22 Sets

By studying this lesson you will be able to;

- identify finite and infinite sets,
- write the subsets of a given set,
- identify equivalent sets, equal sets, disjoint sets and universal sets,
- identify the intersection and union of two sets,
- identify the complement of a set,
- represent sets using Venn diagrams.

Introduction to sets

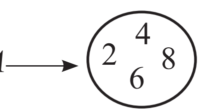
You have learnt earlier that a set is a collection of items that can be clearly identified. The items in a set are called its elements. Curly brackets are used to represent sets in terms of their elements. If a is an element of the set A, we denote this by $a \in A$. Moreover, the number of elements in the set A is denoted by $n(A)$.

Let us recall what was learnt previously about the different ways in which a set can be expressed.

1. Describing the set using a common characteristic by which the elements can be clearly identified
2. Writing the elements within curly brackets
3. Using a Venn diagram

As an example, let us consider how to write the set of all even numbers between 0 and 10 using the above 3 methods in the given order. Let us name this set A.

1. A = {even numbers between 0 and 10}
2. $A = \{2, 4, 6, 8\}$
3. A



The set which has no elements is known as the **null set**. The null set is denoted by { } and also by $\emptyset$. The number of elements in the null set is 0. Therefore, $n(\emptyset) = 0$.

Example 1

If P = {even prime numbers between 5 and 10}, then find $n(P)$.
Since there are no even prime numbers between 5 and 10, $P = \varnothing$ and therefore $n(P) = 0$.

Review Exercise

1. Determine whether each of the following collections is a set.

- **i.** Multiples of four between 0 and 30
- **ii.** The districts of Sri Lanka
- **iii.** Students who are good at mathematics
- **iv.** Triangular numbers
- **v.** The 10 largest integers

2. Write each set given below in terms of its elements and write the number of elements in each set too.

- **i.** A = {multiples of 5 between 0 and 20};
- **ii.** B = {letters of the word "RECONCILLIATION"}
- **iii.** C = {prime numbers between 2 and 13}
- **iv.** D = {numbers between 0 and 20 which are a product of two prime numbers}

3. D = {whole numbers between 5 and 10}.

- **i.** Write the elements of D.
- **ii.** Find $n(D)$.

4. Express the null set in three different ways using the above mentioned first method (That is, by the descriptive method using a special characteristic.)

22.1 Finite sets, infinite sets, equivalent sets and equal sets

Finite sets and infinite sets

Two sets which have been expressed in terms of a common characteristic by which the elements of the set can be clearly identified, are given below.

A = {multiples of 3 between 0 and 20}
B = {multiples of 5}

Let us write the elements of each set within curly brackets.

$A = \{3, 6, 9, 12, 15, 18\}$ $\quad$ $B = \{5, 10, 15, 20, ...\}$

The number of elements in set A is 6. That is, the number of elements in this set is a specific number. Sets with a specific number of elements (that is, sets with a finite number of elements), are known as **finite sets**.

The number of elements in set B however, cannot be stated definitely. That is, the number of elements in this set is infinite. Three dots have been placed at the end of the list of numbers within curly brackets to denote that the set B has an infinite number of elements. Sets that have an infinite number of elements are known as **infinite sets**.

Example 1

For each of the sets given below, write the elements and write whether it is a finite set or an infinite set.

P = {positive multiples of 6 less than 30}
Q = {polygons}

$P = \{6, 12, 18, 24\}$ $\quad$ $n(P) = 4$
Q = {triangle, quadrilateral, pentagon, hexagon, … }

Since the number of elements in the set P is finite, P is a finite set. Since the number of elements in the set Q is infinite, Q is an infinite set.

Equal sets

Consider the two sets given below.

A = {even numbers between 0 and 10}

B = {digits of the number 48268}

These two sets can be written as follows in terms of their elements.

$A = \{2, 4, 6, 8\}$
$B = \{2, 4, 6, 8\}$

Although the two sets A and B have been described differently, when they are written in terms of their elements, we get the same set. Sets which have the same elements are known as **equal sets**. Accordingly, A and B are equal sets. If two sets A and B are equal then we write $A = B$.

