24 Probability

By studying this lesson you will be able to;

- identify random experiments,
- write the sample space of a random experiment,
- identify the equally likely outcomes of a random experiment,
- find the probability of an event in a sample space when the outcomes are equally likely.

24.1 Random experiment

Let us consider the experiment of an ordinary coin being tossed. When a coin is tossed, we know that the outcome will be either "head turns up" or "tail turns up". That is, we know all the possible outcomes before the experiment is conducted. However, we cannot say with certainly whether head will turn up or tail will turn up. Furthermore, this experiment can be repeated any number of times under the same conditions. Another feature is that we will not be able to identify a pattern in the outcomes when the experiment is repeated. Experiments with the above features are called **random experiments**.

Random experiments have the following common characteristics.

- The experiment can be repeated any number of times under the same conditions.
- All the possible outcomes of the experiment are known before the experiment is carried out.
- The outcome of the experiment cannot be stated with certainty before the experiment is carried out.
- When the experiment is repeated, a pattern cannot be recognized in the outcomes.

Let us consider another example.

Even though all the outcomes of the experiment of rolling an unbiased cubic die with its faces numbered from 1 to 6 and recording the number on the face that turns up are known, it is not possible before carrying out the experiment to state with certainty which outcome will occur. Moreover, this experiment can be repeated any number of times under the same conditions, but a pattern in the outcomes cannot be expected. Therefore, rolling an unbiased cubic die and observing the outcome is a random experiment.

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Exercise 24.1

1. For each of the following experiments, in the column to the right, mark "✓" if it is a random experiment and "×" if it is not a random experiment.

Experiment	Random/not random
1. Rolling an unbiased tetrahedral die with its faces numbered from 1 to 4, and recording the number on the face which touches the table.	
2. Drawing a bead from a bag which contains beads of one color and recording its colour.	
3. Throwing a ball at a target and observing whether it hits the target or not.	
4. Planting 5 radish seeds and recording the number of seeds that germinate in 5 days.	
5. Checking whether a door opens when a key picked at random from a bunch of three keys is used.	
6. Tossing a ball in the air and observing whether it falls to the ground.	
7. Drawing out two cards from a box containing three cards, each with one of the numbers 1, 3 and 5 written on it, and observing whether the sum of the two numbers on the two cards that are drawn is an odd number.	

24.2 Sample Space

All the possible outcomes of a random experiment can be written as a set. This set which consists of all the possible outcomes of a random experiment is called its sample space. It is usually denoted by S.

For example,

in the experiment of tossing a coin and observing the side that turns up, the set of all possible outcomes, that is, the sample space is S = {Head, Tail}. Here $N(S) = 2$.

Similarly, the sample space of the experiment of observing the number which turns up when an unbiased cubic die with its faces numbered from 1 to 6 is rolled is

S = {1, 2, 3, 4, 5, 6}

Here $N(S) = 6$.

Example 1

Write the sample space for the experiment of rolling an unbiased tetrahedral die with its faces numbered from 1 to 4 and recording the number on the face which touches the table.

$S = \{1, 2, 3, 4\}$

$n(S) = 4$

Example 2

Write the sample space for the experiment of drawing a bead from a bag which contains two black beads and three white beads marked B_1, B_2, W_1, W_2, W_3 respectively, which are identical in all other aspects. What is the value of $n(S)$?

$S = \{ B_1, B_2, W_1, W_2, W_3 \}$

$n(S) = 5$

Example 3

There are two cards such that R is written on one side and Y is written on the other side. The cards are tossed simultaniously and the letters turned up is recorded. Write the sample space of this experiment.

Getting R on both cards is denoted by (R, R) and getting R on one card and Y on the other is denoted as (R, Y) etc.. Accordingly,

$S = \{(RR), (RY), (YR), (YY)\}$

Note: An event is a subset of the sample space of a random experiment.

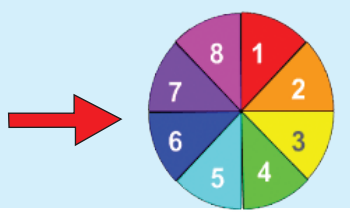
Exercise 24.2

1. Write the sample space of each of the following experiments.

i. Randomly drawing a pen from a bag which contains one pen each of the colors blue, red, black and green and recording the colour. (Assume that the pens are identical in all aspects except the colour)

ii. Recording the number on the card that is drawn at random from a bag containing eleven identical cards numbered 5 to 15.

iii. Recording the number the arrow points to, when the disk shown in the figure is spun and allowed to a stop freely.



