18 Circumference of a Circle

By studying this lesson you will be able to;

- find the diameter of a circle using different methods,
- find the circumference of a circle and the perimeter of a semicircle using formulae,
- solve problems related to the circumference of a circle.

Do the following exercise to recall what you have learnt about circles.

Review Exercise

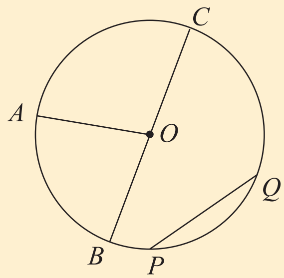
1. a. Fill in the blanks using suitable words.

i. The locus of the points on a plane which are at a constant distance from a fixed point is a. ……………..

ii. The point right at the middle of a circle is known as its ....

b. Copy the two columns A and B given below and using the given figure, join the relevant pairs.

A	B
Point O	Radius
OA	Diameter
BC	Centre
OB	Chord
PQ	



2.

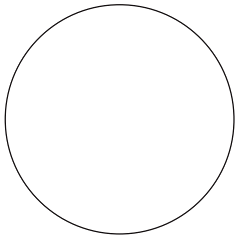
i. What is the length of the diameter of a circle of radius 5 cm?

ii. What is the length of the radius of a circle of diameter 7 cm?

iii. If the radius of a circle is r and diameter is d, write an equation expressing the relationship between d and r.

Measuring the diameter and the circumference of a circle

The total length of the boundary of a circle, or the perimeter of a circle is known as its **circumference**.



A circular ring made from a metal wire of length 25 cm is shown in the above figure. Since the length of the wire is 25 cm, the perimeter or the circumference of the circle is 25 cm.

We cannot directly determine the diameter of a circle.

Do the following activities to identify different methods of finding the diameter of a circle.

Activity 1

(*a*) - Measuring the diameter of a circle using a straight edge with a cm/mm scale.

Step1: Draw any circle using the pair of compasses and mark its centre.

Step2: Draw a diameter and measure its length using a straight edge with a cm/mm scale.

(*b*) - Measuring the diameter by means of an axis of symmetry of a circular lamina.

Step 1: Draw a circle on a piece of paper using an object like a coin or a bangle and cut it out.

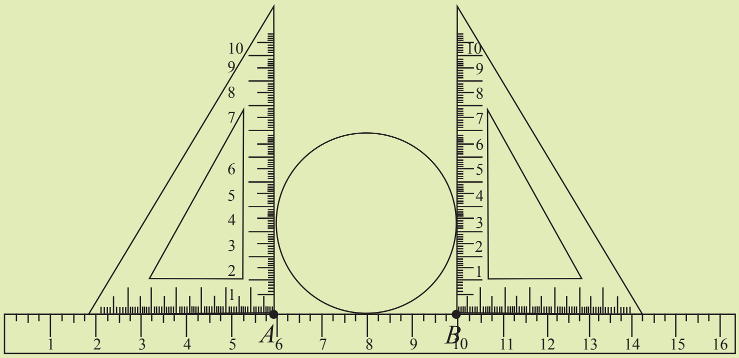
Step 2: Fold the circular lamina into two equal parts (such that the two parts coincide) and mark the axis of symmetry on it.

Step 3: Since the axis of symmetry is a diameter of the circle, measure the length of the axis of symmetry and obtain the length of the diameter.

(*c*)- Measuring the diameter using set squares.

Step 1: Take a ruler, two set squares, a circular coin, a bangle and a cylindrical can.

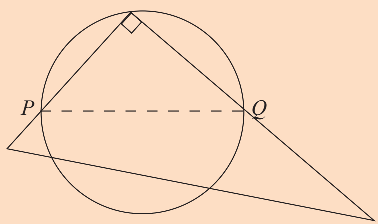
Step 2: Place the bangle and the set squares as shown in the figure, touching the ruler. Find the diameter of the bangle using the two readings denoted by *A* and *B*.



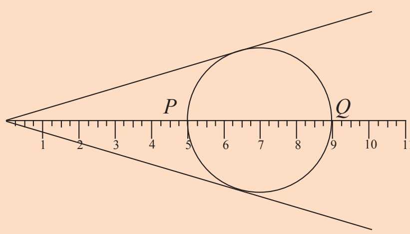
Step 3: Find the diameters of the remaining circular objects too by doing the above activity and note them down in your exercise book.

