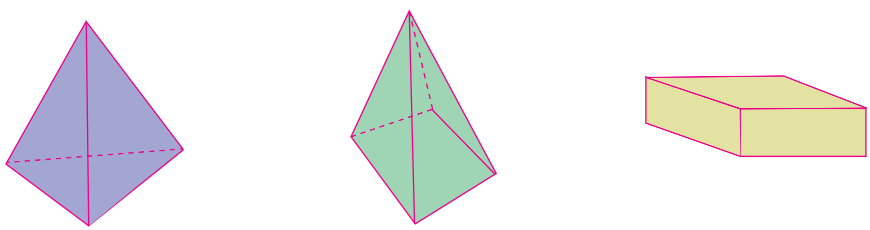
17 Formulae

By studying this lesson you will be able to;

- change the subject of a formula,
- find the value of a variable term in a formula when the values of the other variables are given.

Introducing formulae

In grade 8 you learnt Euler's relationship which is an equation expressing the relationship between the number of edges, number of vertices and number of faces in a solid.



This relationship is the following.

$$\text{Number of edges} = \text{Number of vertices} + \text{Number of faces} - 2$$

By taking the number of edges as E, the number of vertices as V and the number of faces as F, this relationship can be expressed as follows.

$$E = V + F - 2$$

A relationship between two or more quantities expressed as an equation, is known as a "formula".

The quantities in a formula are known as variables. In general, a formula is written with just one variable which is called the "subject of the formula" on one side of the equal sign (usually the left hand side), and the remaining variables on the other side. For example, in the formula $E = V + F - 2$ stated above, E is the subject.

Let us consider another formula. Temperature can be expressed in degrees Celsius or degrees Fahrenheit. The relationship between these two units is given below.

$$F = \frac{9}{5}C + 32$$

Here, F denotes the temperature in Fahrenheit and C denotes the temperature in Celsius. The subject of this formula is F.

Some formulae that are frequently used in Science and Mathematics are given below.

$p = 2(a + b)$

$v = u + at$

$s = \frac{n}{2}(a + l)$

$y = mx + c$

$C = 2\pi r$

$A = \pi r^2$

17.1 Changing the subject of a formula

E is the subject of the formula $E = V + F - 2$. If required, we can make either V or F the subject of this formula. This can be done in a manner similar to solving equations by using axioms.

As an example, let us make V the subject of the formula $E = V + F - 2$.
V is on the right hand side of this equation. F and -2 are also on the same side of the equation as V. To remove the terms F and -2 from the right hand side, let us add $-F$ and $+2$ to both sides of the equation.

We then obtain, $E + (-F) + 2 = V + F - 2 + (-F) + 2$.

Now, by simplifying both sides we obtain,
$E - F + 2 = V$ (since $F + (-F) = 0$ and $-2 + 2 = 0$)

The subject V appears on the right hand side.
Since the subject is usually written on the left hand side, we re-write the above equation as follows with V on the left hand side.

$V = E - F + 2$

The following examples show how the subject of formulae of various forms are changed.

Example 1

Make a the subject of the formula $v = u + at$.

Here the variable a is multiplied by the variable t. Therefore, we need to first make the term at the subject.
Subtracting u from both sides of $v = u + at$ we obtain

$$v - u = u + at - u$$
$$v - u = at$$

Now by dividing both sides by t to make a the subject we obtain,

$$\frac{v-u}{t} = \frac{at}{t}$$

By simplifying this we get the formula $a = \dfrac{v-u}{t}$ with a as the subject.

Example 2

Make n the subject of the formula $S = \dfrac{n}{2}(a + l)$.

$$S = \frac{n}{2}(a + l).$$

Here, the variable n which is to be made the subject is divided by 2 and the result is multiplied by $(a + l)$. Therefore both sides of the formula need to be multiplied by 2 and divided by $(a + l)$ to make n the subject.