Equivalent sets

If the number of elements in the two sets A and B are equal, that is, if $n(A) = n(B)$, then the sets A and B are known as equivalent sets.

If A and B are equivalent sets, we denote this by $A \sim B$.

Example 2

$X = \{$odd numbers between 0 and 10$\}$
$Y = \{$vowels in the English alphabet$\}$

By writing the elements of these sets, show that they are equivalent sets.

$x = \{1, 3, 5, 7, 9\}$ $n(X) = 5$
$y = \{a, e, i, o, u\}$ $n(Y) = 5$

Since $n(X) = n(Y)$, X and Y sets are equivalent sets.

Note :- Although all pairs of equal sets are equivalent, all pairs of equivalent sets need not be equal.

Exercise 22.1

1. From the sets given below, select and write the finite sets and the infinite sets separately.

i. $A = \{$multiples of 5 from 0 to 50$\}$
ii. $B = \{$integers$\}$
iii. $C = \{$numbers that can be written using only the digits 0 and 1$\}$
iv. $D = \{$digits of the number 25265$\}$
v. $E = \{$positive integers which are not prime$\}$

2. Write each of the following sets in terms of their elements and then write all pairs of equal sets and all pairs of equivalent sets.

P = {positive multiples of 3 below 10}
Q = {letters of the word "net"}
R = {odd numbers between 0 and 10}
S = {digits of the number 3693}
T = {vowels in the English alphabet}
v = {letters of the word "ten"}

3. Write 3 examples of finite sets.

4. Write 3 examples of infinite sets.

5. Write three sets which are equivalent to the set $\{2, 3\}$.

22.2 The universal set and subsets

Subsets

When two sets A and B are considered, if all the elements in set B are also in set A, then set B is known as a **subset** of set A.

As an example, let us consider the two sets given below which are expressed in terms of their elements.

$A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6\}$

Since all the elements in set B are in set A, set B is a subset of set A. This is denoted by $B \subset A$ or $A \supset B$, and is read as " B is a subset of A".

Now let us consider another set C. If $C = \{1, 2, 7\}$, then not every element in C belongs to A. Therefore C is not a subset of A. This is denoted by $C \not\subset A$.

Example 1

P = {multiples of 6 between 0 and 20}
Q = {multiples of 3 between 0 and 20}

Write the elements of each of the above sets and select the subset.
$P = \{6, 12, 18\}$

$Q = \{3, 6, 9, 12, 15, 18\}$
As all the elements of P are in Q, P is a subset of Q.

Example 2

Write all the subsets of the set $X = \{1, 2\}$.

It is evident that $\{1\}$ and $\{2\}$ are two subsets. Observe that $\{1, 2\}$ is also a subset. In fact, if two sets A and B are equal, then A is a subset of B and B is a subset of A. Furthermore, the null set is considered to be a subset of every set.

Since the null set and the set itself are subsets of the given set, $\{ \}$ and $\{1, 2\}$ are subsets of the above set X.

Accordingly, the above set X has 4 subsets which are $\{ \},\{1\},\{2\},\{1, 2\}$.

Example 3

Write all the subsets of the set $Y = \{3, 5, 7\}$.

$\{ \}$, $\{3\}$, $\{5\}$, $\{7\}$, $\{3, 5\}$, $\{3, 7\}$, $\{5, 7\}$, $\{3, 5, 7\}$
There are 8 subsets.

Universal sets

In a study conducted on the students of your school, several subsets may come under consideration.

The following can be given as examples.

{Students in grade 9}
{Female students}
{Students sitting for the G.C.E (O/L) examination this year}

The elements of all the above sets are contained in the set of all students of the school. This set can be considered as the **universal set** relevant to the above study.

Let us consider another example.

When we consider the sets of even numbers, odd numbers, triangular numbers and prime numbers, we see that they are all subsets of the set of integers. Therefore the set of integers can be considered as the universal set.

