10 Direct Proportion

By studying this lesson you will be able to;

- identify direct proportions,
- solve problems related to direct proportion using the unitary method,
- solve problems related to direct proportion using the definition,
- write the relationship between two directly proportional quantities in the form $y = kx$,
- solve problems related to the conversion of foreign currencies using the knowledge on direct proportions.

10.1 Introduction to direct proportion

The way the price of a certain type of pen varies depending on the quantity of pens is given in the following table.

Number of pens	Price (Rs)
1	15
2	30
3	45
4	60
5	75
6	90

It is clear from the above table that the price increases as the number of pens increases. Let us consider the number of pens and the price as two quantities.

Based on the above example, a few ratios of different amount of pens and the ratios of the corresponding prices are shown in the following table. Observe that these ratios are equal.

Ratio of two amounts of pens	Ratio of the corresponding prices
1 : 2	15 : 30 = 1 : 2
1 : 3	15 : 45 = 1 : 3
2 : 3	30 : 45 = 2 : 3
3 : 5	45 : 75 = 3 : 5
2 : 5	30 : 75 = 2 : 5

Two distinct quantities are said to be in direct proportion if they increase or decrease in the same ratio.

Therefore, if two quantities are in direct proportion, then when one quantity increases, the other quantity will also increase in the same ratio.
Similarly, if two quantities are in direct proportion and one quantity decreases, then the other quantity will also decrease in the same ratio.

Exercise 10.1

1. For each of the cases given below, write whether the two quantities are directly proportional or not.

a. The number of books and the price

b. The distance travelled by an object moving at a constant speed and the time taken for the journey.

c. The speed of a vehicle and the time taken to travel a certain distance

d. The length of a side of a square and its perimeter

e. The length of a side of a square and its area

f. The number of people needed to finish a task and the number of days taken for it

g. The number of units of electricity consumed by a household and the monthly bill

10. 2 Solving problems related to direct proportion using the unitary method

Suppose we want to find the price of 5 cakes of a certain type of soap, given that the price of 3 cakes of soap of that type is Rs 120.

As you have learnt in previous grades, we can first find the price of one cake of soap and thereby easily find the price of 5 cakes of soap.

Price of 3 cakes of soap = Rs 120
Price of 1 cake of soap = Rs 120 ÷ 3
= Rs 40
Price of 5 cakes of soap = Rs 40 × 5
= Rs 200

This method of calculation can also be explained as follows.

There are two quantities. They are the number of cakes of soap and the price. Initially the price of one cake of soap is found. It is Rs. 40. To find the price of five cakes of soap, the price of one cake of soap is multiplied by 5. Here the price of one cake of soap is clearly the constant value of the following fraction.

$$\frac{\text{price of 3 cakes of soap}}{\text{number of cakes of soap}}$$

The method of solving a problem based on the value of a unit is called the unitary method.

Let us learn how to solve problems related to direct proportion using the unitary method by considering a few examples.

Example 1

If a person walking at a constant speed takes 5 minutes to walk 800 m, calculate the distance he walks in 12 minutes.

Distance walked in 5 minutes in metres = 800
Distance walked in 1 minute in metres = 800 ÷ 5
= 160
Distance walked in 12 minutes in metres = 160 × 12
= 1 920
∴ The distance walked in 12 minutes is 1920 m.

Example 2

If the mass of 10 identical balls used in a cricket match is 3 kg, what is the mass of 3 such balls?

Mass of 10 balls in kilogrammes = 3
Mass of 1 ball in grammes = 3000 ÷ 10
= 300
Mass of 3 balls in grammes = 300 × 3
= 900
∴ The mass of 3 balls is 900g.

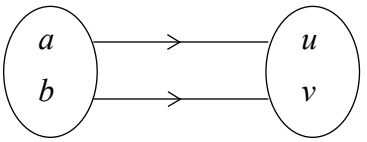
Do the following exercise using the unitary method.

Exercise 10.2

1. If the price of 8 oranges is Rs 320, find the price of 5 such oranges.
2. If the price of 5 m of a certain fabric is Rs 750, find the price of 12 m of that fabric.
3. If the mass of a parcel containing 15 apples is 3.6 kg, find the mass of a parcel containing 8 such apples.
4. If a printing machine makes 240 copies in 5 minutes, determine the number of copies it makes in 12 minutes.
5. If a motor vehicle moving at a constant speed travels 12 km in 15 minutes, calculate the distance it travels in 40 minutes.
6. If a motorbike can travel 90 km on 2 *l* of petrol, find the distance it can travel on 5 *l* of petrol.
7. If the time taken for a tank of capacity 1000 litres to be filled using a pump that releases water at a constant rate is 5 minutes, find the time taken in seconds to fill a tank of capacity 1600 litres.