By multiplying both sides by 2 we obtain,

$$2S = \cancel{2}^{1} \times \frac{n}{\cancel{2}_{1}} \times (a + l)$$
$$2S = n(a + l)$$

Now, by dividing both sides by $(a + l)$ we obtain

$$\frac{2S}{a+l} = \frac{n\,\cancel{(a+l)}^{1}}{\cancel{(a+l)}_{1}}$$

$$\frac{2S}{a+l} = n$$

$$n = \frac{2S}{a+l}$$

Example 3

Make n the subject of the formulal $l = a + (n - 1)d$.

$l = a + (n - 1)d$

Let us consider the variable n which is to be made the subject. Observe that the right hand side of the formula is formed by subtracting 1 from n to obtain $(n - 1)$, then multiplying $(n - 1)$ by d to obtain $(n - 1)d$ and finally adding a to $(n - 1)d$.

To make n the subject, we need to perform the inverse operations corresponding to the arithmetic operations performed in the above three steps (i.e., the inverse operation "addition" of the operation "subtraction, the inverse operation "division" of the operation "multiplication", etc.), starting from the last step and moving up.

Expressed in another way, this means that we make n the subject of the formula by using the relevant axioms.

Therefore, let us first subtract a from both sides of the equation and simplify.

$$l = a + (n - 1)d$$
$$l - a = a + (n - 1)d - a$$
$$l - a = (n - 1)d$$

Now let us divide both sides by d and simplify.

$$\frac{l - a}{d} = \frac{(n - 1)\cancel{d}^{1}}{\cancel{d}_{1}}$$
$$\therefore \quad \frac{l - a}{d} = n - 1$$

Finally let us add 1 to both sides and simplify.

$$\frac{l - a}{d} + 1 = n - 1 + 1$$
$$\frac{l - a}{d} + 1 = n$$
$$n = \frac{l - a}{d} + 1$$

If required, you may simplify the right hand side further, using a common denominator. However it is not essential to do this.

Exercise 17.1

1. Make r the subject of the formula $C = 2\pi r$.
2. Make c the subject of the formula $a = b - 2c$.
3. Make t the subject of the formula $v = u + at$.

4. In the formula $y = mx + c$,

i. make c the subject.

ii. make m the subject.

5. Make c the subject of the formula $a = 2(b + c)$.

6. Make C the subject of the formula $F = \frac{9}{5}C + 32$.

7. In the formula $l = a + (n - 1)d$,

i. make a the subject.

ii. make d the subject.

8. Make y the subject of the formula $\frac{x}{a} + \frac{y}{b} = 1$.

9. Make r_2 the subject of the formula $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$.

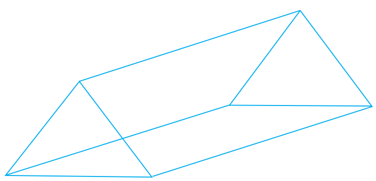
10. Make x the subject of the formula $ax = m(x - t)$.

11. Make a the subject of the formula $P = \frac{at}{a - t}$.

17.2 Substitution

Suppose that the values of all the variables in a formula except one are given. By substituting these values in the formula, the value of the unknown can be found.

Let us determine the number of edges in a solid with straight edges, which has 6 vertices and 5 faces.



The triangular prism shown above is an example of such a solid.

We can find the number of edges by substituting the values $V = 6$ and $F = 5$ in the formula $E = V + F - 2$.

Substituting $V = 6$ and $F = 5$ in the formula we obtain,

$E = 6 + 5 - 2$

$\;\;\; = 9$

Therefore, the solid has 9 edges.

Let us consider more examples.

There are two methods that can be used to find the value of an unknown variable when the values of the remaining variables are given. The first method is to substitute the given values in the formula as it is, and then find the value of the unknown. The second method is to first make the unknown of which the value is to be determined the subject of the formula, and then find its value by substituting the given values.

Let us now consider how the value of an unknown in a formula is found using these two methods.

Example 1

Determine the number of vertices there are in a solid that has 7 faces and 12 edges.

