19 Pythagorean Relation

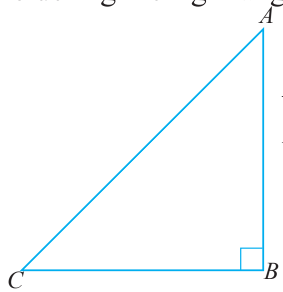
By studying this lesson, you will be able to;

- develop the Pythagorean relation by means of a right angled triangle,
- solve problems related to the Pythagorean relation.

Right angled triangle

If an angle of a triangle is 90°, it is called a right angled triangle. The side which is opposite(in front of) the right angle and which is the longest side of the triangle is called the hypotenuse. The other two sides are called the sides which include the right angle.

Considering the right angled triangle ABC given below;



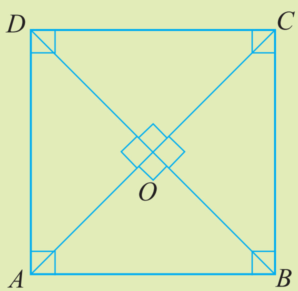
$A\hat{B}C = 90^{\circ}$

AC is the hypotenuse,

AB and BC are the sides which include the right angle.

Activity 1

Complete the table given below by identifying all the right angled triangles in the figure.

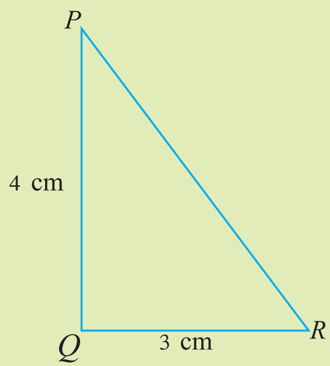


Triangle	Hypotenuse	Sides that include the right angle
AOB	AB	AO, BO
………	………	………
………	………	………
………	………	………
………	………	………
………	………	………

19.2 The Pythagorean relation

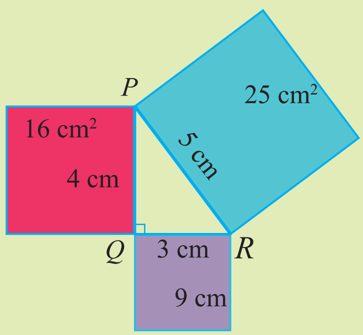
A Greek mathematician named Pythagoras introduced the relationship between the sides of a right angled triangle. Let us understand this relationship through an activity.

Activity 1



Draw a right angled triangle PQR with QR = 3 cm and QP = 4 cm. You may use a set square to do this. Measure the hypotenuse PR and verify that it is 5 cm in length. Cut three squares of side length 3 cm, 4 cm and 5 cm and paste them on the sides RQ, QP and PR respectively, as shown in the figure given below.

Now let us calculate the area of each square as shown below.



The area of the square pasted on QR = 3 cm × 3 cm = 9 cm^2
The area of the square pasted on QP = 4 cm × 4 cm = 16 cm^2
The area of the square pasted on PR = 5 cm × 5cm = 25 cm^2

Observe the relationship between the above values as given below.

the area of the square on PR	=	the area of the square on QR	+	the area of the square on PQ

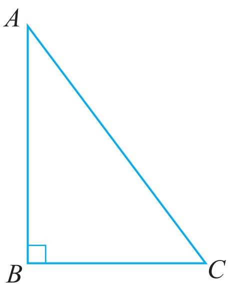
Repeat the above activity by taking the lengths of the sides of the triangle which include the right angle as 6 cm and 8 cm to verify the above relationship for these values too.

The Pythagorean relation for a right angled triangle can be expressed as follows.

> The area of the square drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the squares drawn on the remaining two sides.

Though the Pythagorean relation is shown using areas, we can write it simply in terms of the lengths of the sides of the triangle. Let us see how this is done.

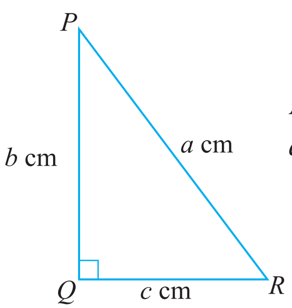
Writing the Pythagorean relation in terms of the lengths of the side



The area of the square drawn on $AB = AB \times AB = AB^2$
The area of the square drawn on $BC = BC \times BC = BC^2$
The area of the square drawn on $AC = AC \times AC = AC^2$
Therefore, according to the Pythagorean relation;

$$AC^2 = AB^2 + BC^2$$

We can express this as follows too.



