# 26 Algebraic Fractions

**By studying this lesson you will be able to;**

- identify algebraic fractions,
- add and subtract algebraic fractions with integral denominators (equal/ unequal denominators)
- add and subtract algebraic fractions with equal algebraic denominators.

We have already learnt to add and subtract numerical fractions and simplify, expand and factorize algebraic expressions.

Do the following review exercise to recall what has been learnt earlier.

## Review Exercise

**1.** Simplify.

**i.** $\frac{2}{5} + \frac{1}{5}$ **ii.** $\frac{5}{7} - \frac{2}{7}$ **iii.** $\frac{1}{9} - \frac{7}{9} + \frac{4}{9}$ **iv.** $\frac{12}{13} - \frac{2}{13} - \frac{1}{13}$

**2.** Fill in each box with the appropriate number.

**i.** $\frac{1}{2} - \frac{1}{4}$

$= \frac{1 \times \square}{2 \times 2} - \frac{1}{4}$

$= \frac{\square - 1}{4}$

$= \frac{\square}{4}$

**ii.** $\frac{3}{4} - \frac{2}{3}$

$= \frac{3 \times \square}{4 \times 3} - \frac{\square \times 4}{3 \times 4}$

$= \frac{\square - \square}{12}$

$= \frac{\square}{12}$

**iii.** $\frac{4}{5} - \frac{3}{10} - \frac{1}{3}$

$= \frac{4 \times \square}{5 \times 6} - \frac{3 \times \square}{10 \times 3} - \frac{1 \times 10}{3 \times \square}$

$= \frac{\square - \square - 10}{30}$

$= \frac{\square}{30}$

$= \frac{\square \div 5}{30 \div 5}$

$= \frac{\square}{6}$

3. Simplify the algebraic expressions given below.

a. $2x + 3x$
b. $3y - y$
c. $5a + 4a + a$
d. $5x + 3y + x + 3y$
e. $3y + 2 - y - 2$
f. $4n - 1 + 5 - 2n$
g. $-3y + 2 - y - 3 + 2y$
h. $5xy - 6xy + 3x + y$

4. Expand and simplify.

a. $2(x + y) + 3x$
b. $3(2x - 4y) + 12y$
c. $-(4 - 3x) - 1$
d. $2(3x - 2) + 3(x + 2)$
e. $3(m + 1) - 2(2m - 1)$
f. $x(x - y) + 2xy$

5. For each of the statements given below, if it is true, mark a "✓" and if it is false, mark a "×" in the box to the right of the statement.

a. The value of $\frac{2}{3} + \frac{1}{4}$ is the same as the value of $\frac{2+1}{3+4}$ . ☐

b. To obtain the sum or the difference of two fractions, their numerators should be equal; if they are not equal, then they should be made equal. ☐

c. The numerator of the sum of two unit fractions is the sum of the denominators of the original two fractions and the denominator is the product of the denominators of the original two fractions. ☐

d. When adding or subtracting two fractions with unequal denominators, the common denominator that should be used is the L. C. M. of the denominators of the original two fractions. ☐

e. By multiplying the numerator and the denominator of a fraction by the same number, we can convert it into its simplest equivalent form. ☐

f. By dividing the numerator and the denominator of a fraction by the same number, we can convert it into its simplest equivalent form. ☐

g. $-3x - 2x$ can be considered as $(-3x) + (-2x)$ . ☐

h. To expand $-3(2x - 5)$, the terms $2x$ and $-5$ need to be multiplied by 3. ☐

i. When $-x - x$ is simplified we obtain $2x$ . ☐

j. When $3x + 4y$ is simplified we obtain $7xy$. ☐

## Introduction to algebraic fractions

If the numerator or denominator or both the numerator and the denominator of a fraction contain an algebraic term or expression, then that fraction is known as an algebraic fraction.