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iv. A bag contains 4 milk flavoured toffees and 3 orange flavoured toffees of the same size and shape. Randomly drawing a toffee and recording its flavour.

v. Recording the sides that turn up when a coin is tossed twice.

24.3 Equally likely outcomes

When the sample space of a random experiment is considered, if each outcome is equally likely to occur, then that experiment is called an experiment with equally likely outcomes. The outcomes of such an experiment are called equally likely outcomes.

Let us consider a cubic die with its faces numbered 1, 2, 3, 4, 5 and 6. Let us assume that the material it is made of is uniformly distributed throughout the die. Then it is clear that due to symmetry, each face of the die has an equal chance of turning up when the die is rolled. Similarly for a coin. Objects such as these which are symmetrical and are made of a material which is uniformly distributed are called unbiased or fair objects. Experiments such as these, of tossing a fair coin or rolling an unbiased die are considered as important examples when it comes to explaining the theory of probability.

Consider the experiment of rolling a cuboidal die with its faces marked, 1, 2, 3, 4, 5 and 6 and recording the number on the face that turns up. Here, the likelihood of the different sides turning up may not be the same. Therefore such a die is not considered to be a fair die. In such experiments, the outcomes are not equally likely.

Now let us consider another experiment.

It is clear that in the experiment of an unbiased cubic die with four faces painted red and two faces painted blue being rolled and the colour of the face that turns up being recorded, the chance of a red face turning up is greater than a blue face turning up. Therefore, the outcomes of this experiments are not equally likely.

Exercise 24.3

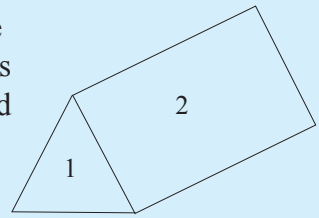
1. For each of the following experiments, determine whether the outcomes are equally likely or not.

i. The four faces of an unbiased tetrahedral die are painted in four different colours, namely, red, blue, yellow and green. Recording the colour of the face which turns up when it is rolled.

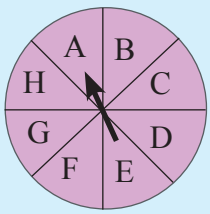
ii. Recording the side which turns up when a fair coin is tossed.

iii. Recording the number on the card which is drawn at random from 10 identical cards which are numbered 1, 1, 1, 1, 2, 2, 2, 3, 3 and 4.

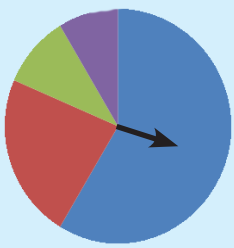
iv. Recording the number on the side which touches the ground when a prism as shown in the figure, with its sides marked with the numbers 1, 2, 3, 4 and 5 is rolled once.



v. Recording the colour of the card drawn randomly from a bag which contains 3 red cards and 4 blue cards which are identical in all other aspects.

vi. 

Recording the letter to which the indicator (arrow) which is fixed at the center of a circular disk points, when the disc which has been divided into 8 equal sectors and named A, B, C, D, E, F, G and H as shown in the figure, is spun and allowed to stop freely.

vii. 

Recording the color on which the indicator (arrow) which is fixed to the centre of the disc falls when it is spun and allowed to stop freely. Here the disc is divided into unequal sectors and shaded with different colours, and placed on a horizon table top.

24.4 Probability of an event when the outcomes are equally likely

You have learnt that the probability of an outcome of a random experiment with equally likely outcomes is given by the following.

$$\text{Probability of a selected outcome} = \frac{1}{\text{total number of outcomes in the sample space of the random experiment}}$$

Consider the experiment of rolling a fair die. Here the probability of a selected outcome is $\frac{1}{6}$ For example, the probability of getting 3 is $\frac{1}{6}$.

Now consider the event of getting an even number. Its probability can be calculated as follows. Since there are three even numbers and three odd numbers, and the

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outcomes of this experiment are equally likely, the probability of getting an even number is $\frac{3}{6}$.

The probability of an event in the sample space of a random experiment with equally likely outcomes is given by the following.

$$\text{Probability of the event} = \frac{\text{Number of elements in the event}}{\text{Number of elements in the sample space}}$$

This can be written using symbols as follows.