We need to use the formula $E = V + F - 2$ here. The values of F and E are given and we need to find V. We can use either of the above mentioned two methods to find V. That is, we can first substitute the given values in the formula $E = V + F - 2$ and then find the value of V by solving the resulting equation, or we can first make V the subject of the formula and then substitute the given values and simplify.

Let us consider both methods.
Let us take the number of edges as E, the number of vertices as V and the number of faces as F.

Method 1

$$E = V + F - 2$$

Substituting $E = 12$ and $F = 7$ we obtain

$$12 = V + 7 - 2$$
$$12 = V + 5$$
$$12 - 5 = V$$
$$7 = V$$
$$V = 7$$

$\therefore$ The number of vertices is 7.

Method 2

First make V the subject of the formula and then substitute the values.

$$E = V + F - 2$$
$$E + 2 = V + F$$
$$E + 2 - F = V$$
$$V = E + 2 - F$$
$$V = 12 + 2 - 7$$
$$V = 7$$

$\therefore$ The number of vertices is 7.

Note: One reason for changing the subject of a formula is because the value of the unknown can then be found easily by directly substituting the given values.

Example 2

Convert 35ºC into Fahrenheit using the formula $C = \frac{5}{9}\ (F - 32)$.
Consider that the temperature in degrees Celsius is denoted by C and the temperature in degrees Fahrenheit is denoted by F.

$$C = \frac{5}{9}\ (F - 32)$$

Substituting $C = 35$,

$$35 = \frac{5}{9}\ (F - 32)$$

Multiplying both sides by 9

$$35 \times 9 = 5\ (F - 32)$$

Dividing both sides by 5

$$\frac{{}^{7}\cancel{35} \times 9}{\cancel{5}_{1}} = F - 32$$

$$63 = F - 32$$
$$63 + 32 = F$$
$$95 = F$$
i.e., $F = 95$

The given tempreature is 95ºF.

Exercise 17.2

1. Find the value of a when $b = 7$ and $c = 6$ in the formula $a = (b + c) - 2$.

2. Find the value of C when $F = 104$ in the formula $C = \frac{5}{9}\ (F - 32)$.

3. Find the value of m when $y = 11$, $x = 5$ and $c = -4$ in the formula $y = mx + c$.

4. Find the value of r when $A = 88$ and $\pi = \frac{22}{7}$ in the formula $A = 2\pi r$.

5. Find the value of d when $l = 22$, $a = -5$ and $n = 10$ in the formula $l = a + (n - 1)d$.

6. Find the value of n when $S = -330$, $a = 4$ and $l = -48$ in the formula $S = \frac{n}{2}\ (a + l)$.

Miscellaneous Exercise

1. Consider the formula $P = C\,(1 + \frac{r}{100})$.

(i) Make r the subject of the above formula.
(ii) Find the value of r when $P = 495$ and $C = 450$.

2. Consider the formula $\frac{y - c}{x} = m$.

(i) Make x the subject of the above formula.
(ii) Find the value of x when $y = 20$, $c = -4$ and $m = 3$.

3. Consider the formula $ax = bx - c$.

(i) Make x the subject of the above formula.
(ii) Find the value of x when $a = 3$, $b = 4$ and $c = 6$.

4. Consider the formula $a = \frac{bx + c}{b}$.

(i) Make b the subject of the above formula.
(ii) Find the value of b when $a = 4$, $c = 5$ and $x = 3$.

5. Find the value of f in the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ when $v = 20$ and $u = 5$.

6. Find the value of b in the formula $\frac{a}{b} = \frac{p}{q}$ when $a = 6$, $p = 3$ and $q = 4$.

7. Consider the formula $S = \frac{n}{2}\,(a + l)$.
(i) Make l the subject of the above formula.
(ii) Find the value of l when $S = 198$, $n = 12$ and $a = 8$.

8. Consider the formula $y = mx + c$.

(i) Make m the subject of the above formula.
(ii) Find the value of m when $y = 8$, $x = 9$ and $c = 2$.