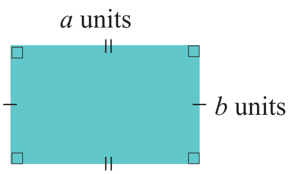
23 Area

By studying this lesson you will be able to;

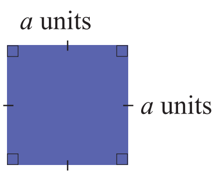
- find the area of a parallelogram,
- find the area of a trapezium,
- find the area of a circle.

Area

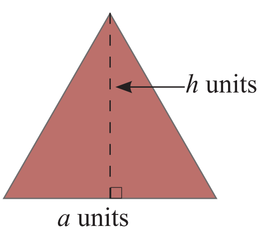
Area can be considered as a quantity that defines the spread of a surface. In grades 7 and 8 you learnt how to find the area of a square lamina, a rectangular lamina and a triangular lamina. Let us recall what was learnt.



If the area of a rectangular lamina of length a units and width b units is A square units, then $A = a \times b$.



If the area of a square lamina of side length a units is A square units, then $A = a^2$.



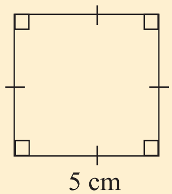
If the area of a triangular lamina of base a units and corresponding perpendicular height h units is A square units, then $A = \frac{1}{2} \times a \times h$.

Do the following review exercise in order to establish these facts further.

Review Exercise

1. Find the area of each plain figure shown below.

i



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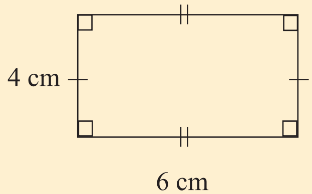

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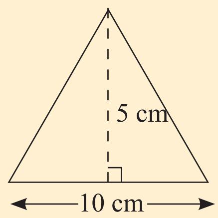

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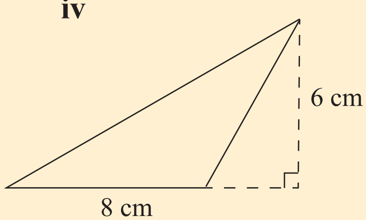

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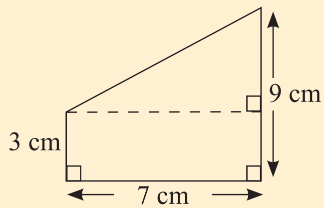

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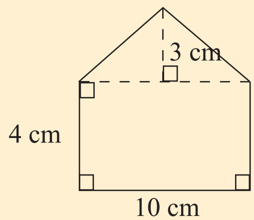
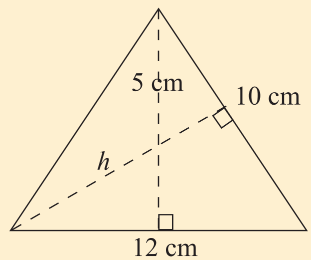


2. In the triangle shown in the figure, the perpendicular height corresponding to the base of length 12 cm is 5 cm, and the perpendicular height corresponding to the base of length 10 cm is h cm.



i. Find the area of the triangle.
ii. Find the value of h.

3. (*a*) What is the perimeter of an equilateral triangular lamina of side length 12 cm?

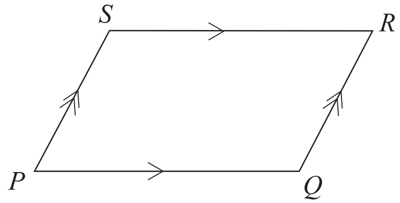
(*b*) Consider a square lamina with the same perimeter as that of the above triangular lamina.

i. Find the side length of the square lamina.
ii. Find the area of the square lamina.

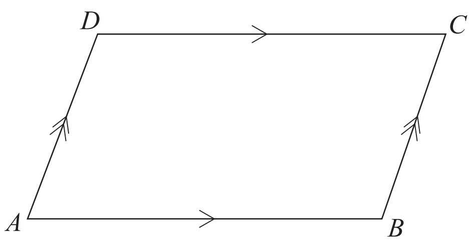
The area of a parallelogram

A quadrilateral with opposite sides parallel to each other is called a parallelogram. We learnt in grade 8 that the opposite sides of a parallelogram are equal. Accordingly, in the parallelogram *PQRS*,

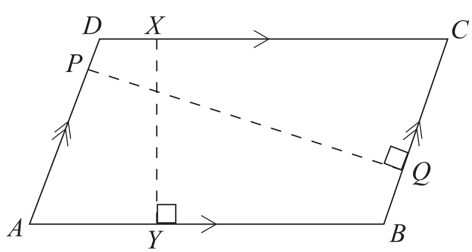
$PQ // SR$ and $PS // QR$.
$PQ = SR$ and $PS = QR$



23.1 The base and height of a parallelogram



Any side of the parallelogram given in the figure can be considered as the base. How the height of the parallelogram corresponding to each base is defined is explained below.

