21 Inequalities

By studying this lesson you will be able to;

- solve inequalities of the form $x \pm a \gtrless b$,
- solve inequalities of the form $ax \gtrless b$,
- find the integral solutions of an inequality,
- represent the solutions of an inequality on a number line.

In Sri Lanka, a person who is 55 years or older is considered to be a senior citizen. Accordingly, if we denote the age of a senior citizen by t, then we can indicate this by the inequality $t \geqslant 55$. This means that the value of t is always greater or equal to 55.

Let us recall what was learnt in grade 8 regarding inequalities.

$x > 3$ is an inequality. This means that the values that x can take are the values that are greater than 3. However, if we write $x \geq 3$, this means that the values that x can take are 3 or a value greater than 3.

Similarly, $x < 3$ means that the values that x can take are the values that are less than 3, and $x \leq 3$ means that the values that x can take are 3 or a value less than 3.

For example, the solution set of $x > 3$ is the set of all numbers which are greater than 3. The set of integral solutions of this inequality is $\{4, 5, 6, 7, \ldots\}$.

The three dots mean that all the integers which belong to the pattern indicated by the first few values belong to the solution set. Therefore, the above inequality has infinitely many integral solutions.

Although in mathematics it is important to represent all the solutions as a set, when indicating the integral solutions of an inequality, it is sufficient to just write the values. For example, the integral solutions of the inequality $x > 3$ can be written as 4, 5, 6, … .

The solution set of an inequality which has an algebraic term, is the set of all values that the algebraic term can take. Let us recall what has been learnt earlier regarding the solution set of an inequality, and how this is represented on a number line, by considering the following examples.

Example 1

Consider the inequality $x > 3$.

i. Write the set of **integral solutions** of the above inequality.
ii. Represent the **integral solutions** of the above inequality on a number line.

i. $\{4, 5, 6, 7, 8, ...\}$

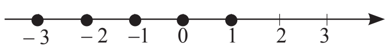
ii.


Example 2

Consider the inequality $x \leq 1$.

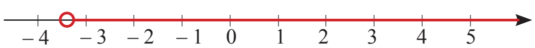
i. Write the set of integral solutions of the above inequality.
ii. Represent the integral solutions of the above inequality on a number line.

i. $\{..., -3, -2, -1, 0, 1\}$

ii.


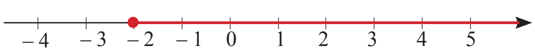
Example 3

Represent the solution set of the inequality $x > -3\frac{1}{2}$ on a number line.



Example 4

Represent the solution set of the inequality $x \geq -2$ on a number line.



Example 5

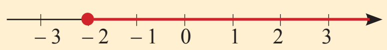
Represent the **solution set** of the inequality $-3 < x \leq 3\frac{1}{2}$ on a number line.

Review Exercise

1. For each of the following inequalities, represent the **set of integral solutions on** a number line.

i. $x > 2$ **ii.** $x \geq -1$ **iii.** $x < 4$ **iv.** $x \leq -2.5$ **v.** $x > 1\frac{1}{2}$

2. For each of the following, write an inequality that has the values represented on the number line as its solution set.

i. 

ii. 

iii. 

iv. 

3. Represent the solution set of each of the following inequalities on a number line.

i. $-1 < x < 2$ **ii.** $-2 \leq x < 3$ **iii.** $-3 < x \leq 1$

iv. $x < -1$ or $x \geq 2$ **v.** $x \leq -3$ or $x > 0$

21.1 Inequalities of the form $x \pm a \gtrless b$

The following notice has been placed near a certain bridge.

"This bridge can only bear loads of less than 10 tons"

Suppose a lorry of mass 4 tons carrying a certain load wishes to cross this bridge. If we take the mass of the load carried by the lorry to be x tons, then the lorry can safely cross the bridge only if $x + 4 < 10$. In other words, if the mass of the load carried by the lorry is x tons, then the lorry can safely cross the bridge, only if the inequality $x + 4 < 10$ is satisfied.

We can find the mass of the load that the lorry can safely carry across the bridge by solving the inequality $x + 4 < 10$.

Solving an inequality means, obtaining an inequality equivalent to the given inequality such that only x (or the given variable) is on one side of the inequality.

