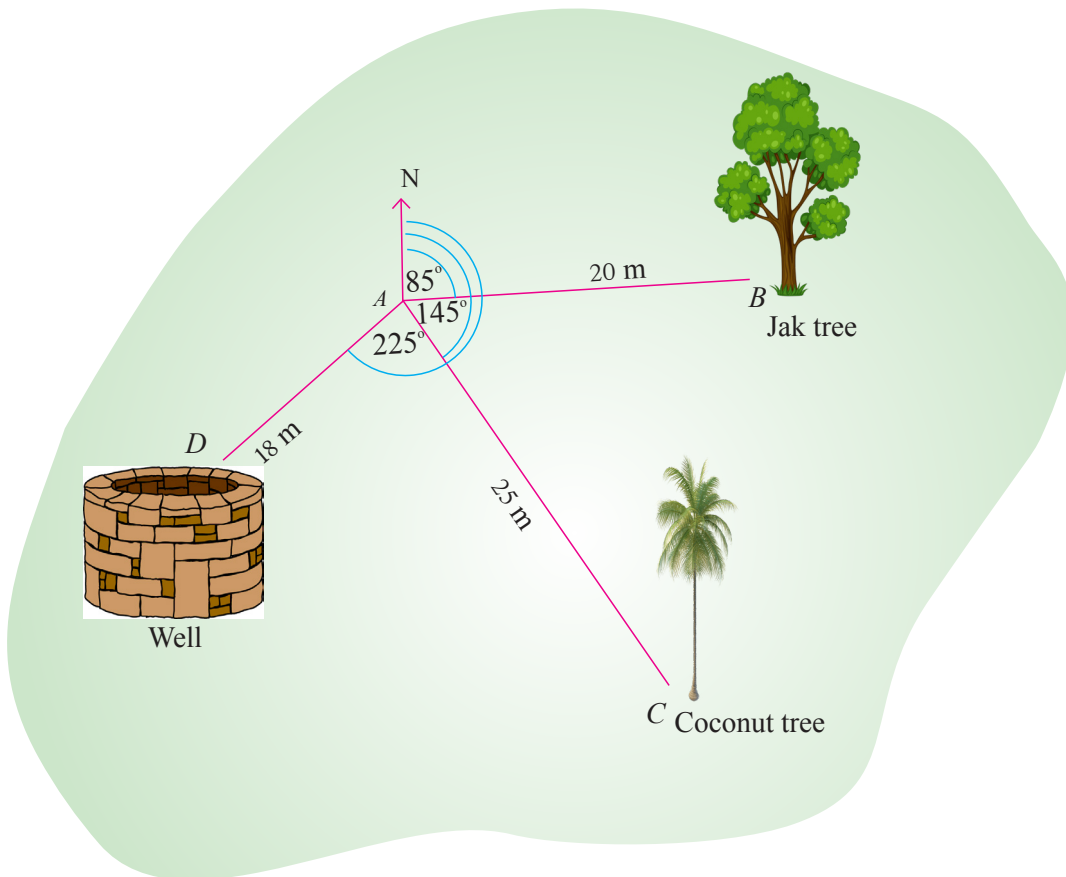
Any location in a horizontal plane can be described in terms of bearings and distances. A clinometer can be used to determine bearings.

### Clinometer



- Place a compass on the horizontal tabletop of a table kept at  $A$ . Suppose for example that we want to describe the location of  $B$  with respect to location  $A$  and mark the direction of North on the tabletop.
- Place the clinometer on the tabletop such that the "0" on the clinometer is towards the North.
- Rotate the straw until the location  $B$  is observed through the straw and measure the clockwise angle of rotation from the direction of North. By writing it using three digits, the bearing of  $B$  is obtained.
- By measuring the distance from  $A$  to  $B$  using a measuring tape, the position of  $B$  can be described in terms of the distance and bearing from  $A$ .

In the following figure the bearings of  $B$ ,  $C$  and  $D$  with respect to  $A$  are given.