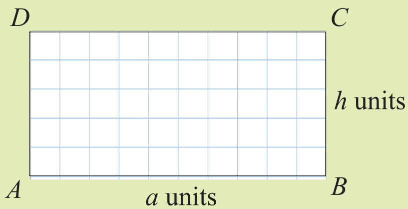


Suppose *AB* is considered as the base of the parallelogram. *AB* and *DC* (the side opposite *AB*) are parallel to each other. According to the figure, the perpendicular distance between these two sides is *XY*. Therefore, *XY* is the perpendicular height corresponding to the base *AB*. When *BC* is considered as the base, the perpendicular distance between the parallel sides *BC* and *AD* according to the figure is *PQ*. Therefore, *PQ* is the perpendicular height corresponding to the base *BC*.

Through the following activity, let us understand how to construct a formula for the area of a parallelogram.

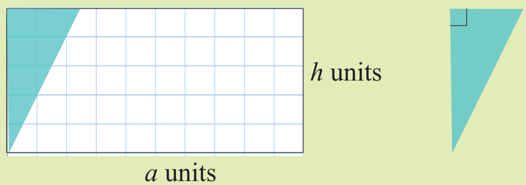
Activity 1

Step 1: In your exercise book, draw a rectangle which is equal in size to that given in the figure. Let us consider its length to be a units and its width to be h units.

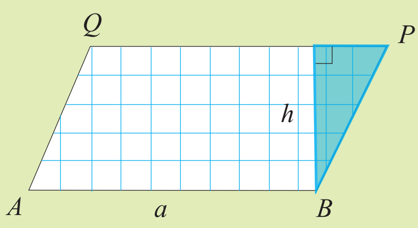


Step 2: Draw another rectangle with the same measurements on another square ruled paper and cut it out.

Step 3: Take that rectangle and cut off the shaded portion as shown below.



Step 4: Create a parallelogram by pasting the triangular portion that was cut off in your exercise book, as shown in the figure. Name it $ABPQ$.



Step 5: Find the area of the rectangle drawn initially, in terms of a and h.

Understand that the area of the parallelogram and the area of the initial rectangle are equal to each other.

Area of parallelogram $ABPQ$ = Area of rectangle $ABCD$

$= a \times h$ square units

Observe that h is the perpendicular height corresponding to the base AB.

According to these facts, a formula for the area of a parallelogram can be given as below.

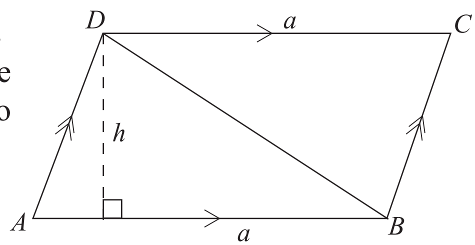
Area of a parallelogram = Length of the base × Perpendicular height corresponding to the base

Let us now consider another method by which the areas of a parallelogram can be found.

The area of a parallelogram can be found by finding the areas of triangles as well.

Consider the parallelogram $ABCD$.

Suppose the length of the base AB is a units and the corresponding height is h units. The parallelogram $ABCD$ is divided into two triangles ABD and BCD by the diagonal DB.



$$\text{The area of triangle } ABD = \frac{1}{2} \times a \times h$$

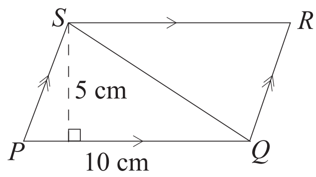
$$\begin{aligned}\text{The area of triangle } BCD &= \frac{1}{2} \times DC \times h \\ &= \frac{1}{2} \times a \times h \text{ (Since } AB = DC)\end{aligned}$$

$$\begin{aligned}\text{Area of parallelogram } ABCD &= \text{Area of triangle } ABD + \text{Area of triangle } BCD \\ &= \frac{1}{2} \times a \times h + \frac{1}{2} \times a \times h \\ &= \frac{ah}{2} + \frac{ah}{2} = \frac{2ah}{2} \\ &= ah\end{aligned}$$

$\therefore$ The area of the parallelogram *ABCD* is *ah* square units.

Example 1

Find the area of the parallelogram $PQRS$.



