13 Rounding off and Scientific Notation

By studying this lesson you will be able to;

- identify the scientific notation and write numbers up to the millions period in scientific notation,
- convert numbers expressed in scientific notation to normal form,
- identify the rules related to rounding off numbers,
- round off a given number to the nearest ten, nearest hundred, nearest thousand and nearest decimal place,
- solve problems related to rounding off.

Introduction

➢ It is the opinion of scientists that dinosaurs are a species of animals that lived on earth 140 000 000 years ago.

➢ The atomic radius of the Hydrogen atom is 0.000 000 000 053 m.

➢ The distance from the sun to the earth is 149 600 000 000 m.

➢ The speed of light is 299 790 000 meters per second.

The above are four instances where numbers have been used to provide information. Using the information in the last two statements, let us find the time taken for a light

ray from the sun to approach earth.
Time =149 600 000 000 ÷ 299 790 000 seconds.

Since there are many digits in each of these numbers, they are lengthy. Therefore more space is required to write them and computations such as the above become difficult. Since a calculator can display only a limited number of characters, it is difficult to do such calculations even with an ordinary calculator. Therefore the need arises to represent such numbers in a more concise way to facilitate calculations.

In this lesson we will learn a method of writing these numbers in a concise way so that it is easy to manipulate them. Let us first do the below given review exercise to recall the facts that have been learnt in previous grades which are relevant to this lesson.

Review Exercise

1. Complete the following table.

Number	As a power of 10
1	$1 = 10^0$
10	$10 = 10^1$
100	$10 \times 10 = 10^{\cdots}$
1000	..... × ..... ×..... = $10^{\cdots}$
10000	..... = $10^{\cdots}$
100000	..... = ...
.....	..... = 10^6
.....	$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 =$ .....

2. Fill in the table given below with the following numbers according to the instructions given in the table.

5.37, 87.5, 0.75, 4.02, 1.01, 10.1, 4575, 0.07, 9, 12.3, 2.7, 9.9

Numbers that are between 1 and 10	
Numbers that are not between 1 and 10	

13.1 Scientific Notation

> The number of students sitting for the G.C.E (O/L) examination this year exceeds 700 000.
>
> - A news item

Several ways in which the six digit number mentioned in the above news item can be expressed are given below.

i. $700 \times 1000 \longrightarrow 700 \times 10^3$

ii. $70 \times 10\,000 \longrightarrow 70 \times 10^4$

iii. $7 \times 100\,000 \longrightarrow 7 \times 10^5$

From the above, the last form is used often as it can be easily written and is the most concise form. It is a product of two parts. The first part is 1 or a number between 1 and 10 while the second part is a power of 10.

7×10^5

Number between 1 and 10 or 1 — Power of 10

Writing a number with many digits in this manner as a product of two numbers, where one is between 1 and 10 or 1 and the other is a power of 10, is known as the scientific notation.

> If A is a number between 1 and 10 or 1 and n is an integer, then $A \times 10^n$ is a number written in scientific notation (Here $1 \leq A < 10$).

Let us write 280 000 in scientific notation.

Taking the first couple of digits in 280 000 and writing it as a number between 1 and 10 we get 2.8.

$\therefore$ $280\,000 = 2\,80000$

$= 2.8 \times 100\,000$

$= 2.8 \times 10^5$

Therefore 280 000 expressed in scientific notation is 2.8×10^5.

Example 1

Write the following numbers in scientific notation.

a. 20 000 **b.** 4240 **c.** 1 million **d.** 3.47 **e.** 34.7
f. 6 **g.** 289.325 **h.** 2491.32

a. $20\,000 = 2.0 \times 10\,000$
$= \underline{\underline{2 \times 10^4}}$

b. $4240 = 4.24 \times 1000$
$= \underline{\underline{4.24 \times 10^3}}$

c. 1 million $= 1000\,000$
$= \underline{\underline{1 \times 10^6}}$

d. $3.47 = 3.47 \times 1$
$= \underline{\underline{3.47 \times 10^0}}$ (since $1 = 10^0$)

e. $34.7 = 3.47 \times 10$
$= \underline{\underline{3.47 \times 10^1}}$

f. $6 = 6 \times 1$
$= \underline{\underline{6 \times 10^0}}$

g. $289.325 = 2.89325 \times 100$
$= \underline{\underline{2.89325 \times 10^2}}$

h. 2491.32

$2491.32 = 2.49132 \times 10^3$

By shifting the decimal point 3 places to the left, we obtain 2.49132×10^3.