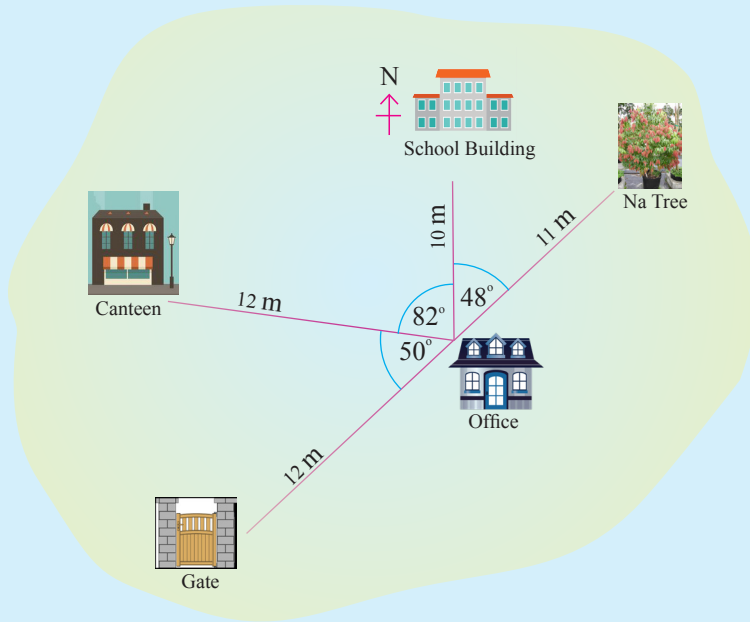


Object which was observed	Bearing	Distance
Jak tree ( $B$ )	$085^\circ$	20 m
Coconut tree ( $C$ )	$145^\circ$	25 m
Well ( $D$ )	$225^\circ$	18 m

Do the following exercise to broaden your knowledge on this topic.

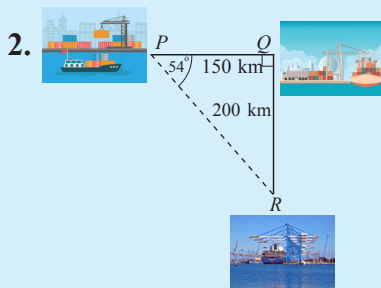
**Exercise 27.2**

1. A rough school plan is shown below.



Using it describe the following.

- i. The location of the Na tree with respect to the School Office
- ii. The location of the gate with respect to the School Office
- iii. The location of the Canteen with respect to the School Office



$P$ ,  $Q$  and  $R$  denote three harbours located in the same ocean.  $Q$  is to the East of  $P$ . Describe the route in terms of the bearing and distance that a ship needs to take to journey,

- i. from harbour  $P$  to harbour  $R$  through  $Q$ .
- ii. directly from harbour  $P$  to harbour  $R$ .

3. A pilot of a certain air plane which is scheduled to fly from Colombo to a certain airport has been instructed to fly 100 km on a bearing of  $020^\circ$  and then another 100 km on a bearing of  $080^\circ$ .

- i. Represent this information in a rough sketch

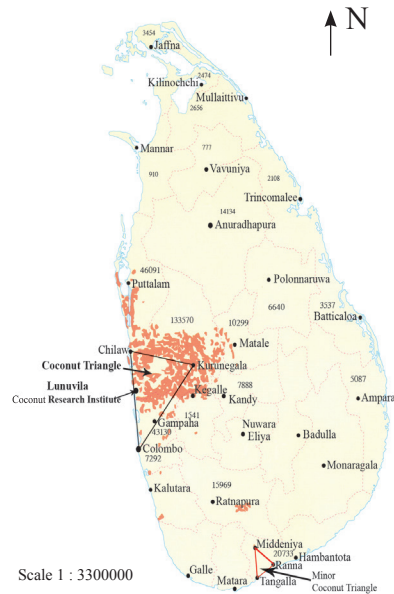
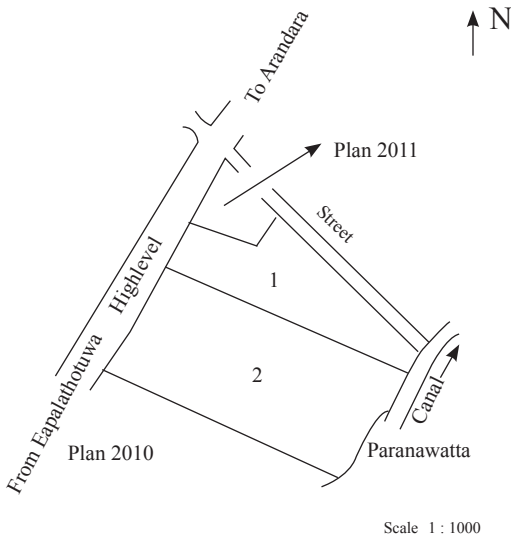
- ii. Write a description of the route that the pilot needs to take if he is to fly back to Colombo from that airport along the same path.

### 27.3 Scale diagrams in a horizontal plane

Given below are two examples of scale diagrams in a horizontal plane.