Different methods of finding the diameter

1. Keep the right angled corner of a piece of paper on a circle as shown in the figure. The distance between the two points(*P* and *Q*) where the arms of the 90^{o} angle meet the circle is the length of the diameter of the circle.



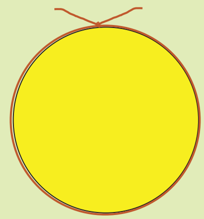
2. Make an instrument as shown in the figure: Draw an angle and its bisector on a Bristol board and calibrate the bisector from the vertex. The length of the diameter of a circle can be measured as shown in the figure.



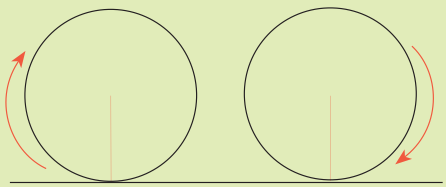
18.1 Measuring the circumference of a circle

Do the following activities in order to find out the methods used to measure the circumference of a circular lamina such as a coin.

Activity 2



1. Mark a point on a piece of thread and with the thread stretched, place it around the circular lamina until it reaches the initial point that was marked. Mark the final point on the thread when it coincides with the initial point, and measure the length between the two points marked on the thread to find the circumference of the coin.



2. Draw a straight line on a piece of paper. Keep it on a flat surface. Mark a point on the circumference of the circular lamina and also on the straight line. Place the circular lamina on the straight line such that the two marks coincide and then role it along the straight line until the point marked on the circular lamina touches the line again. The length of the circumference is obtained by measuring the distance the circular lamina has moved along the straight line.

Developing a formula for the circumference of a circle

Do the following activity to identify the relationship between the circumference and the diameter of a circle.

Activity 3

Complete the table given below by measuring the circumference and the diameter of objects with circular faces, using the methods introduced above.

Object	Diameter d	Circumference c	$\frac{c}{d}$ up to two decimal places
1. Circular lamina made of cardboard			
2. Rs 2 coin			
3. Circular lid of a tin			
4. Compact Disk (CD)			

Compare the values you obtained for $\frac{c}{d}$ in the above activity with the values obtained by your friends and write your conclusion regarding the value of $\frac{c}{d}$.

Through the above activity you would have obtained a value for $\frac{c}{d}$ which is approximately 3.14 for every object you considered. Mathematicians have discovered that $\frac{c}{d}$ is a constant value for all circles. This constant value is denoted by π. It has been shown that this value is 3.14 to the nearest second decimal and is approximately equal to the fraction $\frac{22}{7}$.

Accordingly,

$$\frac{c}{d} = \pi .$$

That is,

$$c = \pi d.$$

This is a formula giving the relationship between the circumference and the diameter of a circle. A formula giving the relationship between the radius and the circumference of a circle can be derived as follows.

Since $d = 2r$ we obtain $c = \pi \times 2r$.

i,e.,

$$c = 2\pi r$$

If the circumference of a circle is denoted by c, the diameter by d and the radius by r, then,

$$\boxed{c = \pi d}$$

$$\boxed{c = 2\pi r}$$

Example 1

Find the circumference of a circle of radius 7 cm.

Use $\frac{22}{7}$ for the value of π.

Circumference $c = 2\pi r$

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{1}{\cancel{7}}$$

$$= 44$$

$\therefore$ the circumference is 44 cm.

Exercise 18.1

1. Find the circumference of the circle with the measurement given below. Use $\frac{22}{7}$ for the value of π.

i. radius 7 cm

ii. diameter 21 m

iii. radius 10.5 cm

iv. diameter $17\frac{1}{2}$ m

v. radius $\frac{7}{2}$ m

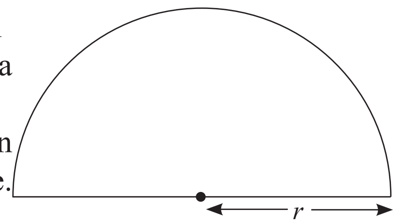
vi. diameter 28 cm

vii. radius 15.4 cm

viii. diameter $3\frac{1}{9}$ m

18.2 Perimeter of a semicircular lamina

When a circular lamina is separated into two equal parts along a diameter, each part is known as a semicircular lamina (in short, semicircle).
The length of the curved line of a semicircle is known as the arc length. It is exactly half the circumference.