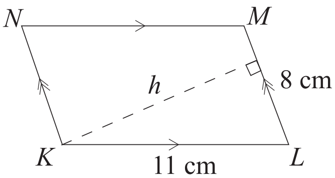
The area of the parallelogram $PQRS = 10 \times 5$

$= 50$

Therefore, the area of the parallelogram is 50 cm^2.

Example 2

If the area of the parallelogram $KLMN$ is 48 cm^2, find the value of h.



The area of the parallelogram $KLMN = 48$ cm^2

$$\text{Therefore, } 8 \times h = 48$$

$$h = \frac{48}{8}$$

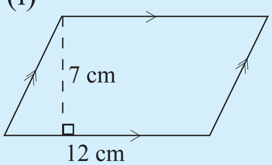
$$h = 6$$

Therefore, $h = 6$ cm.

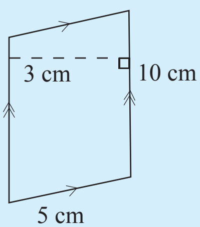
Exercise 23.1

1. Find the area of each parallelogram given below.

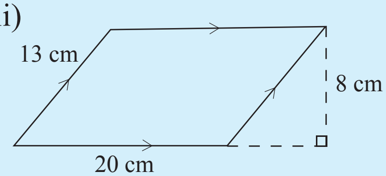
(i)



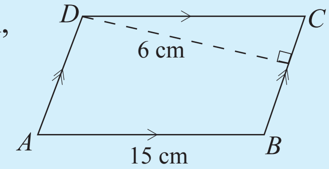
(ii)



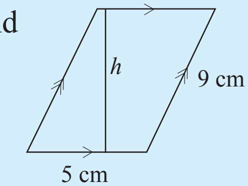
(iii)



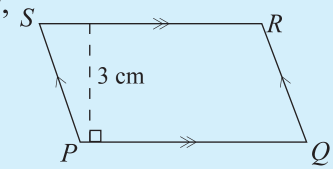
2. If the perimeter of the parallelogram $ABCD$ is 52 cm, find its area.



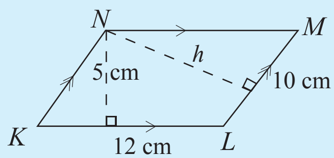
3. If the area of the parallelogram in the figure is 35 cm^2, find the value of h.



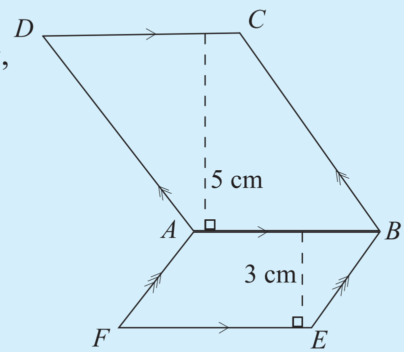
4. If the area of the parallelogram $PQRS$ is 105 cm^2, calculate the length of the side PQ.



5. **i.** Find the area of the parallelogram $KLMN$.
ii. Find the value of h.

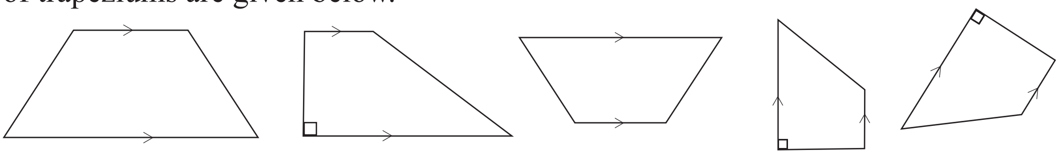


6. If the area of the parallelogram $ABCD$ is 30 cm^2, find the area of the parallelogram $ABEF$.



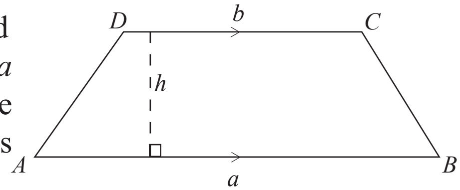
23.2 The area of a trapezium

A quadrilateral with one pair of sides parallel is called a trapezium. Several figures of trapeziums are given below.

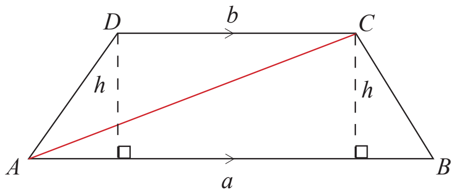


Let us develop a formula for the area of a trapezium.

Let us take the lengths of the parallel sides AB and DC of the trapezium given in the figure as a units and b units respectively and the perpendicular distance between these two sides as h units.



Let us find the area of the trapezium by adding the areas of the two triangles obtained by drawing the diagonal AC of the trapezium.



