15 Equations

By studying this lesson you will be able to;

- solve linear equations containing brackets,
- solve linear equations containing fractions,
- solve simultaneous linear equations when the coefficient of one unknown is equal in both equations.

Linear equations

Do the following exercise to recall the facts that you have learnt in previous grades on solving linear equations.

Review Exercise

Solve the following linear equations.

a. $x + 12 = 20$ **b.** $x - 7 = 2$ **c.** $5 + m = 8$

d. $2x = 16$ **e.** $-3x = 6$ **f.** $2p + 1 = 5$

g. $3b - 7 = 2$ **h.** $\frac{x}{2} = 3$ **i.** $\frac{2p}{2} = 6$

j. $\frac{m}{5} - 1 = 8$ **k.** $2(x + 3) = 11$ **l.** $3(1 - x) = 9$

15.1 Solving linear equations with two types of brackets

You may have observed that there are some equations with brackets in the review exercise. In this lesson we expect to learn how to solve linear equations with two types of brackets. Let us first consider how to construct a linear equation with several brackets and find its solution.

Note: There are several types of brackets that we use.

( ) → Parentheses

{ } → Curly Brackets

[ ] → Square Brackets

When applying brackets, the usual practice is to first use parentheses, then curly brackets and finally square brackets.

"The result of adding three to a certain number and subtracting one from twice this value, and finally multiplying the resulting value by five and adding two is equal to 47".
Let us consider how to construct an equation using the above information and then solve it.

If the number is x, when 3 is added, we obtain $x + 3$.

Twice this expression can be written as $2(x + 3)$ using parentheses.

The expression that is obtained when 1 is subtracted from this is $2(x+3) - 1$.

Using curly brackets to write five times this expression we obtain,

$$5\{2\,(x + 3) - 1\}$$

It is given that when 2 is added to this expression it is equal to 47. Therefore,

$$5\{2\,(x + 3) - 1\} + 2 = 47$$

Now, by solving this equation, let us find the value of the number (x).

First, by simplifying the expression with parentheses we obtain

$$5\{2x + 6 - 1\} + 2 = 47.$$

When we simplify the expression within curly brackets we obtain,

$$5\{2x + 5\} + 2 = 47.$$

Now, simplifying the expression with curly brackets we obtain

$$10x + 25 + 2 = 47.$$
$$10x + 27 \quad = 47$$

Subtracting 27 from both sides we obtain,

$$10x + 27 - 27 = 47 - 27.$$

That is, $10x = 20$.
Dividing both sides by10 we obtain,

$$\frac{10x}{10} = \frac{20}{10}$$
$$x = 2$$

Therefore, the number is 2.

Let us consider a few more examples of equations with brackets to improve our skills of solving such equations.

Example 1

Solve $2\{3(2x-1)+4\} = 38$.

$$\frac{2\{3(2x-1)+4\}}{2} = \frac{38}{2} \quad \text{(dividing both sides by 2)}$$

$$3(2x-1)+4 = 19$$

$$6x-3+4 = 19 \text{ (simplifying the expression with parentheses)}$$

$$6x+1 = 19$$

$$6x+1-1 = 19-1 \text{ (subtracting 1 from both sides)}$$

$$6x = 18$$

$$\frac{6x}{6} = \frac{18}{6} \quad \text{(dividing both sides by 6)}$$

$$\underline{\underline{x = 3}}$$

Example 2

Solve $5\{4(x+3)-2(x-1)\} = 72$.

$$5\{4(x+3)-2(x-1)\} = 72$$

$$5\{4x+12-2x+2\} = 72 \quad \text{(simplifying the expression with parentheses)}$$

$$5\{2x+14\} = 72$$

$$10x+70 = 72 \quad \text{(simplifying the expression with curly brackets)}$$

$$10x+70-70 = 72-70 \text{ (subtracting 70 from both sides)}$$

$$\frac{10x}{10} = \frac{2}{10} \quad \text{(dividing both sides by 10)}$$

$$\underline{\underline{x = \frac{1}{5}}}$$

Exercise 15.1

Solve the following equations.

a. $2\{2(x-1)+2\} = 18$

b. $5\{3(x+2)-2(x-1)\} = 60$

c. $6+2\{x+3(x+2)\} = 58$

d. $5\{2+3(x+2)\} = 10$

e. $2\{3(y-1)-2y\} = 2$

f. $7x+5\{4-(x+1)\} = 17$

15.2 Solving linear equations containing fractions

Now, let us consider how to construct a linear equation with fractions and find its solution.

A vendor bought a stock of mangoes to sell. He discarded 10 fruits which were rotten. The rest he divided into 12 equal piles of 5 mangoes each.

Let us construct an equation using the above data.

If the vender bought x mangoes to sell,
when 10 mangoes are discarded, the remaining amount is $x - 10$.
The number of piles that can be made when the remaining mangoes are divided into groups of 5 is $\frac{x-10}{5}$. It is given that the number of piles is 12.