A **universal set** is a set which contains all the elements under consideration. Universal sets are denoted by ε.

As another example, suppose that the numbers 1, 2, 3, 4, 5 and 6 are written on the 6 sides of a cubic die. By rolling this die once, the score that can be obtained is one of the numbers 1, 2, 3, 4, 5 and 6.Therefore, we obtain $\{1, 2, 3, 4, 5, 6\}$ as the set of possible outcomes. This is the universal set of all possible outcomes that can be obtained when a die is rolled once.
This can be expressed as $\varepsilon = \{1, 2, 3, 4, 5, 6\}$. A few subsets of this universal set are given below.

A = {odd numbers}	$A = \{1, 3, 5\}$
B = {values greater than 4}	$B = \{5, 6\}$
C = {even prime numbers}	$C = \{2\}$

Example 4

Write a universal set for A; $A = \{2, 4, 6, 8\}$
ε = {numbers between 1 and 10}

Exercise 22.2

1. Write 8 subsets of the set $A = \{2, 5, 8, 10, 13\}$.

2. Determine whether each of the following statements is true or not.

i. $\{1, 2, 3\} \subset$ {numbers divisible by 5}
ii. $\{4, 9, 16\} \subset$ {square numbers}
iii. {cylinder} $\subset$ {polygons}
iv. {red} $\subset$ {colours of the rainbow}
v. {solution of $2x - 1 = 7$} $\subset$ {even numbers}

3. Write a universal set for the set A; $A = \{a, e, i, o, u\}$

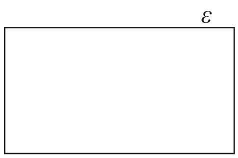
4. For each of the following parts, name a suitable universal set, such that the given sets are subsets.

i. {5, 10, 15, 20, 25} , {10, 100, 100, ...}
ii. {countries with more than 90% literacy},{countries which are not bordered by an ocean}
iii. {January, March, May, August}, {months with 31 days}
{months during which the members of your family celebrate their birthdays}

22.3 Venn diagrams

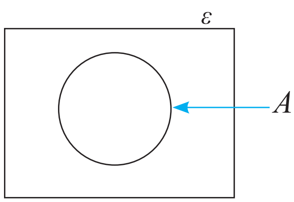
You have learnt how to represent sets in a Venn diagram in earlier grades. In Venn diagrams, sets are represented by closed figures.

The universal set is represented in a Venn diagram by a rectangle as shown below.



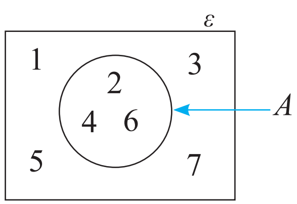
The subsets of a universal set are represented using round or oval shaped figures (circles or ellipses)

We represent a subset A within the universal set as follows.

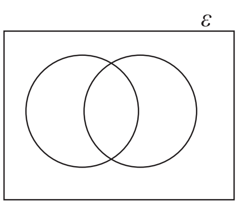


Example 1

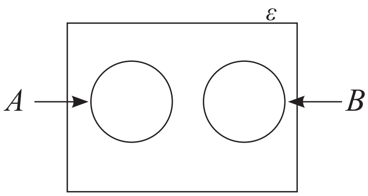
$\varepsilon = \{1, 2, 3, 4, 5, 6, 7\}$
$A = \{2, 4, 6,\}$
Represent the above sets in a Venn diagram.



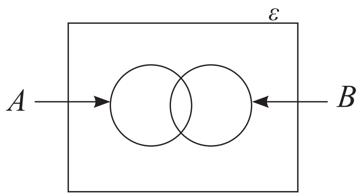
Two subsets of a universal set are generally represented as below.



Special instances of two subsets of a universal set are shown below.