**Example 1**

Write 5 algebraic fractions that have an algebraic term only in the numerator.

$$\frac{x}{2}, \frac{3x}{5}, \frac{7y}{20}, \frac{6mn}{3}, \frac{2t^2}{5}$$

**Example 2**

Write 5 algebraic fractions that have an algebraic expression only in the numerator.

$$\frac{x+1}{5}, \frac{2x-1}{3}, \frac{x+y}{2}, \frac{m-n}{7}, \frac{3m-2n-1}{10}$$

**Example 3**

Write 5 algebraic fractions that have an algebraic term only in the denominator.

$$\frac{3}{x}, \frac{2}{3m}, \frac{5}{2y}, \frac{4}{3xy}, \frac{5}{m^2}$$

**Example 4**

Write 5 algebraic fractions that have an algebraic expression only in the denominator.

$$\frac{3}{2x+1}, \frac{2}{a+b}, \frac{5}{2m-n}, \frac{4}{3x-2y}, \frac{1}{3x+cy+2}$$

**Example 5**

Write 5 algebraic fractions which have an algebraic term in both the numerator and the denominator.

$$\frac{a}{c}, \frac{2a}{d}, \frac{2m}{3n}, \frac{4x}{5y}, \frac{2xy}{3pq}, \frac{2x^2}{5y^2}$$

**Example 6**

Write 5 algebraic fractions that have an algebraic expression in the numerator and an algebraic term in the denominator.

$$\frac{x+1}{2x},\ \frac{2a+b}{c},\ \frac{3a+d}{4a},\ \frac{2x-1}{c},\ \frac{4x^2y-a^2}{b}$$

**Example 7**

Write 5 algebraic fractions that have an algebraic term in the numerator and an algebraic expression in the denominator.

$$\frac{x}{2x+5},\ \frac{a}{5b+d},\ \frac{3c}{a+b},\ \frac{4xy}{5x-3},\ \frac{a^2}{a-b}$$

**Example 8**

Write 5 algebraic fractions that have algebraic expressions in both the numerator and the denominator.

$$\frac{x+1}{2x-1},\ \frac{x+y}{3x+2y},\ \frac{3x-4}{x+1},\ \frac{4m-3n}{5m+2n},\ \frac{4x-y}{2x+3y-4}$$

## 26.1 Adding and subtracting algebraic fractions with equal integral denominators

We can add and subtract algebraic fractions in the same way that we added and subtracted fractions with whole numbers in the numerator and denominator.

**Example 1**

Express $\frac{5x}{9} + \frac{2x}{9}$ as a single fraction.

$\frac{5x}{9} + \frac{2x}{9} = \frac{5x+2x}{9}$ (since the denominators of both fractions are equal)

$= \frac{7x}{9}$

**Example 2**

Simplify $\frac{5y}{7} - \frac{3y}{7}$ .

$\frac{5y}{7} - \frac{3y}{7} = \frac{5y - 3y}{7}$ (since the denominators of both fractions are equal)

$= \frac{2y}{7}$

**Example 3**

Simplify $\frac{4x}{15} + \frac{7x}{15} - \frac{2x}{15}$ .

$\frac{4x}{15} + \frac{7x}{15} - \frac{2x}{15} = \frac{11x - 2x}{15}$ (since the denominators of both fractions are equal)

$= \frac{9x}{15} = \frac{3x}{5}$ (by dividing by 3, the highest common factor of 9 and 15)

**Example 4**

Simplify $\frac{x + 1}{5} + \frac{x + 2}{5}$ .

$\frac{x + 1}{5} + \frac{x + 2}{5} = \frac{x + 1 + x + 2}{5}$ (since the denominators of both fractions are equal)

$= \frac{x + x + 1 + 2}{5}$

$= \frac{2x + 3}{5}$

**Example 5**

Simplify $\frac{2b + 3}{7} - \frac{b + 2}{7}$ .