Hence, the arc length of a semicircle of radius $r = \frac{1}{\cancel{2}} \times (\overset{1}{\cancel{2}}\pi r)$

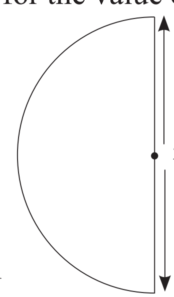
$= \pi r$

It is clear from the figure that, to find the perimeter of a semicircle, the diameter should be added to the arc length.

$\therefore$ The perimeter of a semicircle $= \pi r + 2r$

Example 1

Find the perimeter of the semicircle shown in the figure. Use $\frac{22}{7}$ for the value of π.



Arc length of a semicircle of diameter $d = \frac{1}{2}\pi d$

$\therefore$ Arc length of the semicircle of diameter 21 cm $= \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \overset{3}{\cancel{21}}$

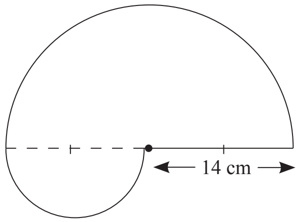
$= 33$

$\therefore$ The perimeter of the figure $= 33 + 21 = 54$ cm

Example 2

A compound figure, consisting of two semicircular laminas of radius 14 cm and diameter 14 cm respectively is shown in the figure. Find its perimeter.

Use $\frac{22}{7}$ for the value of π.



$$\text{Arc length of a semicircle of radius } r = \frac{1}{2} \times 2\pi r$$

$$\therefore \text{ Arc length of the semicircle of radius 14 cm} = \frac{1}{\cancel{2}_1} \times \cancel{2}^1 \times \frac{22}{\cancel{7}_1} \times \cancel{14}^2 \text{ cm} = 44 \text{ cm}$$

$$\text{Arc length of a semicircle of diameter } d = \frac{1}{2}\pi d$$

$$\therefore \text{ Arc length of the semicircle of diameter 14 cm} = \frac{1}{\cancel{2}_1} \times \frac{\cancel{22}^{11}}{\cancel{7}_1} \times \cancel{14}^2 \text{ cm} = 22 \text{ cm}$$

$$\therefore \text{Perimeter of the figure} = 44 + 22 + 14 \text{ cm}$$

$$= \underline{\underline{80 \text{ cm}}}$$

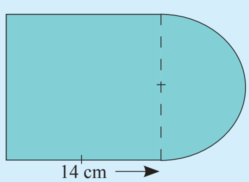
Exercise 18.2

1. Find the perimeter of the semi circular lamina with the measurement given below. Use $\frac{22}{7}$ for the value of π.

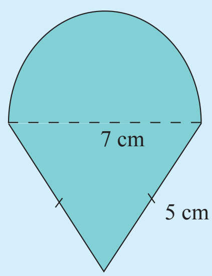
i. r =14cm   ii. d =7 cm

2. Find the permeter of the shaded part of each of the figures given below. The curved parts in the figures are semicireles.Use $\frac{22}{7}$ for the value of π.

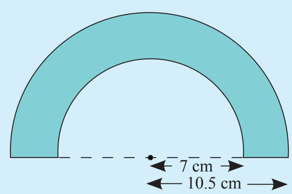
i.



ii.



iii.



18.3 Problems related to the circumference of a circle

Example 1

A wheel of radius 35 cm moves along a straight road.

i. Find in metres, the distance it moves during one full rotation.

ii. What is the distance it moves in metres during 100 rotations?

iii. How many rotation does the wheel undergo when travelling a distance of 1.1 km? (Use $\frac{22}{7}$ for the value of π.)

i. During one full rotation of the wheel, it moves a distance which is equal to its circumference.

$$\text{Circumference} = 2 \times \frac{22}{\cancel{7}_1} \times \cancel{35}^5 \text{ cm} = 220 \text{ cm}$$

$\therefore$ The distance travelled during one full rotation = $\underline{\underline{2.2 \text{ m}}}$

ii. Distance travelled during 100 rotations $= 2.2 \text{ m} \times 100$

$= \underline{\underline{220 \text{ m}}}$

iii. Distance travelled = 1.1 km

= 1100 m

Distance travelled during one rotation of the wheel = 2.2 m

Therefore, number of rotations $= \frac{1100}{2.2}$

$= \underline{\underline{500}}$

Example 2

A circular frame is made by joining the two ends of a wire of length 66 cm. Find its radius.