Exercise 13.1

1. Complete the following table according to the given examples.

	Number	1 or a number between 1 and 10 × a power of 10	Scientific notation
	48	4.8×10	4.8×10^1
a.	8		
b.	99		
c.	78		
	548	5.48×100	5.48×10^2
d.	999		
e.	401		
f.	111		
	34 700	3.47×10000	3.47×10^4
g.	54 200		
h.	49 40000		
i.	10 00000		

2. Write each of the following numbers in scientific notation.

a.	200	**f.**	340000
b.	254	**g.**	6581200
c.	1010	**h.**	7.34
d.	5290	**i.**	18.5
e.	74300	**j.**	715.8

3. A few important facts about Sri Lanka are given below. Write the numbers which are related to these facts in scientific notation.

The height of Piduruthalagala mountain is 2524 m.
The area of Sinharaja forest is 9300 hectares.
The length of Mahaweli river is 335 km.
The total area of Sri Lanka is 65 610 km^2.

13.2 Writing a number between 0 and 1 in scientific notation

Consider the pattern given below.

$10\,000 = 10^4$
$1000 = 10^3$
$100 = 10^2$
$10 = 10^1$
$1 = 10^0$
$0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$

$0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$

$0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

It is clear that,

when writing 0.1 as a power of 10 the index is –1

when writing 0.01 as a power of 10 the index is –2

when writing 0.001 as a power of 10 the index is –3.

0.75 is a number which is less than 1. When it is written in terms of a number between 1 and 10, it should be written as 7.5 divide by 10. The way this is done mathematically can be expressed as follows.

Since $0.75 \times 10 = 7.5$,

$$0.75 = \frac{7.5}{10}$$

$$= \frac{7.5}{10^1} \quad (\text{Since } 10 = 10^1)$$

$$= \underline{\underline{7.5 \times 10^{-1}}} \quad \left(\text{Since } \frac{1}{10^1} = 10^{-1}\right)$$

Accordingly, the number 0.75 has been expressed as the product of a number between 1 and 10 and a power of 10.

$\therefore$ 0.75 expressed in scientific notation is 7.5×10^{-1} .

In the same manner, let us write 0.0034 in scientific notation.

Since $0.0034 \times 1000 = 3.4$,

$$0.0034 = \frac{3.4}{1000}$$

$$= \frac{3.4}{10^3}$$

$$= \underline{\underline{3.4 \times 10^{-3}}}$$

Note: When a number between 0 and 1 is written in scientific notation, the index of the power of 10 is a negative integer.

Example 1

Express each of the following numbers in scientific notation.

a. 0.8453 **b.** 0.047 **c.** 0.000017

a. $0.8453 = 8.453 \div 10$

$$= \frac{8.453}{10}$$

$$= \frac{8.453}{10^1}$$

$$= \underline{\underline{8.453 \times 10^{-1}}}$$

b. $0.047 = 4.7 \div 100$

$$= \frac{4.7}{100}$$

$$= \frac{4.7}{10^2}$$

$$= \underline{\underline{4.7 \times 10^{-2}}}$$

c. 0.000017

$$= 1.7 \div 100000$$

$$= \frac{1.7}{10^5}$$

$$= \underline{\underline{1.7 \times 10^{-5}}}$$

Exercise 13.2

1. Copy the following table and complete it.

Number less than 1	Expressed in terms of a number between 1 and 10	Scientific notation
a. 0.041	$\frac{4.1}{100} = \frac{4.1}{10^2}$	4.1×10^{-2}
b. 0.059		
c. 0.0049		
d. 0.000 135	$\frac{1.35}{10000} = \frac{1.35}{10^4}$	..... $\times 10^{-4}$
e. 0.000 005		
f. 0.000 003 9		
g. 0.111345		

2. Write each of the following numbers in scientific notation.

a. 0.08
b. 0.543
c. 0.0004
d. 0.0019
e. 0.00095
f. 0.000 000 054

3. Express each of the following numbers in scientific notation.

The radius of an atom is 0.000 0000 01 cm.
The mass of one cubic centimetre of air is 0.00129 g.
The mass of one cubic centimetre of hydrogen is 0.000 088 9 g.