$\frac{2b + 3}{7} - \frac{b + 2}{7} = \frac{2b + 3 - (b + 2)}{7}$ (the algebraic expression to be subtracted must be written within brackets)

$= \frac{2b + 3 - b - 2}{7}$

$= \frac{2b - b + 3 - 2}{7}$

$= \frac{b + 1}{7}$

**Example 6**

Simplify $\frac{7c+1}{8} - \frac{2c+1}{8} - \frac{c-2}{8}$ .

$$\frac{7c+1}{8} - \frac{2c+1}{8} - \frac{c-2}{8} = \frac{7c+1-(2c+1)-(c-2)}{8}$$

$$= \frac{7c+1-2c-1-c+2}{8}$$

$$= \frac{4c+2}{8}$$

$$= \frac{2(2c+1)}{8}$$

$$= \underline{\underline{\frac{2c+1}{4}}}$$

**Exercise 26.1**

**1.** Simplify and write the answer in the simplest form.

**a.** $\frac{a}{5} + \frac{a}{5}$ **b.** $\frac{3d}{15} + \frac{2d}{15}$ **c.** $\frac{2t}{3} - \frac{t}{3}$

**d.** $\frac{7k}{8} - \frac{3k}{8}$ **e.** $\frac{3k}{7} + \frac{2k}{7} + \frac{k}{7}$ **f.** $\frac{5h}{9} - \frac{2h}{9} - \frac{h}{9}$

**g.** $\frac{7v}{10} - \frac{3v}{10} + \frac{v}{10}$ **h.** $\frac{x}{8} - \frac{3x}{8}$ **i.** $\frac{p}{9} - \frac{4q}{9} - \frac{5p}{9}$

**2.** Simplify and write the answer in the simplest form.

**a.** $\frac{3y+1}{5} + \frac{2y+2}{5}$ **b.** $\frac{4m-1}{7} + \frac{3m-2}{7}$ **c.** $\frac{5n+3}{8} + \frac{2n-1}{8}$

**d.** $\frac{5c-2}{10} + \frac{3c+4}{10}$ **e.** $\frac{6d+1}{10} - \frac{2d-3}{10}$

**f.** $\frac{3x+1}{6} - \frac{2x-3}{6} + \frac{x+4}{6}$

## 26.2 Adding and subtracting algebraic fractions with unequal integral denominators

Now let us consider how to simplify expressions of algebraic fractions with unequal integral denominators such as $\frac{x}{6} + \frac{3x}{4}$ . These types of fractions can be simplified in the same way that numerical fractions are simplified. A common multiple of the denominators of the fractions can be taken as the common denominator. However simplification is made easier by taking the least common multiple of the denominators.

For example, the denominators of the above two fractions are 6 and 4. Their least common multiple is 12. Therefore, initially, the above fractions need to be converted into fractions with denominator 12. To convert $\frac{x}{6}$ into a fraction with denominator 12, the denominator and numerator of $\frac{x}{6}$ need to be multiplied by 2. (Observe that 2 is obtained from $\frac{12}{6}$ ). Similarly, to convert $\frac{3x}{4}$ into a fraction with denominator 12, we need to multiply the numerator and denominator of $\frac{3x}{4}$ by 3. (Observe that 3 is obtained from $\frac{12}{4}$). Accordingly, we may write the following to simplify the given expression.

$$\frac{x}{6} + \frac{3x}{4} = \frac{2}{2} \times \frac{x}{6} + \frac{3}{3} \times \frac{3x}{4}$$

When we simplify the numerator and the denominator of these fractions we obtain the following.

$$\frac{2x}{12} + \frac{9x}{12}$$

Now since both fractions have a common denominator, we can write the above as follows.

$$\frac{2x + 9x}{12}$$

By simplifying this we get $\frac{11x}{12}$.

Accordingly, $\frac{x}{6} + \frac{3x}{4} = \frac{11x}{12}$.

**Example 1**

Simplify $\frac{2y}{5} + \frac{y}{4}$.