According to the Pythagorean relation
$a^2 = b^2 + c^2$

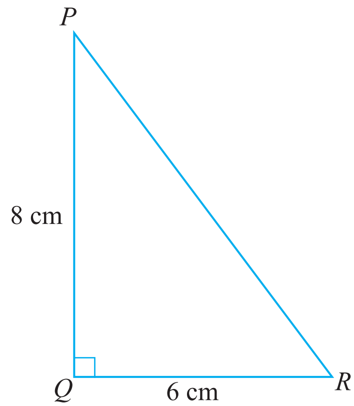
Example 1

In the right angled triangle *PQR*, $PQ = 8$ cm and $QR = 6$ cm. Find the length of *PR*.

By applying the Pythagorean relation to the right angled triangle *PQR*,

$$PR^2 = PQ^2 + QR^2$$
$$PR^2 = 8^2 + 6^2$$
$$= 64 + 36$$
$$= 100$$
$$PR = \sqrt{100} = 10$$

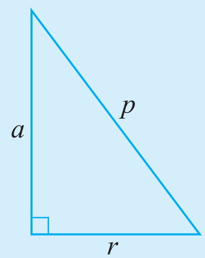
$\therefore$ the length of $PR = 10$ cm.



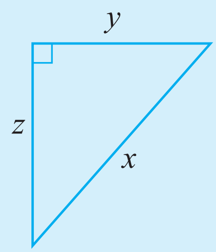
Exercise 19.1

1. Write the Pythagorean relation for each right angled triangle using the given lengths.

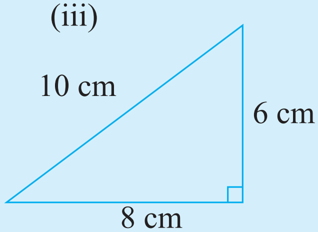
(i)



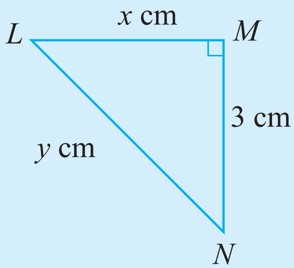
(ii)



(iii)

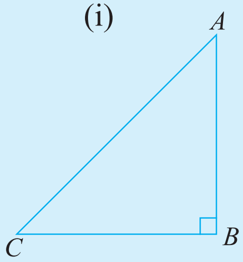


(iv)



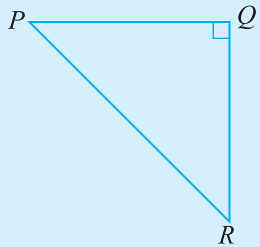
2. Fill in the blanks in the expressions related to the figures shown below.

(i)



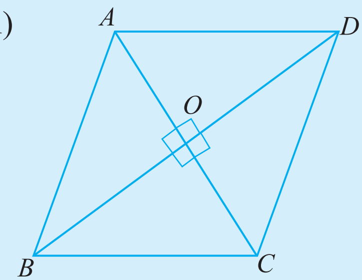
$AC^2 = AB^2 + \ldots\ldots\ldots$

(ii)



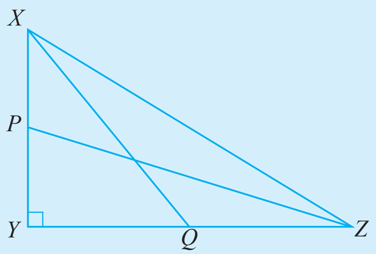
$PR^2 = \ldots\ldots\ldots + \ldots\ldots\ldots$

(iii)

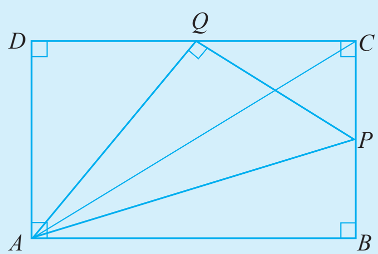


a. $AD^2 = \ldots\ldots\ldots + \ldots\ldots\ldots$
b. $\ldots\ldots\ldots = BO^2 + \ldots\ldots\ldots$
c. $\ldots\ldots\ldots = BO^2 + OC^2$
d. $DC^2 = \ldots\ldots\ldots + \ldots\ldots\ldots$

3. Identify all the right angled triangles in each figure given below and write the Pythagorean relation for each triangle that is identified.