Survey Plane No: 103

Coconut Cultivation



In every scale diagram, the scale to which the diagram is drawn is given, and the direction of North is marked. It is very important to understand what is meant by the scale (ratio) given in the scale diagram. For example, a scale of 1 : 500 000 means that a distance of 500 000 cm is represented by 1 cm in the scale diagram. In other words, the distance between two points on the scale diagram is  $\frac{1}{500\,000}$ th of the actual distance between the two points. Moreover, since 500 000 cm is equal to 5 km, the actual distance represented by 1 cm in the scale diagram is 5 km.

Now let us learn how to draw scale diagrams by considering some examples.

#### Example 1

The vertices of a triangular floor area are  $A$ ,  $B$  and  $C$ . The positions of the vertices with respect to a point  $P$  located in this area is given below.

With respect to  $P$ ,

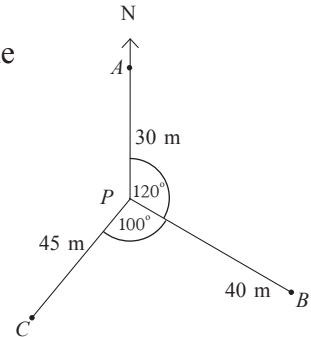


- $A$  is located 30 m away on a bearing of  $000^\circ$
- $B$  is located 40 m away on a bearing of  $120^\circ$
- $C$  is located 45 m away on a bearing of  $220^\circ$

Draw a scale diagram of the floor area using this information and find its perimeter.

**Step 1:** Mark the direction of North at the top right hand corner of the sheet of paper.

**Step 2:** Draw a rough sketch as shown, based on the information that is given.



**Step 3:** To represent the distances 30 m, 40 m and 45 m, select the scale of 1 cm representing 10 m, that is, the scale of 1:1000. (Here, the scale should be selected according to the size of the sheet of paper. Moreover, by selecting a value such as 1000, anyone who is examining the scale diagram can easily get an idea of the actual distances represented in it.)

**Step 4:** For each distance that is to be represented in the scale diagram, calculate the corresponding length using the selected scale.

$$PA = 3000 \times \frac{1}{1000} \text{ cm} = 3 \text{ cm}$$

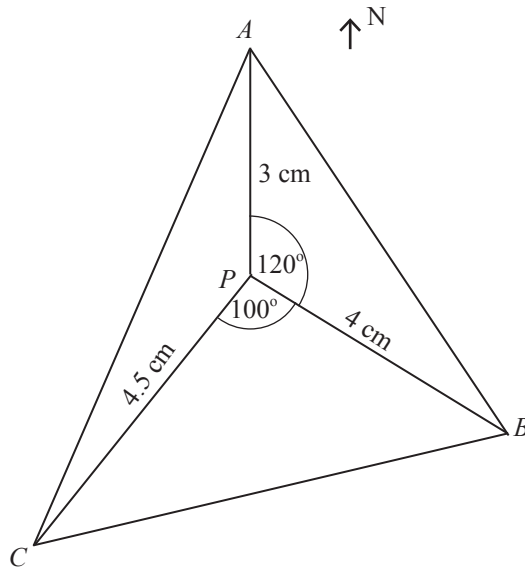
$$PB = 4000 \times \frac{1}{1000} \text{ cm} = 4 \text{ cm},$$

$$PC = 4500 \times \frac{1}{1000} \text{ cm} = 4.5 \text{ cm}$$

**Step 5:** Using a straight edge with a cm scale and a protractor, draw the scale diagram with a pencil as shown below.

- First draw the line segment  $AP$  of length 3cm upwards.
- Draw the line segment  $PB$  of length 4 cm which makes an angle of  $120^\circ$  clockwise with  $PA$ .
- Draw the line segment  $PC$  of length 4.5 cm which makes an angle of  $100^\circ$  clockwise with  $PB$ .

- Draw the line segments  $AB$ ,  $BC$  and  $AC$ .



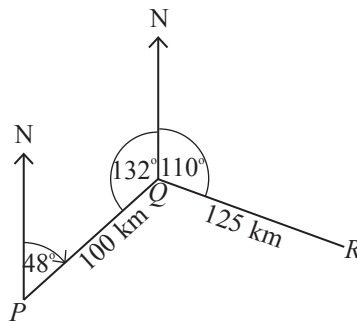
**Step 6:** Measure the lengths of  $AB$ ,  $BC$  and  $AC$ . You will see that  $AB = 6$  cm,  $AC = 7.1$  cm and  $BC = 6.5$  cm. Therefore the perimeter of the scale diagram is  $6 + 7.1 + 6.5 = 19.6$  cm.