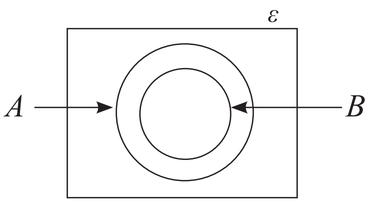


When the two sets A and B have no elements in common

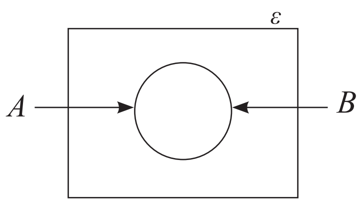


When A and B have common elements

6



When B is a subset of A



When A and B are equal

You will learn more about these four instances and the corresponding regions under the next section on the intersecton of sets, union of sets and disjoint sets.

22.4 Intersection of sets, union of sets and disjoint sets

Intersection of sets

When two or more sets are considered, the set consisting of the elements which are common to all the sets is known as their intersection. When two sets A and B are considered, their intersection is denoted by $A \cap B$.

As an example, let us consider the pair of sets given below,

$A = \{1, 2, 3, 4, 5, 7\}$

$B = \{2, 5, 6, 7\}$

The set consisting of the elements common to both A and B is $\{2, 5, 7\}$.

Therefore, the intersection of the sets A and B is $A \cap B = \{2, 5, 7\}$.

Example 1

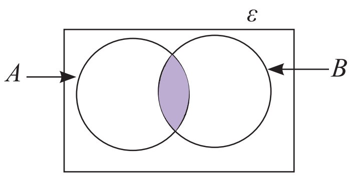
M = {students of Kannangara Vidyalaya who play cricket}

N = {students of Kannagara Vidyalaya who play football}

Write the set $M \cap N$ in descriptive form.

$M \cap N$ = {students of Kannangara Vidyalaya who play both cricket and football}

Now let us consider how the intersection of two sets is represented in a Venn diagram.
Suppose the two sets A and B have common elements.
That is, A and B have a non - empty intersection. There should be a region to represent the set of elements common to both these sets. The Venn diagram depicting this is given below.



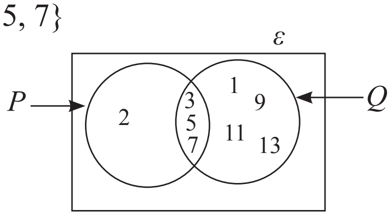
The shaded region in the figure is common to both the sets A and B. Therefore the elements that are common to these two sets can be included here.

Let us consider how to represent two sets with a non-empty intersection in a Venn diagram, through the following example.

Example 2

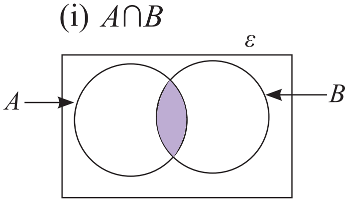
Write the following two sets by listing their elements and then find their intersection. Represent all the elements in a Venn diagram.

P ={prime numbers between 0 and 10}
Q ={odd numbers between 0 and 15}
$P = \{2, 3, 5, 7\}$
$Q = \{1, 3, 5, 7, 9, 11, 13\}$
$\therefore\ P \cap Q = \{3, 5, 7\}$

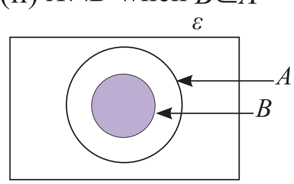


The intersecton of two sets under different conditions are given below.

(i) $A \cap B$

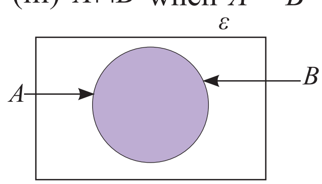


(ii) $A \cap B$ when $B \subset A$



$A \cap B = B$ when $B \subset A$

(iii) $A \cap B$ when $A = B$

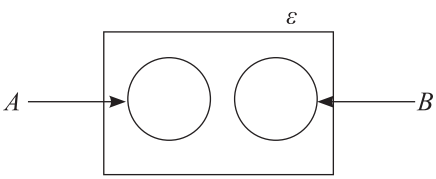


$A \cap B = A = B$ when $A = B$

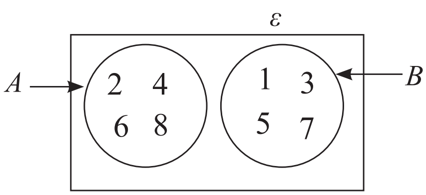
Let us consider how two sets which do not have common elements represented in a Venn diagram.