When solving an inequality, we can adopt the procedure followed in solving equations to a large extent.

For example, we can subtract 4 from both sides of the above inequality $x + 4 < 10$. Accordingly,

$x + 4 - 4 < 10 - 4.$

When we simplify this we obtain

$x < 6.$

Therefore, for the lorry to safely cross the bridge, the load that it carries should be less than 6 tons.

Example 1

Solve the inequality $x + 2 < 7$ and represent the integral solutions on a number line.

$x + 2 < 7$

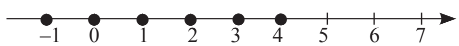
$x + 2 - 2 < 7 - 2$ (subtracting 2 from both sides)

$\underline{\underline{x < 5}}$

The integral solutions of this inequality are the integers that are less than 5. That is, the values 4, 3, 2, 1, 0, –1, –2, … .

These integral solutions can also be expressed as a set as $\{4, 3, 2, 1, 0, -1, -2, \ldots\}$
The solution set can be represented on a number line as follows.

Integral solutions of x



Example 2

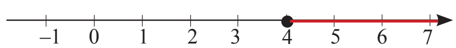
Solve the inequality $x - 3 \geq 1$ and represent the solution set on a number line.

$x - 3 \geq 1$

$x - 3 + 3 \geq 1 + 3$ (adding 3 to both sides)

$\underline{\underline{x \geq 4}}$

Now let us represent the solution set on a number line.



All the solutions (numbers that are greater or equal to 4) are represented here. It is important to remember that not only the integral solutions, but solutions such as 4.5 and 5.02 are also included.

Example 3

The maximum mass that a bag can carry is 6 kg. Nimal puts x packets of rice of mass 1 kilogramme each and 2 packets of sugar of mass 1 kilogramme each into this bag. This information can be represented by the inequality $x + 2 \leq 6$.

i. Solve this inequality.
ii. What is the maximum number of packets of rice that Nimal can carry in this bag?

i. $x + 2 \leq 6$

$x + 2 - 2 \leq 6 - 2$

$\underline{\underline{x \leq 4}}$

ii. Therefore, the maximum number of packets of rice that can be carried in this bag is 4.

Exercise 21.1

1. Solve each of the following inequalities and write the set of integral solutions.

i. $x + 3 > 5$ **ii.** $x - 4 < 1$ **iii.** $x - 7 \geq - 6$ **iv.** $2 + x \leq - 4$
v. $7 + x > 5$

2. Solve each of the following inequalities and represent the solution set on a number line.

i. $x + 1 > 3$ **ii.** $x - 3 \leq 1$ **iii.** $6 + x \geq 2$ **iv.** $x - 7 < - 7$
v. $x + 5 > - 1$

3. Sakindu has 60 rupees. He buys a book for x rupees and a pen for 10 rupees. The total value of the items he bought can be expressed in terms of an inequality as $x + 10 \leq 60$. Solve this inequality and determine the maximum price that the book could be.

4. The maximum number of people that can travel in a certain van is 15. If 3 people get into the van from one location and x number of people from another location, this information can be represented by the inequality $x + 3 \leq 15$.

i. Solve the above inequality.
ii. What is the maximum number of people that can get into the van from the second location?

5. The sum of the ages of Githmi and Nethmi does not exceed 30. Githmi is 14 years old. If Nethmi's age is taken as x years, this information can be represented by the inequality $x + 14 \leq 30$. Solve this inequality and find the maximum age that Nethmi could be.

21.2 Inequalities of the form $ax \gtreqless b$

The price of two books of the same type is more than 40 rupees. If we take the price of one of these books as x rupees, we can represent this information by the inequality $2x > 40$ involving x. By solving this inequality, we can find the price that each book could be.

When solving this type of inequalities there are some important facts that we should keep in mind.

Consider the following inequalities.

i. The inequality $3 < 4$ is true.
$2 \times 3 < 2 \times 4$ (multiplying both sides by 2)
The inequality $6 < 8$ is true.

ii. The inequality $8 > 6$ is true.
$\frac{8}{2} > \frac{6}{2}$ (dividing both sides by 2)
The inequality $4 > 3$ is true.