**Step 7:** Using the scale  $1$  cm  $\longrightarrow$   $10$  m, calculate the actual length.  
Perimeter of the floor =  $10 \times 19.6 = 196$  m

### Example 2

A ship journeying from harbour  $P$  approaches harbour  $Q$  after travelling  $100$  km on a bearing of  $048^\circ$ . It then travels  $125$  km on a bearing of  $110^\circ$  and approaches harbour  $R$ . Draw a scale diagram and describe the position of  $R$  with respect to  $P$ .

**Step 1:** Based on the information given, draw a rough sketch as shown below.



**Step 2:** Mark a point  $P$  on a sheet of paper and mark the direction of North upwards.

- Since the bearing of  $Q$  from  $P$  is  $048^\circ$ , the angle that  $PQ$  makes with the direction of North at  $P$  is  $48^\circ$  in the clockwise direction.
- Since the bearing of  $R$  from  $Q$  is  $110^\circ$ , the angle that  $QR$  makes with the direction of North at  $Q$  is  $110^\circ$  in the clockwise direction.

Since the direction of North at  $P$  and the direction of North at  $Q$  are parallel, the angle formed between the direction of North at  $Q$  and  $PQ$  is  $132^\circ$  (allied angles)

$$\begin{aligned}\text{Therefore, } \hat{PQR} &= 360^\circ - (132^\circ + 110^\circ) \\ &= 360^\circ - 242^\circ \\ &= 118^\circ\end{aligned}$$

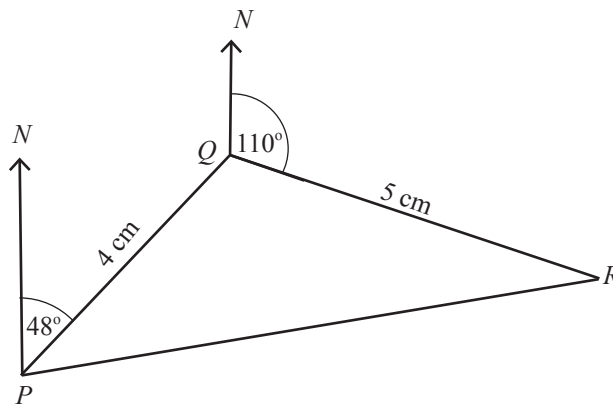
**Step 3:** Select the scale of 1 cm representing 25 km, that is, 1 : 2 500 000 to represent the distances 100 km and 125 km in the scale diagram. (If there is space on the sheet of paper, the scale of 1 : 1 250 000 can also be used).

**Step 4:** According to the selected scale, calculate the lengths of by which  $PQ$  and  $QR$  are to be represented in the scale diagram.

$$PQ = \frac{100}{25} \text{ cm} = 4 \text{ cm}, \quad QR = \frac{125}{25} \text{ cm} = 5 \text{ cm}$$

(When drawing scale diagrams, the magnitudes of the angles do not change.)

**Step 5:** Draw the scale diagram using a straight edge, a protractor and a pencil, based on the above measurements.



**Step 6:** When  $PR$  is measured, we obtain  $PR = 7.7$  cm. When  $\hat{NPR}$  is measured, we obtain  $\hat{NPR} = 82^\circ$ .

**Step 7:** Using the scale, calculate the actual length of  $PR$ .

$$\begin{aligned}\text{Actual length of } PR &= 7.7 \times 25 \text{ km} \\ &= 192.5 \text{ km}\end{aligned}$$

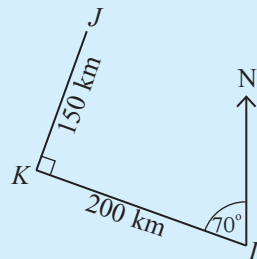
**Step 8:** The position of  $R$  can be described as follows.  
 $R$  is situated 192.5 km from  $P$  on a bearing of  $082^\circ$ .

**Exercise 27.3**

1. A rough sketch of the route of a ship travelling from harbour  $L$  to harbour  $K$  and then from harbour  $K$  to harbour  $J$  is given.

i. Find the following based on this rough sketch.

- Bearing of  $K$  from  $L$
- Bearing of  $J$  from  $K$
- The lengths of  $LK$  and  $KJ$  in a scale diagram drawn to the scale of 1 cm representing 50 km.



ii. Using the above scale, draw a scale diagram of the route of the ship.