$\frac{2y}{5} + \frac{y}{4} = \frac{4 \times 2y}{4 \times 5} + \frac{5 \times y}{5 \times 4}$ (Since the L. C. M. of 5 and 4 is 20, equivalent fractions with 20 as the denominator are obtained.)

$= \frac{8y}{20} + \frac{5y}{20}$

$= \frac{8y + 5y}{20} = \frac{13y}{20}$

**Example 2**

Simplify $\frac{2t}{3} - \frac{t}{2}$.

$\frac{2t}{3} - \frac{t}{2} = \frac{2 \times 2t}{2 \times 3} - \frac{3 \times t}{3 \times 2}$ (since the L.C.M. of 3 and 2 is 6, equivalent fractions with 6 as the denominator are obtained)

$= \frac{4t}{6} - \frac{3t}{6}$

$= \frac{4t - 3t}{6}$

$= \frac{t}{6}$

**Example 3**

Simplify $\frac{3v}{2} - \frac{4v}{5} + \frac{3v}{4}$.

$\frac{3v}{2} - \frac{4v}{5} + \frac{3v}{4} = \frac{10 \times 3v}{10 \times 2} - \frac{4 \times 4v}{4 \times 5} + \frac{5 \times 3v}{5 \times 4}$ (since the L.C.M. of 2, 4 and 5 is 20, equivalent fractions with 20 as the denominator are obtained)

$= \frac{30v}{20} - \frac{16v}{20} + \frac{15v}{20}$

$= \frac{29v}{20}$

You may have observed in the above examples that when the denominators are not equal, it is easy to simplify by taking the L.C.M. of the unequal denominators as the common denominator.

Now let us consider instances where we have to multiply an algebraic expression by a number. Here it is important to remember to write the algebraic expression within brackets.

**Example 4**

Simplify $\frac{x + 1}{2} + \frac{2x + 1}{3}$.

$\frac{x + 1}{2} + \frac{2x + 1}{3} = \frac{3\,(x + 1)}{3 \times 2} + \frac{2\,(2x + 1)}{2 \times 3}$ (writing the algebraic expression within brackets; L.C.M. of 2 and 3 is 6)

$$= \frac{3x+3}{6} + \frac{4x+2}{6} \quad \text{(expanding)}$$

$$= \frac{7x+5}{6}$$

**Example 5**

$$\frac{5y-1}{6} - \frac{3y-2}{4}$$

$$\frac{5y-1}{6} - \frac{3y-2}{4} = \frac{2\,(5y-1)}{2 \times 6} - \frac{3\,(3y-2)}{3 \times 4} \quad \text{(L.C.M. of 4 and 6 is 12)}$$

$$= \frac{2\,(5y-1)}{12} - \frac{3\,(3y-2)}{12}$$

$$= \frac{2\,(5y-1) - 3\,(3y-2)}{12}$$

$$= \frac{10y - 2 - 9y + 6}{12} \quad \text{(expanding by multiplying by 2 and by } -3)$$

$$= \frac{y+4}{12}$$

**Example 6**

$$\frac{3m+2n}{5} - \frac{2m-n}{10} - \frac{3m-2n}{15}$$

$$= \frac{6\,(3m+2n)}{6 \times 5} - \frac{3\,(2m-n)}{3 \times 10} - \frac{2\,(3m-2n)}{2 \times 15} \quad \text{(L.C.M. of 5, 10 and 15 is 30)}$$