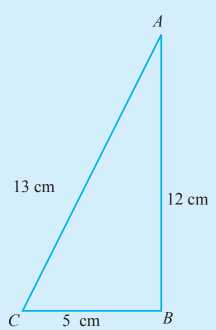


i.



ii.

4. Fill in the blanks in the statements given below which are related to the right angled triangle in the figure.



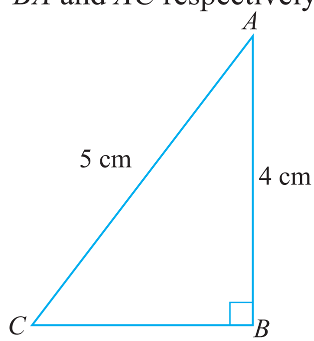
The longest side of the triangle is ....
The area of the square drawn on the side AB = $12 \times 12 = 144$ cm^2
The area of the square drawn on the side BC = ... = ... cm^2
The area of the square drawn on the side AC = ... = ... cm^2
The sum of the areas of the squares drawn on the sides BC and BA = ... cm^2 .
$\therefore$ The areas of the square drawn on AC is .... (equal/not equal) to the sum of the areas of the squares drawn on BC and BA.

Now let us consider some problems which can be solved using the Pythagorean relation.

Example 2

A 5m long straight wooden rod is in a vertical plane with one end touching the top of a 4m high vertical wall and the other end in contact with the horizontal ground a certain distance away from the foot of the wall. Find the horizontal distance from the foot of the wall to the point where the rod is in contact with the ground.

We can draw a rough sketch of this as shown below. Here the wall and the wooden rod are represented by BA and AC respectively.



Applying the Pythagorean relation to the right angled triangle ABC

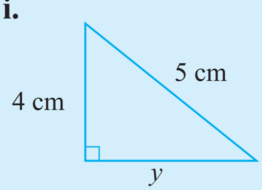
$$AC^2 = AB^2 + BC^2$$
$$5^2 = 4^2 + BC^2$$
$$25 = 16 + BC^2$$
$$\therefore BC^2 = 9$$
$$BC = \sqrt{9} = 3$$

$\therefore$ The horizontal distance from the foot of the wall to the wooden rod is 3m.

Exercise 19.2

1. Find the length of each side indicated by an algebraic symbol in each figure.

i.



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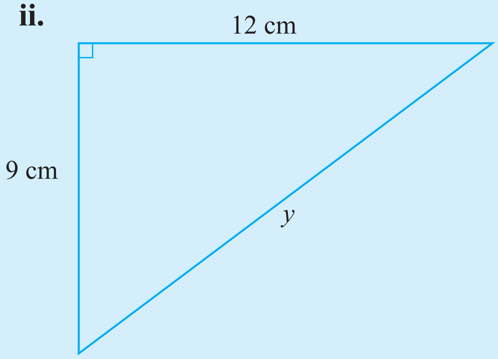

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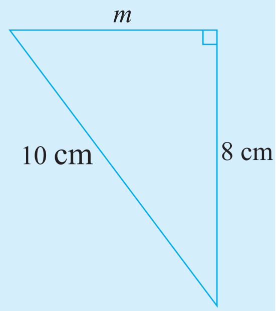
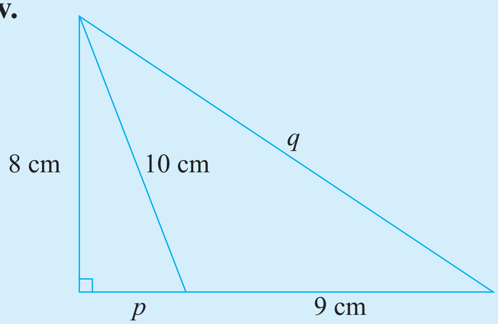
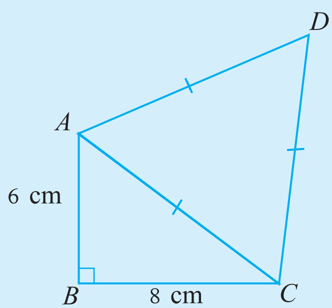


iv.

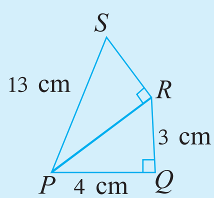


2. Find the perimeter of each figure given below.

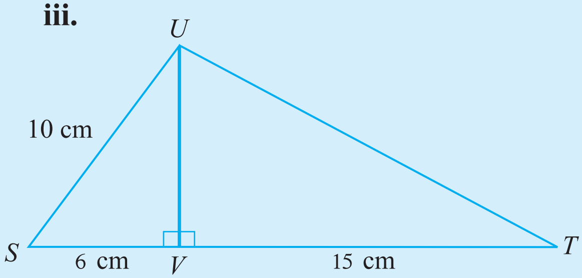
i.