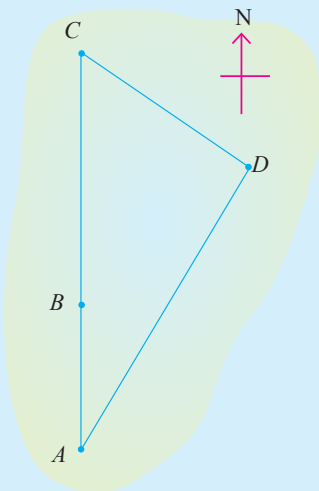
iii. Using the scale diagram,

- find the distance from harbour  $L$  to harbour  $J$
- find the bearing of harbour  $L$  from harbour  $J$ .

vi. Using the Pythagorean relation, calculate the distance from harbour  $L$  to harbour

$J$  and check whether the answer you obtained in (iii) (a) above is correct.

2.



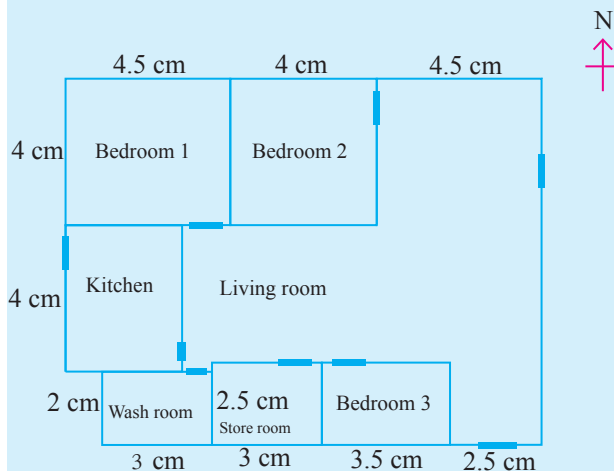
A portion of a map drawn to the scale of 1 : 50 000 is shown here. While the cities  $A$ ,  $B$  and  $C$  are situated on the same straight line,  $C$  lies directly north of  $A$ .

- Measure and write the lengths of the line segments  $AB$ ,  $BC$ ,  $CD$  and  $AD$  and the magnitudes of the angles  $\hat{ACD}$ ,  $\hat{ADC}$  and  $\hat{CAD}$ .
- Calculate the actual distances of  $AB$ ,  $BC$ ,  $CD$  and  $AD$ .
- Describe the locations of  $B$ ,  $C$  and  $D$  with respect to  $A$  in terms of bearings and distances from  $A$ .

3. The School Office is located at a distance of 10 m and on a bearing of  $025^\circ$  from the school flag post. The Main Hall is located at a distance of 12 m and on a bearing of  $310^\circ$  from the school flag post.

- Draw a rough sketch based on the above information.
- Draw a scale diagram based on the sketch, using the scale of 1 cm representing 2 m.

- iii. Using the scale diagram, find the shortest distance between the Office and the Main Hall.
- iv. Describe the location of the Main Hall with respect to the Office.
4. A pilot flies a plane 80 km on a bearing of  $150^\circ$  and then 150 km on a bearing of  $200^\circ$  and arrives at airport  $B$  from airport  $A$ .
- Draw a rough sketch based on the above information.
  - Draw a scale diagram using a suitable scale and find,
    - the bearing of  $B$  from  $A$
    - the distance from  $A$  to  $B$
    - the bearing of  $A$  from  $B$ .
5. The floor plan of a house to be constructed, which is drawn to scale is shown below. Answer the questions given below using the scale diagram.



- If the actual length of bedroom 2 is 4 m, express the scale to which this plan is drawn, as a ratio.
  - Find the actual breadth of the house.
  - Find the actual area of the washroom in square meters.
6. A person standing on a straight road that runs from East to West across a carnival ground, observes a flag post on a bearing of  $115^\circ$ . When he travels 220 m to the East along the road, he see the flag post on a bearing of  $210^\circ$ .
- Describe the final location of the person with respect to the flag post.
  - By drawing a scale diagram, find the shortest distance from the flag post to the road.



7. A portion of a road map of Sri Lanka drawn to the scale of 1 : 1 000 000 is shown here. The main “A” road is highlighted in red.
- i. Find the actual length in kilometres that is represented by 1 cm in the map.
  - ii. With the aid of a string, find the length of the portion of the “A” road which falls between *X* and *Y* in the scale diagram and find the actual distance from *X* to *Y* along this road in kilometres.