If the number of elements in the sample space S is $n(S)$, the number of elements in the event A is $n(A)$ and the probability of event A occurring is $p(A)$, then

$$p(A) = \frac{n(A)}{n(S)}$$

Now let us learn more by considering some examples.

Example 1

Consider the experiment of observing the side that turns up when an unbiased coin is tossed once.

i. Write the sample space of this experiment and find $n(S)$.
ii. If the event A is "head" turns up, write the elements in A and find $n(A)$.
iii. Find $p(A)$, the probability that head turns up.

i. S = {head, tail}
$n(S) = 2$

ii. A = {head}
$n(A) = 1$

iii. $p(A) = \frac{n(A)}{n(S)}$

$p(A) = \frac{1}{2}$

Example 2

Consider the experiment of recording the number on the face that touches the table when an unbiased tetrahedral die with its faces numbered 1, 2, 3 and 4 is rolled.

i. Find the probability of getting 2.
ii. Find the probability of getting an even number.
iii. Find the probability of getting a number greater than 1.

Since the sample space is $S = \{1, 2, 3, 4\}$, $n(S) = 4$.

i. Probability of getting $2 = \frac{1}{4}$

ii. If B is the event of getting an even number,

since $B = \{2, 4\}$, $n(B) = 2$.

$$\therefore p(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}.$$

iii. There are 3 numbers greater than 1. (2, 3, 4)

$\therefore$ the probability of getting a number greater than $1 = \frac{3}{4}$

Exercise 24.4

1. Consider the experiment of rolling an unbiased cubic die with its faces numbered from 1 to 6 and recording the number on the face that turns up.

i. Write the sample space S of all the possible outcomes of this experiment.

ii. Find the value of $n(S)$.

iii. If A is the event of an even number turning up, write the elements of A and find $n(A)$.

iv. Find $P(A)$, the probability of A occurring.

v. Find the probability of a prime number turning up.

2. Consider the experiment of drawing a card at random from a bag containing 8 identical cards marked with the letters A, B, C, D, E, F, G and H and recording the letter on it.

i. Write the sample space.

ii. Find the probability of drawing the card with the letter B marked on it.

iii. Find the probability of drawing a card with a vowel marked on it.

iv. Find the probability of drawing a card with the letter K marked on it.

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3. There are 25 identical cards numbered from 1 to 25 in a box. Consider the experiment of drawing a card at random from the box and recording the number on it.

i. Find the probability of drawing the card with 8 marked on it.

ii. Find the probability of drawing a card with a number which is a multiple of 5 marked on it.

iii. Find the probability of drawing a card with an odd number marked on it.

iv. Find the probability of drawing a card with a square number marked on it.

4. 

Consider the experiment of spinning the disc in the figure and recording the colour of the sector in which the arrow head lands when the disc stops spinning.

i. Find the probability of the arrow head landing in the dark blue sector.

ii. Find the probability of the arrow head landing in the red sector.

iii. Find the probability of the arrow head landing in the yellow sector.

5. In a multiple choice question paper, of the 5 answers that are given for a question, only one is correct. A person picks one of the answers randomly for a question to which he does not know the answer. What is the probability of that answer

i. being correct.

ii. being incorrect.

6. In a bag, there are 3 red beads, 2 black beads and 5 white beads which are identical in all other aspects. Consider the experiment of randomly drawing a bead from the bag and recording its colour.

i. Find the probability of drawing a red bead.

ii. Find the probability of drawing a blue bead.

iii. Find the probability of drawing either a red bead or a white bead.

iv. Find the probability of drawing a black bead.

7. Consider the experiment of recording the day of the week on which a student picked at random was born.

i. Find the probability of the student being a person who was born on a Monday.

ii. Find the probability of the student being a person who was born on a Sunday.

iii. Find the probability of the student being a person who was born on either a Wednesday or a Friday.

iv. Find the probability of the student being a person who was born on a day which is neither a Saturday nor a Sunday.

Summary

In a random experiment with equally likely outcomes,

- the probability of a selected outcome $= \dfrac{1}{\text{total number of outcomes in the sample space of the random experiment}}$
- the probability of the event $= \dfrac{\text{number of elements in the event}}{\text{number of elements in the sample space}}$
- $p(A) = \dfrac{n(A)}{n(S)}$

For free distribution.