$$= \frac{6\,(3m+2n)}{30} - \frac{3\,(2m-n)}{30} - \frac{2\,(3m-2n)}{30}$$

$$= \frac{18m + 12n - 6m + 3n - 6m + 4n}{30}$$

$$= \frac{6m + 19n}{30}$$

## Exercise 26.2

**1.** Simplify and give the answer in the simplest form.

**a.** $\frac{a}{3} + \frac{a}{6}$ **b.** $\frac{b}{4} + \frac{b}{12}$ **c.** $\frac{5x}{3} - \frac{x}{6}$

**d.** $\frac{3y}{4} - \frac{5y}{16}$ **e.** $\frac{a}{2} + \frac{a}{3}$ **f.** $\frac{c}{3} - \frac{c}{4}$

**g.** $\frac{3n}{7} + \frac{n}{5}$ **h.** $\frac{3d}{10} + \frac{2d}{15}$ **i.** $\frac{5m}{6} - \frac{3m}{10}$

**2.** Simplify and give the answer in the simplest form.

**a.** $\frac{a}{2} + \frac{a}{3} + \frac{a}{4}$ **b.** $\frac{c}{5} + \frac{3c}{10} + \frac{2c}{15}$

**c.** $\frac{3x}{5} + \frac{x}{6} - \frac{2x}{15}$ **d.** $\frac{3n}{4} - \frac{3n}{8} - \frac{n}{2}$

**3.** Simplify and write in the simplest form.

**a.** $\frac{2a}{5} + \frac{3a-2}{6}$ **b.** $\frac{2b-1}{8} + \frac{3b}{12}$

**c.** $\frac{3c+2}{6} + \frac{2c-1}{9}$ **d.** $\frac{5t-3}{10} - \frac{3t}{15}$

**e.** $\frac{2m-n}{12} - \frac{3m+n}{9}$ **f.** $\frac{3y+1}{10} + \frac{2y-1}{5} + \frac{4-y}{20}$

**g.** $\frac{3x-y}{4} + \frac{2x+y}{6} - \frac{5x-2y}{3}$ **h.** $\frac{3y+2}{3} - \frac{y-1}{4} - \frac{2y-3}{8}$

## 26.3 Adding and subtracting algebraic fractions with the same algebraic denominator

As an example of this type of algebraic fraction we have $\frac{2}{5x} + \frac{1}{5x}$. Although the denominators of these fractions are algebraic terms, since they are equal, we can simplify this in the same way that we simplify numerical fractions.

Accordingly, we can simplify the above as,

$$\frac{2}{5x} + \frac{1}{5x} = \frac{2+1}{5x}$$

$$= \frac{3}{5x}$$

**Example 1**

Simplify $\dfrac{4}{7m} + \dfrac{2}{7m}$.

$$\frac{4}{7m} + \frac{2}{7m} = \frac{4+2}{7m}$$

$$= \frac{6}{7m}$$

**Example 2**

Simplify $\dfrac{5}{6n} - \dfrac{1}{6n}$.

$$\frac{5}{6n} - \frac{1}{6n} = \frac{5-1}{6n}$$

$$= \frac{4}{6n}$$

$$= \frac{2}{3n}$$

(simplifying by dividing by the common factor 2)

**Example 3**

Simplify $\dfrac{3a}{4b} + \dfrac{1}{4b} - \dfrac{a}{4b}$.

$$\frac{3a}{4b} + \frac{1}{4b} - \frac{a}{4b} = \frac{3a+1-a}{4b}$$ (common denominator is $4b$)

$$= \frac{2a+1}{4b}$$

**Example 4**

Simplify $\dfrac{3}{x+1} + \dfrac{2}{x+1}$.

Although the denominators are algebraic expressions, since they are equal, this can be simplified in the same manner as above.

$$\frac{3}{x+1} + \frac{2}{x+1} = \frac{3+2}{x+1}$$

$$= \frac{5}{x+1}$$

**Example 5**

Simplify $\dfrac{7}{x-3} - \dfrac{4}{x-3}$.

$$\frac{7}{x-3} - \frac{4}{x-3} = \frac{7-4}{x-3}$$ (common denominator is $x - 3$)

$$= \frac{3}{x-3}$$

## Exercise 26.3

**1.** Simplify and give the answer in the simplest form.

**a.** $\frac{5}{a} + \frac{2}{a}$ **b.** $\frac{8}{x} + \frac{2}{x}$ **c.** $\frac{3}{y} - \frac{1}{y}$

**d.** $\frac{4}{3y} - \frac{2}{3y}$ **e.** $\frac{3}{5t} + \frac{2}{5t}$ **f.** $\frac{h}{2k} + \frac{5h}{2k}$

**g.** $\frac{7}{2n} + \frac{3}{2n} - \frac{1}{2n}$ **h.** $\frac{8}{3v} - \frac{4}{3v} - \frac{1}{3v}$ **i.** $\frac{5}{m} + \frac{2}{m} + \frac{1}{m}$