Disjoint sets

If two sets have no elements in common, then they are known as disjoint sets. In other words, if two sets A and B are such that $A \cap B = \emptyset$, then A and B are disjoint sets.
Disjoint sets can be represented in a Venn diagram as shown below.



As an example, let us consider the two sets shown below.
$A = \{2, 4, 6, 8\}$
$B = \{1, 3, 5, 7\}$
Since $A \cap B = \emptyset$, A and B are disjoint sets.



$A \cap B = \emptyset$

Union of sets

When two or more sets are considered, the set which consists of all the elements in these sets is known as the **union of these sets**. When two sets A and B are considered, their union is denoted by $A \cup B$.

As an example, let us consider the pair of sets given below.

$A = \{1, 3, 5, 7, 8\}$
$B = \{2, 3, 4, 6, 7, 8\}$

The union of the sets A and B is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Accordingly, the union of the sets A and B is $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Example 1

P = {prime numbers between 0 and 10}
P = {odd numbers between 0 and 10}

Write these sets in terms of their elements and write $P \cup Q$ also in terms of its elements. Furthermore, express the union in terms of a common characteristic of its elements.

$P = \{2, 3, 5, 7\}$
$Q = \{1, 3, 5, 7, 9\}$

$P \cup Q = \{1, 2, 3, 5, 7, 9\}$

The union expressed in terms of a common characteristic;
$P \cup Q$ = {numbers between 0 and 10 which are either prime or odd}

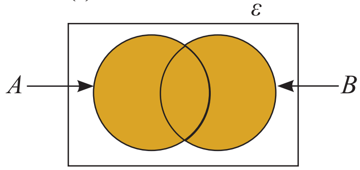
Example 2

X = {students in KannangaraVidyalaya who play cricket}
Y = {students in KannangaraVidyalaya who play football}

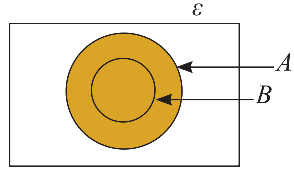
$X \cup Y$ = {students in Kannangara Vidyalaya who play either cricket or football or both}

Now, let us see how to represent the union of sets in a Venn diagram.

(i) $A \cup B$

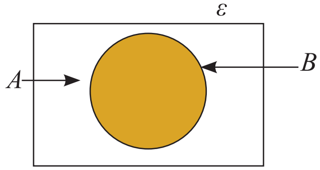


(ii) $A \cup B$ when $B \subset A$



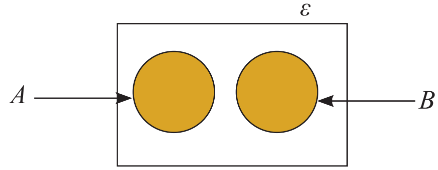
$A \cup B = A$

(iii) $A \cup B$ when $A = B$



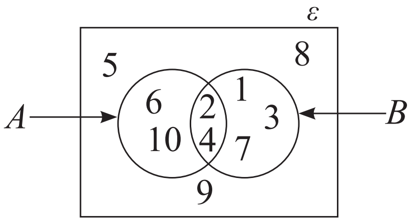
$A \cup B = A = B$ when $A = B$

(iv) $A \cap B = \emptyset$



Example 3

Answer the given questions based on the following Venn diagram.



i. Write set A in terms of its elements.
ii. Write set B in terms of its elements.
iii. Write the universal set ε in terms of its elements.
iv. Express $A \cap B$ in terms of its elements.
v. Express $A \cup B$ in terms of its elements.