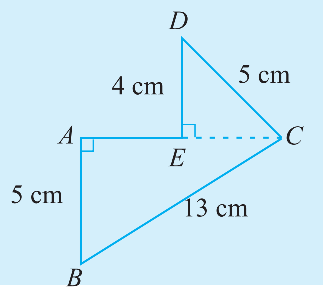
ii.



iii.

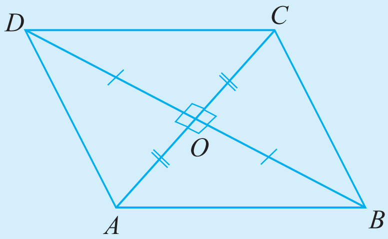


iv.



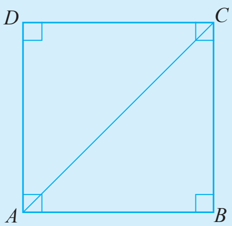
3.

i.



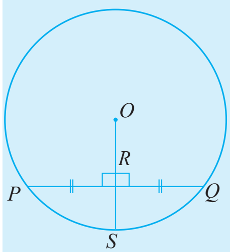
The diagonals *BD* and *AC* of the rhombus *ABCD* bisect each other perpendicularly at *O*. Moreover, *BD* = 16cm and *AC* = 12cm. Find the perimeter of the rhombus.

ii.



If the length of the diagonal *AC* of the square *ABCD* is 10cm, find the area of the square.

iii.



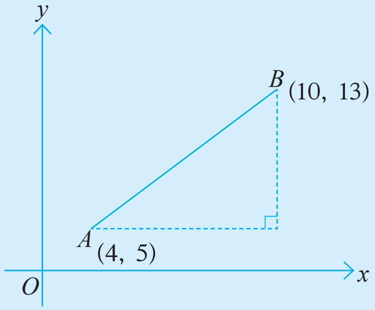
In the circle with centre *O* shown in the figure, the midpoint of the chord *PQ* is *R*. Moreover, *OR* produced meets the circle at *S*. If $O\hat{R}P = 90^\circ$,$O\hat{R}P$ PQ=12 cm and *OR*= 8 cm, find

i. the length of *RQ,*
ii. the radius of the circle,
iii. the length of *RS.*

4. In the triangle *ABC*, $A\hat{B}C = 90^\circ$, *AB* = 8 cm and *BC* = 6 cm. The mid points of *BC* and *BA* are *R* and *P*. Find the perimeter of the quadrilateral *APRC*.

Miscellaneous Exercise

1.

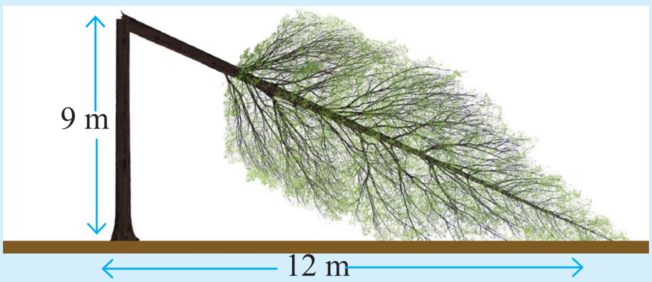


Find the shortest distance between the point $A = (4, 5)$ and $B = (10, 13)$ located on a Cartesian plane.

2. The city Q is located 5 km east of the city P and the city R is located 12 km north of the city Q. Find the distance between the two cities P and R.

3. To keep a 16m tall flag post vertical, one end of a supportive cable is attached to the top of the post while the other end is fixed to a point on the ground (horizontal), 12m from the foot of the flag post. Another cable is fixed from the opposite direction, with one endattached to the flag post, 12m above its foot, and the other to the ground, 9m from its foot. Calculate the total length of the cable that has been used.

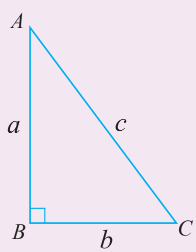
4.



A tree which was struck by a tornado is shown in the figure. Find the height of the tree before it was struck.

Summary

In the ABC right angled triangle



$$AC^2 = AB^2 + BC^2$$

$$c^2 = a^2 + b^2$$