**j.** $\frac{8}{7xy} - \frac{8}{7xy} + \frac{8}{7xy}$

**2.** Simplify and give the answer in the simplest form.

**a.** $\frac{5}{m+3} + \frac{2}{m+3}$ **b.** $\frac{8}{n+5} + \frac{3}{n+5}$ **c.** $\frac{4}{a+b} + \frac{6}{a+b}$

**d.** $\frac{4x}{x+2y} + \frac{x+y}{x+2y}$ **e.** $\frac{9h}{x+y} - \frac{7h-2}{x+y}$ **f.** $\frac{3x+y}{x-3y} - \frac{2x+4y}{x-3y}$

## 26.4 Simplifying algebraic fractions with algebraic expressions in the numerator and the denominator

**Example 1**

Simplify $\frac{5x}{2x+1} + \frac{3x}{2x+1}$.

$$\frac{5x}{2x+1} + \frac{3x}{2x+1} = \frac{5x+3x}{2x+1} \quad \text{(the common denominator is } 2x+1\text{)}$$

$$= \frac{8x}{2x+1}$$

**Example 2**

Simplify $\frac{7y}{3y-1} - \frac{2y}{3y-1}$.

$$\frac{7y}{3y-1} - \frac{2y}{3y-1} = \frac{7y-2y}{3y-1} \quad \text{(the common denominator is } 3y-1\text{)}$$

$$= \frac{5y}{3y-1}$$

**Example 3**

Simplify $\frac{2x-1}{5x+1}+\frac{3x+2}{5x+1}$ .

$$\frac{2x-1}{5x+1}+\frac{3x+2}{5x+1}=\frac{2x-1+3x+2}{5x+1}$$ (the common denominator is $5x+1$)

$$=\frac{5x+1}{5x+1}$$

$$=\underline{\underline{1}}$$

**Example 4**

Simplify $\frac{9m-1}{5m-1}+\frac{3m}{5m-1}-\frac{2m+1}{5m-1}$ .

$$\frac{9m-1}{5m-1}+\frac{3m}{5m-1}-\frac{2m+1}{5m-1}=\frac{9m-1+3m-(2m+1)}{5m-1}$$ (algebraic expressions to be subtracted need to be written within brackets)

$$=\frac{9m-1+3m-2m-1}{5m-1}$$ (multiplying by the – sign and expanding)

$$=\frac{10m-2}{5m-1}$$

$$=\frac{2\cancel{(5m-1)}}{\cancel{(5m-1)}}$$ (separating out the common factor in the numerator and simplifying)

$$=\underline{\underline{2}}$$

**Exercise 26.4**

**1.** Simplify and write the answer in the simplest form.

**a.** $\frac{k}{3k-1}+\frac{2}{3k-1}$

**b.** $\frac{2h}{5h-2}-\frac{h}{5h-2}$

**c.** $\frac{3t}{3t-1}-\frac{1}{3t-1}$

**d.** $\frac{2k+1}{5k+1}-\frac{k-2}{5k+1}$

e. $\dfrac{2y}{3y+2} - \dfrac{y}{3y+2} + \dfrac{1}{3y+2}$

f. $\dfrac{2a+1}{5a-2} - \dfrac{3a}{5a-2} - \dfrac{3}{5a-2}$

g. $\dfrac{8m+10}{2m+3} - \dfrac{4m+1}{2m+3} + \dfrac{2m}{2m+3}$

h. $\dfrac{m}{m+n} - \dfrac{m-n}{m+n} - \dfrac{m-n}{m+n}$