i. $A = \{2, 4, 6, 10\}$
ii. $B = \{2, 1, 4, 7, 3\}$
iii. $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
iv. $A \cap B = \{2, 4\}$
v. $A \cup B = \{6, 10, 2, 4, 1, 3, 7\}$

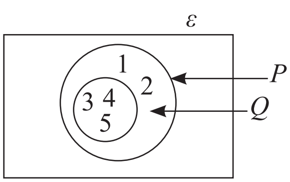
Example 4

$P = \{1, 2, 3, 4, 5\}$
$Q = \{3, 4, 5\}$

i. Represent the above sets in a Venn diagram.
ii. Express $P \cap Q$ and $P \cup Q$ in terms of their elements.

i. $P \cap Q = \{3, 4, 5\}$
$P \cup Q = \{1, 2, 3, 4, 5\}$

ii.



Exercise 22.4

1. The sets *P, Q* and *R* are defined as follows.

$P = \{1, 3, 6, 8, 10, 13\}$
$Q = \{1, 6, 7, 8\}$
$R = \{2, 3, 9, 10, 12\}$

Express each of the following sets in terms of its elements.

i. $P \cap Q$ **ii.** $P \cap R$ **iii.** $Q \cap R$ **iv.** $P \cup Q$ **v.** $P \cup R$ **vi.** $Q \cup R$

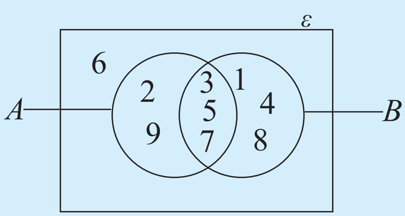
2. The sets *A, B* and *C* are defined as follows.

A = {counting numbers from 1 to 12}
B = {prime numbers less than 10}
C = {factors of 12}

i. Write each of the above sets in terms of its elements.
ii. Write each of the following sets in terms of its elements.

i. $A \cap B$ **ii.** $A \cap C$ **iii.** $B \cap C$ **iv.** $A \cup B$ **v.** $A \cup C$ **vi.** $B \cup C$

3. Consider the Venn diagram given below.



Write each of the following sets in terms of its elements.
i. A
ii. B
iii. $A \cup B$
iv. $A \cap B$

22.5 Complement of a set

Let us consider a subset A of a universal set. The set of elements in the universal set which do not belong to the set A is known as the **complement of A**.

Consider the following example.

If we take,
$\varepsilon = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{2, 4, 6\}$,
then the set consisting of all the elements in the universal set which are not in set A is $\{1, 3, 5, 7\}$.

This set is the complement of the set A. The complement of the set A is denoted by A'. Accordingly,
we can write, $A' = \{1, 3, 5, 7\}$.

Example 1

By considering the given universal set (ε) and its subset B, write the set B' in terms of its elements.

$\varepsilon = \{5, 10, 15, 20, 25, 30, 35\}$
$B = \{10, 20, 30\}$

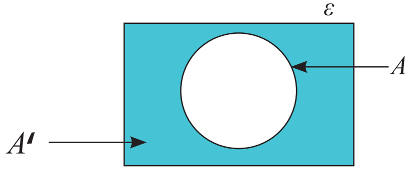
$B' = \{5, 15, 25, 35\}$

Example 2

If $\varepsilon = \{\text{birds}\}$ and
$P = \{\text{birds that make nests}\}$, then write P' in descriptive form.
$P' = \{\text{birds that do not make nests}\}$

Now let us see how to represent the complement of a set in a Venn diagram.

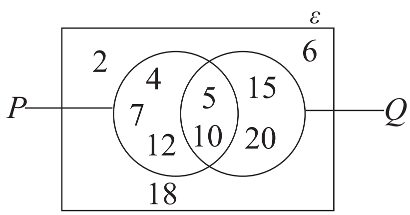
If A is a subset of a universal set, then A' is represented in a Venn diagram as follows.