Let us assume that the radius is r cm.

$$c = 2\pi r$$

$$2 \times \frac{22}{7} \times r = 66$$

$$r = 66^{3} \times \frac{7}{22_{1}} \times \frac{1}{2}$$

$$= \frac{21}{2}$$

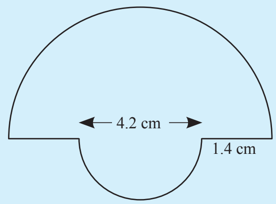
$$= 10.5 \text{ cm}$$

$\therefore$ The radius is 10.5 cm.

Exercise 18.3

Use $\frac{22}{7}$ for the value of π.

1. A lamina composed of two semicircles is shown in the figure. This is pasted on an ornamental box. A gold thread is to be pasted around the lamina.



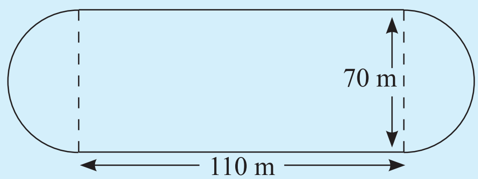
(i) Find the minimum length of thread required to paste around this lamina?

(ii) Find the total length in metres of the thread required to paste around 500 such laminas.

2. The circumference of a circular plot of land is 440 m. Find its radius.

3. The perimeter of a semicircular lamina is 39.6 cm. Find its diameter.

4. A sketch of a playground is shown in the figure. It consists of a rectangular part and two semicircular parts.



(i) Find the perimeter of the playground.

(ii) Show that the distance covered by a runner in completing $2\frac{1}{2}$ rounds of the playground is more than 1 km.

5. A cyclist rides a bicycle along a straight road. The radius of each wheel of the bicycle is 28 cm.

(i) Find the distance the bicycle moves during the period athat the wheels complete one full rotation.

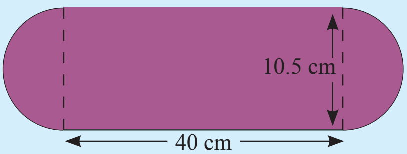
(ii) What is the distance the bicycle moves in meters during the period that the wheels complete 50 rotation?

(iii) The cyclist says that the wheels rotate at least 800 times when it travels a distance of 1500 m. Do you agree with this statement? Give reasons for your answer.

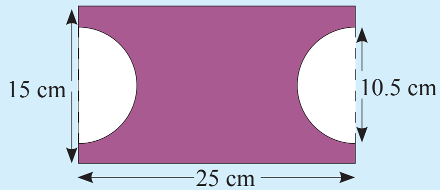
Miscellaneous Exercise

1. Find the perimeter of the shaded part of each figure.

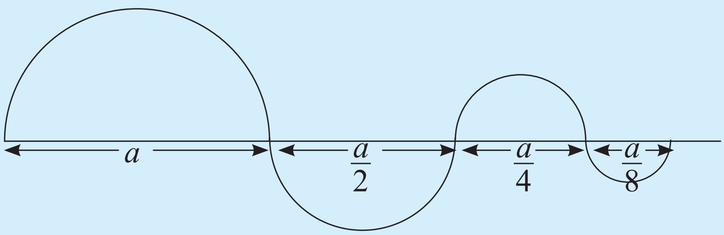
i.



ii.

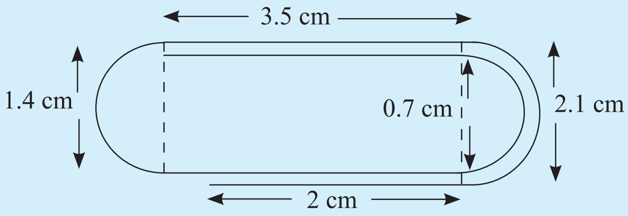


2.



Show that the length of metal wire needed to make the frame shown in the figure which consists of 4 semicircular parts is $\frac{135a}{28}$. (Use $\frac{22}{7}$ for the value of π.)

3. A paper clip with semi circular parts is to be made according to the given measurements. Find the length of the wire needed to make the clip in the figure.



Summary

In acircle of radius r, diameter d and circumference c,

- $c = \pi d$
- $c = 2\pi r$
- The perimeter of a semicircle = $\pi r + 2r$