13.3 Converting numbers expressed in scientific notation to general form

As an example, let us convert the number 5.43×10^4 written in scientific notation to general form.

Method I

$5.43 \times 10^4 = 5.43 \times 10000$
$= 54\,300$
$\therefore\ 5.43 \times 10^4 = 54300$

Method II

Since it is multiplied by 10^4, (that is 10 000) shifting the decimal point 4 places to the right, we obtain 54300.

54 300
54 300

Another example is given below. This is an instance where the index of the power of 10 is a negative number.

Method I

$5.43 \times 10^{-4} = 5.43 \times \frac{1}{10^4}$

$= 5.43 \div 10000$

$= 0.000543$

Method II

Since it is divided by 10^4, shifting the decimal point 4 places to the left we obtain 0.000543.

$0\overset{\curvearrowleft}{.000543}$

Example 1

Convert the following numbers to general form.

(i) 8.9×10^3

(i) $8.9 \times 10^3 = 8.9 \times 1000$

$= \underline{\underline{8900}}$ $\overset{\curvearrowright}{8900}.$

(ii) 8.9×10^{-3}

(ii) $8.9 \times 10^{-3} = 8.9 \times \frac{1}{10^3}$

$= \underline{\underline{0.0089}}$ $\overset{\curvearrowleft}{0.0089}$

Here for example, 8.9×10^3 can be directly written as 8 900. When multiplying, if the index of the power of 10 is a positive integer, then the decimal point should be shifted to the right, the same number of positions as the index (adding zeros if necessary). When multiplying, if the index of the power of 10 is a nagative integer, then the decimal point should be shifted to the left, the same number of positions as the index.

Exercise 13.3

1. Fill in the given blanks to convert each of the following numbers expressed in scientific notation to general form.

i. $5.43 \times 10^3 = 5.43 \times$ .....

$= \underline{\underline{.....}}$

ii. $7.25 \times 10^5 =$ ..... $\times$ .....

$=$ .....

iii. $6.02 \times 10^1 =$ ..... $\times$ .....

$=$ .....

$\underline{\underline{\quad\quad}}$

iv. $5.99 \times 10^{-2} = 5.99 \times \frac{1}{10^{...}}$

$= \frac{5.99}{.....}$

$= \underline{\underline{0.0599}}$

v. $1.06 \times 10^{-6} = 1.06 \times$ .....

$= \frac{1.06}{.....}$

$= \underline{\underline{.....}}$

2. Convert the following numbers to general form.

a. 8.9×10^2
b. 1.05×10^4
c. 7.994×10^5
d. 8.02×10^3
e. 9.99×10^7
f. 7.2×10^{-1}
g. 8.34×10^{-3}
h. 5.97×10^{-4}
i. 9.12×10^{-5}
j. 5.00×10^{-6}

3. Select the larger number from each of the number pairs given below.

a. 2.1×10^4, 3.7×10^4
b. 2.1×10^4, 3.7×10^3
c. 2.1×10^4, 3.7×10^5
d. 2.1×10^4, 2.1×10^{-4}
e. 2.1×10^4, 3.7×10^{-3}
f. 2.1×10^{-4}, 3.7×10^{-3}

4. Write the following numbers in general form.

The area of the earth covered by land is 1.488×10^8 km^2.
The area of the earth covered by the oceans is 3.613×10^8 km^2.
The total surface area of the earth is 5.101×10^8 km^2.

Rounding off numbers

> It is reported that 2 500 people attended the book exhibition held at Sarasvathi hall over the weekend.
>
> - A news item

The number of tickets that were sold over the weekend to those who attended the exhibition mentioned in the news item was 2 483. Accordingly, the actual number of people who attended the exhibition is 2 483. The number 2 500 which is mentioned in the news item is a number which is close to 2 483, easy to remember and has a special feature. Moreover it is sufficient to communicate an idea of the number that attended the exhibition.