When the two sides of an inequality are either multiplied by the same positive number or divided by the same positive number, the inequality sign does not change.

iii. The inequality $2 < 3$ is true.
$2 \times -2 < 3 \times -2$ (multiplying both sides by -2)
The inequality $-4 < -6$ which is obtained is false. However, $-4 > -6$ is true.

iv. The inequality $9 > 6$ is true.
$\frac{9}{-3} > \frac{6}{-3}$ (dividing both sides by -3)
The inequality $-3 > -2$ which is obtained is false. However, $-2 < -3$ is true.

When an inequality is multiplied or divided by a negative number, the inequality sign changes. That is, the sign $<$ changes to $>$ and the sign $\geqslant$ changes to $\leqslant$, etc .

Through the following examples, let us learn how inequalities are solved taking into consideration the above facts.

Example 1

Solve the inequality $2\,x < 12$ and represent the solutions on a number line.

$$2\,x < 12$$

$$\frac{2x}{2} < \frac{12}{2} \quad \text{(dividing both sides by 2)}$$

$$\underline{\underline{x < 6}}$$

Example 2

Solve the inequality $3x \geq 12$.

$$3x \geq 12$$

$$\frac{3x}{3} \geq \frac{12}{3}$$

$$\underline{\underline{x \geq 4}}$$

Example 3

Solve the inequality $-5x \leq 15$.

$$-5x \leq 15$$

$$\frac{-5x}{-5} \geq \frac{15}{-5} \quad \text{(when dividing by a negative number the sign changes)}$$

$$\underline{\underline{x \geq -3}}$$

Example 4

Solve the inequality $\frac{x}{3} < 2$.

$$\frac{x}{3} \times 3 < 2 \times 3 \qquad \text{(multiplying both sides by 3)}$$

$$\underline{\underline{x < 6}}$$

Example 5

Solve the inequality $-\frac{2x}{5} > 6$.

$$-\frac{2x}{5} > 6$$

$$-\frac{2x}{5} \times 5 > 6 \times 5 \quad \text{(multiplying both sides by 5)}$$

$$-2x > 30$$

$$\frac{-2x}{-2} < \frac{30}{-2} \quad \text{(the sign changes when dividing by } -2\text{)}$$

$$\underline{\underline{x < -15}}$$

Exercise 21.2

1. Solve each of the following inequalities and write the integral solutions.

i. $2x > 6$ **ii.** $3x \leq 12$ **iii.** $-5x \geq 10$ **iv.** $-7x < -35$

v. $-2x > -5$ **vi.** $\frac{x}{2} \leq 1$ **vii.** $\frac{x}{4} \geq -2$ **viii.** $-\frac{2x}{3} < 4$

2. Solve each of the following inequalities and represent the solutions on a number line.

i. $4x > 8$ **ii.** $7x \leq 21$ **iii.** $-3x \geq 3$ **iv.** $-2x < -6$

v. $\frac{x}{3} \geq 1$ **vi.** $\frac{x}{6} < -\frac{1}{6}$ **vii.** $\frac{2x}{3} \geq 4$ **viii.** $-\frac{3x}{5} < -\frac{1}{6}$

3. The price of 2 mangoes is less than 50 rupees. If the price of one mango is x rupees, this information can be represented by the inequality $2x \leq 50$. Solve this inequality and find the maximum possible price of a mango.

4. The maximum mass that can be carried by an elevator is 560 kilogrammes. Eight men of mass x kilogrammes each are riding this elevator. This information can be represented by the inequality $8x \leq 560$. Find the maximum mass that each man could be.

5.

(a) The amount of money Mahesh has is less than four times the amount that Ashan has. Mahesh as 68 rupees. If the amount that Ashan has is denoted by x rupees, then this information can be represented by the inequality $4x > 68$. Solve this inequality.

(b) If Ashan has only 5 rupee coins, what is the least amount he could be having?

Summary

- When the two sides of an inequality are either multiplied by the same positive number or divided by the same positive number, the inequality sign does not change.
- When an inequality is multiplied or divided by a negative number, the inequality sign changes. That is, the sign $<$ changes to $>$ and the sign $\geqslant$ changes to $\leqslant$, etc.