Area of triangle $ABC = \frac{1}{2} \times AB \times h$

Area of triangle $ACD = \frac{1}{2} \times DC \times h$

Area of trapezium $ABCD$ = Area of triangle ABC + Area of triangle ACD

$$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times DC \times h$$

$$= \frac{1}{2} \times h \times (AB + DC)$$

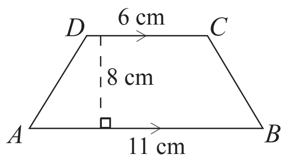
$$= \frac{1}{2} \times (AB + DC) \times h$$

$$= \frac{1}{2} \times (a + b) \times h.$$

$$\text{The area of a trapezium} = \frac{1}{2} \times \left(\begin{array}{c}\text{The sum of the}\\ \text{lengths of the parallel}\\ \text{sides}\end{array}\right) \times \left(\begin{array}{c}\text{The perpendicular}\\ \text{distance between the}\\ \text{parallel sides}\end{array}\right)$$

Example 1

Find the area of the trapezium *ABCD*.

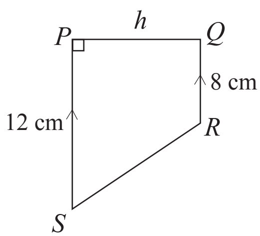


$$\text{The area of the trapezium } ABCD = \frac{1}{2} \times (11 + 6) \times 8$$

$$= \frac{1}{\cancel{2}_1} \times 17 \times \cancel{8}^4$$

$$= \underline{\underline{68 \text{ cm}^2}}$$

Example 2



If the area of the trapezium *PQRS* is 70 cm^2, find the value of *h*.

$$\text{The area of the trapezium } PQRS = \frac{1}{2} \times (12 + 8) \times h$$

$$= \frac{1}{\cancel{2}_1} \times \cancel{20}^{10} \times h$$

Since the area is given as 70 cm^2,

$$10\,h = 70$$

$$h = \frac{70}{10}$$

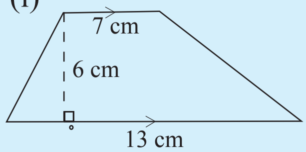
$$h = 7$$

Therefore, $h = 7$ cm.

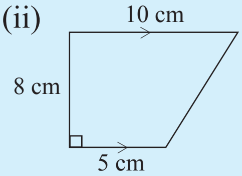
Exercise 23.2

1. Find the area of each trapezium given below.

(i)



(ii)



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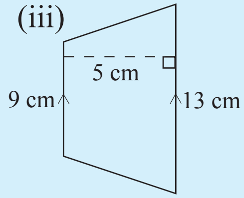
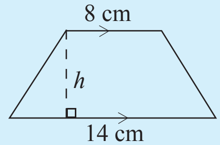
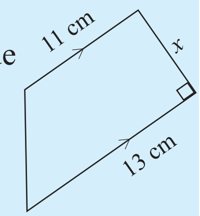


2. If the area of the trapezium in the figure is 88 cm^2, find the value of h.

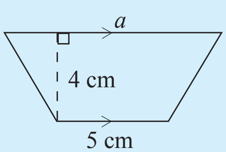


3. If the area of the trapezium in the figure is 60 cm^2, find the value of x.



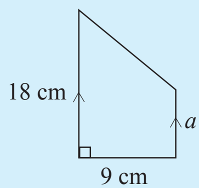
4. Find the length marked as a in each trapezium given below. The area of each trapezium is given below the figure.

i.



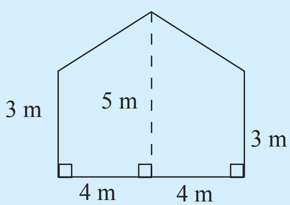
The area is 26 cm^2.

ii.



The area is 135 cm^2.

5.



The side view of a wall is given in the figure. Find the area of the wall according to the given measurements.

6. The area of a trapezium is 30 cm^2. The perpendicular distance between the parallel sides is 3 cm.

i. Give three pairs of integral values that the lengths of the parallel sides can take.

ii. Give three pairs of non – integral values that the lengths of the parallel sides can take.

23.3 The area of a circle

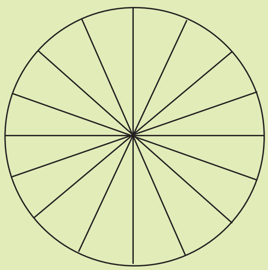
We have learnt how to find the area of a lamina that takes the shape of a rectangle, a square, a triangle, a parallelogram or a trapezium.
Now let us consider how the area of a circular shaped lamina can be found.