Therefore, $\frac{x-10}{5} = 12$

Now, let us solve this equation and find x.

$$\frac{x-10}{5} = 12$$

Multiplying both sides of the equation by 5,

$$5 \times \frac{x-10}{5} = 12 \times 5$$
$$x - 10 = 60$$

Adding 10 to both sides,

$$x - 10 + 10 = 60 + 10$$
$$x = 70$$

Therefore, the vendor bought 70 mangoes to sell.

Let us study the following examples to learn more on solving linear equations with fractions.

Example 1

Solve $\frac{x+3}{2} = 15$.

$$\frac{x+3}{2} = 15$$
$$2 \times \frac{x+3}{2} = 15 \times 2 \text{ (multiplying both sides by 2)}$$
$$x + 3 = 30$$

$$x + 3 - 3 = 30 - 3 \text{ (subtracting 6 from both sides)}$$
$$\underline{\underline{x = 27}}$$

Example 2

Solve $\dfrac{y}{2} - \dfrac{y}{3} = 9$.

$$\frac{y}{2} - \frac{y}{3} = 9$$

$$6 \times \frac{y}{2} - 6 \times \frac{y}{3} = 9 \times 6$$ (multiplying both sides by 6, the L.C.M. of the denominators 2 and 3)

$$3y - 2y = 54$$
$$\underline{\underline{y = 54}}$$

Example 3

Solve $2\left(\dfrac{m}{3} - 1\right) = 10$.

$$2\left(\frac{m}{3} - 1\right) = 10$$

$$\frac{2}{2}\left(\frac{m}{3} - 1\right) = \frac{10}{2} \text{ (dividing both sides by 2)}$$

$$\frac{m}{3} - 1 = 5$$

$$\frac{m}{3} - 1 + 1 = 5 + 1 \text{ (adding 1 to both sides)}$$

$$\frac{m}{3} = 6$$

$$3 \times \frac{m}{3} = 6 \times 3 \text{ (multiplying both sides by 3)}$$

$$\underline{\underline{m = 18}}$$

Note: When solving equations, it is not necessary to write the reason for each simplification.

Exercise 15.2

Solve each of the following equations.

a. $\dfrac{x-2}{5} = 4$ **b.** $\dfrac{y+8}{3} = 5$ **c.** $\dfrac{2a}{3} + 1 = 7$

d. $\dfrac{5b}{2} - 3 = 2$ **e.** $\dfrac{2p+3}{4} = 5$ **f.** $\dfrac{3m-2}{7} = 4$

g. $\dfrac{3x}{2} + \dfrac{x}{4} = 7$ **h.** $\dfrac{2m}{3} - \dfrac{3m}{5} = 1$ **i.** $4\left(\dfrac{3x}{2} - 1\right) = 12$

j. $\dfrac{1}{3}\left(\dfrac{2a}{3} - 3\right) = 2$ **k.** $\dfrac{m-3}{2} + 1 = 4$ **l.** $\dfrac{x+1}{2} + \dfrac{x}{3} = 8$

m. $\dfrac{y+1}{2} + \dfrac{y-3}{4} = \dfrac{1}{2}$ **n.** $\dfrac{x+3}{2} - \dfrac{x+1}{3} = 2$

15.3 Solving simultaneous equations

You have learnt in previous grades and in the earlier section of this lesson how to find the value of the unknown by solving a linear equation.
In this section we will learn how to solve linear equations with two unknowns.

Suppose it is given that the sum of two numbers is 6.

If we take the two numbers as x and y, then we can construct the equation $x + y = 6$, based on the given statement.
Here, x and y are not unique. The following table shows several different pairs of values of x and y which satisfy the above equation.

x	y	$x + y$
-1	7	6
0	6	6
1	5	6
2	4	6
3	3	6
4	2	6
5	1	6
6	0	6

Table 1

By observing the above table, we can conclude that there are infinitely many pairs of values of x and y which satisfy the equation $x + y = 6$.
If there is another relationship between x and y, we can construct another equation and by solving both equations simultaneously we can find the values of x and y that satisfy both equations.

Suppose it is given that the difference of the two numbers is 2. If we take the larger number as x, we can construct the equation $x-y=2$, based on the given statement.

There are infinitely many pairs of values of x and y which satisfy this equation too as can be concluded from observing the following table.

x	y	$x-y$
6	4	2
5	3	2
4	2	2
3	1	2
2	0	2
1	–1	2

Table 2

By observing Tables 1 and 2, you can see that there is only one pair of values of x and y which satisfies both $x+y=6$ and $x-y=2$. This pair is $x = 4$ and $y = 2$. Therefore, the solution of the above two equations is $x = 4$ and $y = 2$.

A pair of equations of this type with two unknowns is known as a pair of simultaneous equations. "Simultaneous" means "occurring at the same time".
Let us learn how to solve pairs of simultaneous equations using several other methods which are shorter, by considering the following examples.