Rounding off a number means representing the value of the number by a value which is close to it, which is simple and, easy to remember and communicate. There are many ways of rounding off numbers. Let us consider a few of them.

13.4 Rounding off to the nearest 10

Representing a number by the multiple of 10 which is nearest to it is known as "rounding off to the nearest 10".

Let us round off 2 483, which is the number of people who attended the exhibition, to the nearest 10. The number 2 483 lies between the two multiples of ten, 2 480 and 2 490. However it is closer to 2 480 than to 2 490. Accordingly, when 2 483 is rounded off to the nearest 10, we obtain 2 480.

We can describe this more generally as follows.

When rounding off 2 481, 2 482, 2 483 and 2 484 to the nearest 10 we obtain 2 480. This is because the multiple of 10 which all these numbers are closest to is 2 480. Similarly, if we round off 2 486, 2 487, 2 488 and 2 489 to the nearest 10 we obtain 2 490. The reason for this is also the same as the above. Even though the remaining number 2 485 is at an equal distance from the two multiples of ten 2 480 and 2 490, when rounding it off to the nearest 10, the convention is to round it off to the nearest 10 which is greater than it, that is, to 2 490. Finally, it is clear that when 2 480 is rounded off to the nearest 10 we obtain 2 480 itself and when 2 490 is rounded off to the nearest 10 we obtain 2 490 itself.

Example 1

Round off to the nearest 10.
i. 273 **ii.** 1428 **iii.** 7196.

i. 270

ii. 1430

iii. 7200

Exercise 13.4

1. Round off each of the following numbers to the nearest 10.

a. 33	**b**. 247	**c**. 3 008
d. 59	**e**. 306	**f**. 4 010
g. 85	**h**. 1514	**i**. 1 895
j. 12 345	**k**. 234 532	**f**. 997 287

2. The height of the mountain Piduruthalagala is 2 524 m. Round off this number to the nearest 10.

3. Write every whole number which when rounded off to the nearest 10 is equal to 140.

4. Write every whole number which when rounded off to the nearest 10 is equal to 80.

What is the smallest whole number which when rounded off to the nearest 10 is 80?
What is the largest whole number which when rounded off to the nearest 10 is 80?

5. When a certain number is rounded off to the nearest 10, the number 260 is obtained. Find separately the least and the greatest value that this number can take.

13.5 Rounding off to the nearest 100 or 1000

"Rounding off to the nearest 100" or "to the nearest 1000" is defined in the same way that "rounding off to the nearest 10" was defined.

For example, the number 7 346 is between the two multiples of hundred, 7 300 and 7 400 and is closer to 7 300 than to 7 400. Therefore when 7 346 is rounded off to the nearest 100, we obtain 7 300. Similarly, if we round off 7 675 to the nearest 100 we obtain 7 700. In general, if we round off a number from 7 300 to 7 349 (both included) to the nearest 100 we obtain 7 300, and if we round off a number from 7 350 to 7 400 (both included) to the nearest 100 we obtain 7 400.

Now, let us consider how to round off numbers to the nearest 1000. For example, when 41 873 is rounded off to the nearest 1000 we obtain 42 000. The reason for this is because 41 873 is closer to 42 000 than to 41 000.

It must be clear to you by this time, what occurs when we round off numbers. Now let us consider a method that can be used to round off numbers easily.

• Let us round off 2 425 to the nearest 100.

2425

The two multiples of 100 between which 2425 lies are 2400 and 2500. The value of 2425 is less than the value of 2450 which is exactly at the centre between these two multiples of 100. Therefore, 2425 is closer to 2400 than to 2500.

Accordingly, when 2425 is rounded off to the nearest 100 we obtain 2400.

• Let us round off 2485 to the nearest 100.

2485

The two multiples of 100 between which 2485 lies are 2400 and 2500. The value of 2485 is greater than the value of 2450 which is exactly at the centre between these two multiples of 100. Therefore, 2485 is closer to 2500 than to 2400.

Accordingly, when 2485 is rounded off to the nearest 100 we obtain 2500.