To do this, let us first engage in the activity given below.

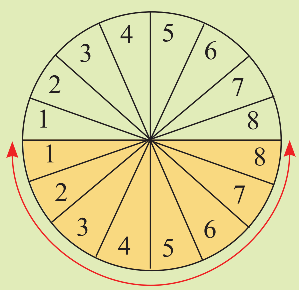
Activity 2

Step 1 : Draw a circle of radius 6 cm on a sheet of paper.

Step 2 : Divide the circle into the maximum possible number of sectors (about 16) by drawing straight lines through the center.



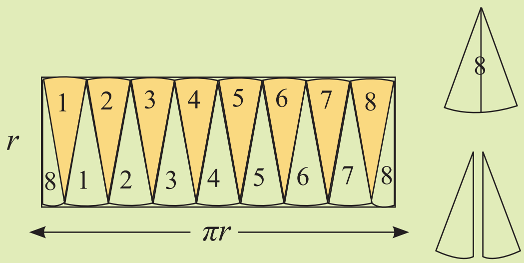
Step 3 : Colour half the circle and number all the sectors consecutively as shown in the figure.



$$\frac{2\pi r}{2} = \pi r$$

Step 4 : Separate out all the sectors by cutting along the drawn lines.

Step 5 : Paste the separated sectors such that a rectangular shape (approximately) is obtained as shown in the figure. (Understand that the accuracy increases as the number of sectors increases.)



Since paper is not wasted, the areas of the circle and the rectangle should be equal. Find the area of the rectangle as shown below by considering the radius of the circle as r.

The length of the rectangle that is obtained = circumference of the circle $\times \frac{1}{2}$

$= 2\pi r \times \frac{1}{2}$

$= \pi r$

The width of the rectangle that is obtained $= r$

The area of the rectangle = length × width

$= \pi r \times r$

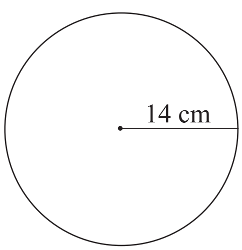
$= \pi r^2$

$\therefore$ Therefore, the area of a circle of radius $r = \pi r^2$

In calculations, we use 3.142 or $\frac{22}{7}$ for the value of π.

Example 1

Find the area of a circular lamina of radius 14 cm.



The area of the circular lamina $= \pi r^2$

$= \frac{22}{\cancel{7}_1} \times \cancel{14}^{2} \times 14$

$= 616$

$\therefore$ Therefore, the area of the circular lamina is 616 cm^2.

Example 2

Calculate the radius of a circular lamina of area 154 cm^2.

The area of the circular lamina $= \pi r^2$

$$= \frac{22}{7} \times r^2$$

The area of the circular lamina is given as 154 cm^2.

$$\frac{22}{7} r^2 = 154$$

Therefore, $\frac{22}{\cancel{7}} r^2 \times \cancel{7} = 154 \times 7$

$$\frac{\cancel{22} r^2}{\cancel{22}} = \frac{1078}{22} = 49$$

$$r^2 = 49$$

Therefore, $r = 7$ or $r = -7$.

However, the radius cannot be a negative value.
Therefore, the radius of the circle is 7 cm.

Exercise 23.3

1. The following are the dimensions of some circular laminas. Find the area of each lamina (Use $\frac{22}{7}$ for the value of π).

i. Radius 14 cm **ii.** Radius 21 cm **iii.** Diameter 7 cm **iv.** Diameter 21 cm

2. The following are areas of some circular laminas. Calculate the radius of each lamina.

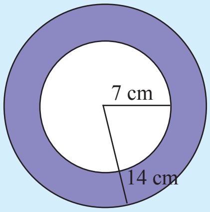
i. 616 cm^2 **ii.** 1386 cm^2 **iii.** $38\frac{1}{2}$ cm

3. Consider the largest circular lamina that can be cut out from a square lamina of area 196 cm^2.

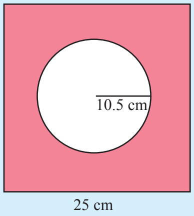
i. What is the radius of this circular lamina?
ii. What is the area of the circular lamina?

4. Find the area of the shaded part in each figure given below.

i.



ii.



5. What is the maximum number of circular laminas of radius 7 cm that can be cut out from a rectangular lamina of length 70 cm and width 14 cm?

Summary

- The area of a parallelogram of base length a and height h is ah.
- The area of a trapezium of which the lengths of the two parallel sides are a and b and the perpendicular distance between the two parallel sides is h is $\frac{1}{2}(a+b)h$.
- The area of a circle of radius r is πr^2.