Example 1

Solve the pair of simultaneous equations $x + y = 6$ and $x - y = 2$.

To facilitate finding the solution, let us label the two equations as ① and ② .

$x + y = 6$ ______ ①
$x - y = 2$ ______ ②

Method I

We can name this method "the method of substitution".

By making x the subject of equation ② , we can write it as

$$x = 2 + y.$$

By substituting this expression for x in equation ① we obtain,

$$2 + y + y = 6.$$
$$2 + 2y = 6$$

This is a linear equation in one unknown.

Let us find the value of y by solving it.

$$2 - 2 + 2y = 6 - 2$$
$$2y = 4$$
$$\frac{2y}{2} = \frac{4}{2}$$
$$\underline{\underline{y = 2}}$$

We can now find the value of x by substituing $y = 2$ in $x = 2 + y$.

$$x = 2 + 2$$
$$\underline{\underline{x = 4}}$$

Method II

This method can be named "the method of elimination".

$x + y = 6$ ______ ①
$x - y = 2$ ______ ②

First, observe that $+\,y$ occurs in equation (1) and $-\,y$ occurs in equation ②.
By adding both equations we get
$x + y + x - y = 6 + 2$

Here we have used the axiom "Quantities which are obtained by adding equal quantities to equal quantities, are equal".

Now we obtain a linear equation in x, since $+\,y$ and $-y$ cancel off.
Let us solve it and find the value of x.

$$2x = 8$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$\underline{\underline{x = 4}}$$

To find the value of y, let us substitute $x = 4$ in equation ①,

$$4 + y = 6$$
$$4 - 4 + y = 6 - 4$$
$$\underline{\underline{y = 2}}$$

$x = 4$
$y = 2$

Note that in the above pair of simultaneous equations, the coefficient of y was 1 in one equation and -1 in the other. That is, the numerical values of these coefficients are equal (when the signs are ignored).

Let us consider a few more examples. We will use the 2nd method to solve them.

Example 2

Solve $2m + n = 10$
$m - n = 2$.

$2m + n = 10 \longrightarrow$ ①
$m - n = 2 \longrightarrow$ ②

Adding ① and ②, $2m + n + m - n = 10 + 2$

$$3m = 12$$
$$\frac{3m}{3} = \frac{12}{3}$$
$$\underline{\underline{m = 4}}$$

By substituting $m = 4$ in ①,

$$2 \times 4 + n = 10$$
$$8 + n = 10$$
$$n = 10 - 8$$
$$\underline{\underline{n = 2}}$$

$m = 4$
$n = 2$

Example 3

Solve $2a + b = 7$
$a + b = 4$.

$2a + b = 7 \longrightarrow$ ①
$a + b = 4 \longrightarrow$ ②

In these equations, the coefficient of b is equal. Therefore, to eliminate b, we must subtract one equation from the other.

① – ②, $2a + b - (a + b) = 7 - 4$ (As there is a subtraction, it is essential to use brackets and write $(a + b)$)

$$2a + b - a - b = 3$$
$$\underline{\underline{a = 3}}$$

By substituting $a = 3$ in ②,

$$3 + b = 4$$
$$b = 4 - 3$$
$$\underline{\underline{b = 1}}$$

Example 4

Solve $x + 2y = 11$
$x - 4y = 5$.

$x + 2y = 11 \longrightarrow (1)$
$x - 4y = 5 \longrightarrow (2)$

Here the coefficients of x are equal. Therefore, let us subtract one equation from the other to eliminate x.

$$(1) - (2),\ x + 2y - (x - 4y) = 11 - 5$$
$$x + 2y - x + 4y = 6$$
$$6y = 6$$
$$\frac{6y}{6} = \frac{6}{6}$$
$$\underline{\underline{y = 1}}$$

By substituting $y = 1$ in (1),
$$x + 2 \times 1 = 11$$
$$x + 2 = 11$$
$$x + 2 - 2 = 11 - 2$$
$$\underline{\underline{x = 9}}$$

Exercise 15.3

1. Solve each of the following pairs of simultaneous equations.

a. $a + b = 5$
$a - b = 1$

b. $x + y = 8$
$2x + y = 2$

c. $m + 2n = 7$
$m - n = 1$

d. $4c - b = 7$
$4c - 2b = 2$

e. $2a + 3b = 16$
$4a + 3b = 26$

f. $3k + 4l = 4$
$3k - 2l = 16$

g. $x + 3y = 12$
$-x + y = 8$

h. $3m - 2n = 10$
$-3m + n = -14$

2. The sum of two numbers is 10 and their difference is 2. Taking the two numbers as x and y, construct a pair of simultaneous equations and solve them.

3. Two pens and a pencil cost Rs 32. A pen and a pencil cost Rs 20. Taking the price of a pen as Rs p and the price of a pencil as Rs q,construct a pair of simultaneous equations and by solving the pair find the price of a pen and the price of a pencil.