• Let us round off 2450 to the nearest 100.

2450

The two multiples of 100 between which 2450 lies are 2400 and 2500. The number 2450 is exactly at the centre between these two multiples of 100. According to the convention, the number which is at the centre is rounded off to the nearest multiple of 100 greater than that number.

Accordingly, when 2450 is rounded off to the nearest 100 we obtain 2500.

• Let us round off 2485 to the nearest 1000.

2485

The two multiples of 1000 between which 2485 lies are 2000 and 3000. The value of 2485 is less than the value of 2500 which is exactly at the centre between these two multiples of 1000. Therefore, 2485 is closer to 2000 than to 3000.

Accordingly, when 2485 is rounded off to the nearest 1000 we obtain 2000.

- Let us round off 2754 to the nearest 1000.

2754

The two multiples of 1000 between which 2754 lies are 2000 and 3000. The value of 2754 is greater than the value of 2500 which is exactly at the centre between these two multiples of 1000. Therefore, 2754 is closer to 3000 than to 2000.

Accordingly, when 2754 is rounded off to the nearest 1000 we obtain 3000.

- Let us round off 12 500 to the nearest 1000.

12500

The two multiples of 1000 between which 12 500 lies are 12 000 and 13 000. The number 12 500 is exactly at the centre between these two multiples of 1000. According to the convention, the number which is at the centre is rounded off to the nearest multiple of 1000 greater than that number.

Accordingly, when 12 500 is rounded off to the nearest 1000 we obtain 13 000.

Exercise 13.5

1. Round off each of the following numbers to the nearest 100.

a. 54 **b.** 195 **c.** 1009 **d.** 2985 **e.** 72324 **f.** 7550

2. Round off each of the following numbers to the nearest 1000.

a. 1927 **b.** 2433 **c.** 19999 **d.** 45874 **e.** 38000 **f.** 90500

3. The number of students in a school is 2 059. Round off this number to the,

i. nearest 10
ii. nearest 100
iii. nearest 1000.

4. When a number is rounded off to the nearest 100, the number 4 500 is obtained.

i. What is the smallest whole number it could be?

ii. What is the largest whole number it could be?

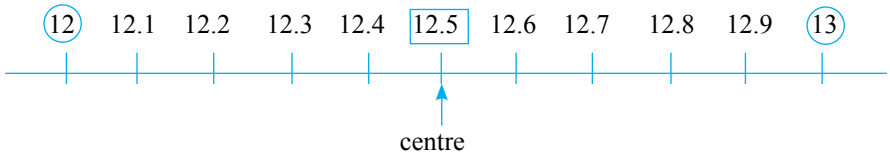
Rounding off decimal numbers

When the mass of a 5 year old child was measured, it was 12.824 kg. If we write this in grammes, it is 12 824 g. This value was obtained because the scale used for this purpose gives the mass to the nearest gramme. However, for practical purposes

the mass is usually required to the nearest kilogramme, to the nearest 10th of a kilogramme or to the nearest 100th of a kilogramme.

It is useful to know how to round off any given decimal number to the nearest whole number, nearest first decimal place, nearest second decimal place, etc. In this lesson we will learn how to round off decimal numbers.

Initially, let us consider how to round off a number with one decimal digit to a whole number.
Let us round off 12.7 to the nearest whole number.



The whole numbers on either side of 12.7 are 12 and 13.

Since the numbers 12.1, 12.2, 12.3 and 12.4 are closer to the whole number 12 than to the whole number 13, when these numbers are rounded off to the nearest whole number we obtain 12. Similarly, since the numbers 12.6, 12.7, 12.8 and 12.9 are closer to 13 than to 12, when these numbers are rounded off to the nearest whole number we obtain 13. Furthermore, as in the above sections, 12.5 rounded off to the nearest whole number is accepted by convention to be 13. Accordingly, when 12.7 is rounded off to the nearest whole number we obtain 13.

Similarly,
12.3 rounded off to the nearest whole number is 12 and
12.5 rounded off to the nearest whole number is 13.

Rounding off to a given decimal place

Round off 3.74 to the nearest first decimal place.