A' is the set of elements which belong to ε but not to A. Therefore the whole region in the Venn diagram which does not belong to A, belongs to A'.

Example 3

Find the following using the information in the Venn diagram.



i. P' **ii.** Q' **iii.** $P \cap Q$ **iv.** $P \cup Q$

i. $P' = \{2, 6, 15, 18, 20\}$

ii. $Q' = \{2, 4, 6, 7, 12, 18\}$

iii. $P \cap Q = \{5, 10\}$

iv. $P \cup Q = \{4, 5, 7, 12, 15, 20, 10\}$

Exercise 22.5

1. ε = {Sakindu, Ravindu, Sanindu, Pavindu, Nithindu}

A = {Sakindu, Pavindu}

B = {Ravindu,Sanindu, Nithindu}

C = {Sakindu, Sanindu, Pavindu}

Write each of the following sets in terms of its elements, based on the above given information.

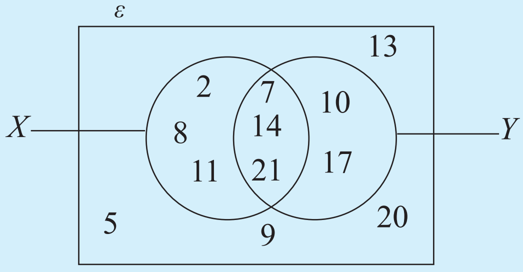
i. A' **ii.** B' **iii.** C'

iv. $A \cap C$ **v.** $A \cap B$ **vi.** $B \cap C$

2. If $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $P = \{1, 3, 5, 7, 9, 10\}$ and $Q = \{2, 4, 5, 7, 8\}$, represent ε, P and Q in a Venn diagram and using the Venn diagram, write each of the sets given below in terms of its elements.

i. P' **ii.** Q' **iii.** $P \cap Q$ **iv.** $P \cup Q$

3. By considering the Venn diagram given below, write each of the given sets in terms of its elements.

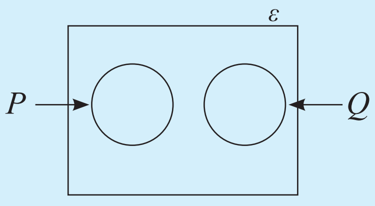


i. X **ii.** Y **iii.** $X \cap Y$
iv. $X \cup Y$ **v.** X' **vi.** Y'

Miscellaneous Exercise

1. Represent the following information in the given Venn diagram.

$\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, \}$
$P = \{2, 4, 6\}$
$Q = \{1, 5, 8\}$

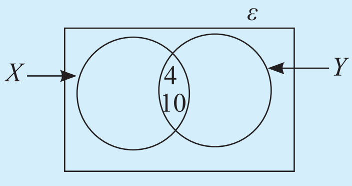


Write each of the sets given below in terms of its elements.

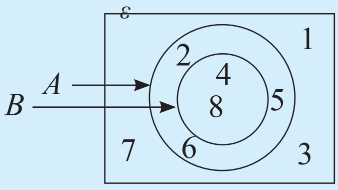
a. $P \cap Q$
b. $P \cup Q$
c. P'
d. Q'

2. Include the elements in the following sets in the given Venn diagram.

$\varepsilon = \{2, 3, 4, 5, 7, 8, 10, 12 \}$
$P = \{2, 4, 10\}$
$Q = \{3, 4, 8, 10\}$



3. Answer the following based on the information in the Venn diagram.



i. Write set A in terms of its elements.
ii. Write set B in terms of its elements.
iii. Write set ε in terms of its elements.
iv. $A \cap B$ set in terms of its elements.
v. $A \cup B$ set in terms of its elements.
vi. A' set in terms of its elements.

Summary

- Sets with a finite number of elements are called finite sets.
- Sets with an infinite number of elements are called infinite sets.
- Sets with the same elements are called equal sets.
- Sets with the same number of elements are called equivalent sets.
- A sets which contains all the elements under consideration is called universal set.