In 3.74, the digit in the first decimal place is 7 and the digit in the second decimal place is 4. When rounding off to the first decimal place, the digit in the second decimal place is considered and the digit in the first decimal place is adjusted accordingly if necessary.
The rule used here for rounding off is similar to that used in the previous sections. Since the number with one decimal digit which is closest to the numbers 3.71, 3.72, 3.73 and 3.74 is 3.7, when these numbers are rounded off to the first decimal place we obtain 3.7. Similarly when the numbers 3.75, 3.76, 3.77, 3.78 and 3.79 are rounded off to the first decimal place we obtain 3.8. Accordingly, 3.74 rounded off to the first decimal place is 3.7.

The rule for rounding off numbers to other decimal places is also the same. Let us consider the following example.

Example 2

Round off

i. 3.784 **ii.** 3.796

to the nearest second decimal place.

When rounding off to the nearest second decimal place, the digit in the third decimal place needs to be considered.

i. 3.784 lies between 3.78 and 3.79. Since 3.784 is closer to 3.78 than to 3.79, when it is rounded off to the nearest second decimal place we obtain 3.78.

ii. 3.796 lies between 3.79 and 3.80. Since 3.796 is closer to 3.80 than to 3.79, when it is rounded off to the nearest second decimal place we obtain 3.80.

Exercise 13.6

1. Round off each of the following numbers to the nearest whole number and to the nearest first decimal place.

i. 5.86 **ii.** 12.75 **iii.** 10.43 **iv.** 123.79
v. 8.04 **vi.** 13.99 **vii.** 101.98 **viii.** 100.51

2. The value of π is 3.14159… . Round off this value to

i. the nearest whole number
ii. the nearest first decimal place
iii. the nearest second decimal place.

3. The diameter of a sphere is 3.741 cm. Round off this value to

i. the nearest first decimal place
ii. the nearest second decimal place.

4. According to a survey plan, the area of a plot of land is 0.785 ha. Round off this value to

i. the nearest first decimal place
ii. the nearest second decimal place.

5. In an animal farm, the mean amount of milk obtained from a healthy cow per day is 5.25 *l*. If there are 45 such animals, round off the amount of milk obtained per a day

i. to the nearest litre

ii. to the nearest first decimal place.

Miscellaneous Exercise

1. Write each of the following groups of numbers in ascending order.

i. 3.10×10^{2} , 3.10×10^{-4} , 3.10×10^{0} , 3.10×10^{5}

ii. 4.78×10^{-2} , 1.43×10^{4} , 9.99×10^{-3} , 2.32×10^{1}

iii. 7.85×10^{0} , 7.85×10^{-4} , 7.85×10^{2} , 7.85×10^{-2}

2. There are 250 labourers working in a factory which pays Rs 1 230 per day as wages to a labourer.

i. Find the amount of money required per day to pay the wages of all these labourers.

ii. Write 1 230 and 250 in scientific notation.

iii. Using the numbers written in (ii) above in scientific notation, find the amount of money required per day for wages.

iv. Compare the values obtained in (i)and (iii) above.

3. The volume of tea produced in a day at a certain tea factory is 1 500 kg. If the factory operates for 30 days during a certain month, show that the volume of tea produced that month is 4.5×10^{4} kg.

4. Fill in the tables given below.

(a)

Expression	The expression obtained when the numbers in the given expression are rounded off to the nearest whole number	The value obtained for the expression by taking the product after rounding off the numbers
59.2×9.97	60×10	600
8.4×5.7	8×6	48
12.3×11.95	..... ×.....	.....
10.15×127.6	..... ×.....	.....
459.7×3.51	..... ×.....	.....
109.5×4.49	..... ×.....	.....

(b)

Expression	Product without rounding off the numbers	Value obtained by rounding off the product to the nearest whole number
59.2×9.97 8.4×5.7 12.3×11.95 10.15×127.6 459.7×3.51 109.5×4.49	590.224	590

Summary

- Scientific notation is a method of expressing a number concisely to facilitate calculations.
- If $1 \leq A < 10$ and $n \in \mathbb{Z}$ then $A \times 10^n$ is a number expressed in scientific notation.