10.3 Solving problems related to direct proportion using the definition

In the first section of this lesson it was explained that if two quantities are directly proportional, then the ratio of any two values of the first quantity is equal to the ratio of the corresponding values of the second quantity. This can be shown algebraically as below. Let us assume that the price of an amount a of a certain item is Rs u and the price of an amount b of the same item is Rs v.



Then we can write $a : b = u : v$.

This can be expressed in terms of fractions as $\frac{a}{b} = \frac{u}{v}$ (or $\frac{b}{a} = \frac{v}{u}$).

Let us learn how to solve problems related to direct proportion using this feature by considering the following examples.

Example 1

If the price of 5 mangoes is Rs 75, find the price of 8 mangoes.

Let us take the price of 8 mangoes as x. Then we can illustrate this information using an arrow diagram as shown below.

Number of mangoes		Price (Rs)
5	→	75
8	→	x

Using this as the base, let us write an algebraic equation as shown below and by solving it, find the value of x; that is, the price of 8 mangoes.

$5 : 8 = 75 : x$

Therefore, $\frac{5}{8} = \frac{75}{x}$

$5x = 75 \times 8$

$x = \frac{75 \times 8}{5}$

$x = 120$

Accordingly, the price of 8 mangoes is Rs 120.

Example 2

Find the price at which an item bought for Rs 500 should be sold to earn a profit of 15%.

Let us write the information in this problem as follows, so that we can use direct proportions. "If the selling price of an item bought for Rs 100 is Rs 115 (since the profit is 15%), find the selling price of an item bought for Rs 500."

Let us assume that the selling price of an item bought for Rs 500 is Rs x.

Purchase price (Rs)		Selling price (Rs)
100	⟶	115
500	⟶	x

$100 : 500 = 115 : x$

$$\frac{100}{500} = \frac{115}{x}$$

$$100x = 115 \times 500$$

$$x = \frac{115 \times 500}{100}$$

$$x = 575$$

Accordingly, the selling price should be Rs 575.

Exercise 10.3

1. For each of the proportions given below, write the suitable value in the blank space.

a. $2 : 5 = 8 : .....$ **b.** $3 : 4 = ... : 20$

c. $5 : 3 = 40 : .....$ **d.** $4 : 1 = ... : 8$

e. $8 : ..... = 24 : 15$ **f.** $..... : 6 = 35 : 30$

2. Solve each problem given below using proportions, by first drawing an arrow diagram and then writing an algebraic equation.

a. If the price of 10 kg of rice is Rs 850, find the price of 7 kg of rice.

b. If the mass of 9 cm^3 of a certain type of metal is 108 g, find the mass of 12 cm^3 of this metal.

c. If the distance travelled in 4 hours by a motorbike moving at a constant speed is 240 km, find the distance travelled by it in 3 hours.

d. Find the amount needed to buy an item worth Rs 800 from a shop which offers a discount of 3%.

e. If a commission of 12% is given when an item is sold, what is the commission given for an item worth Rs 15 000?

f. If the price of 4 pencils is Rs 48, find the number of pencils that can be bought for Rs 132.

g. If the price of 12 bottles is Rs 4800, find the number of bottles that can be bought for Rs 6000.

10.4 Solving problems related to direct proportion algebraically

If the price of 1 pen is Rs 15, then

- the price of 2 pens is Rs 30.
- the price of 3 pens is Rs 45.
- the price of 4 pens is Rs 60.

If we consider the above four instances, it can be observed that if the amount of money spent is divided by the number of pens, the value that is obtained is a constant.

That is, $\frac{\text{money spent}}{\text{number of pens}}$ = constant value.

This constant value is the price of one pen. Accordingly, if the money spent for x pens is y,

we can write $\frac{y}{x} = k$; here k is a constant.

This equation can also be written as $y = kx$.

Let us learn how to solve problems related to direct proportions using the above algebraic equation, by considering the following examples.

Example 1

If the price of 3 exercise books is Rs 75, find the price of 5 such exercise books.
Let us take the number of books as x and the price as y.

Then we can write $y = kx$; where k is a constant. The value of k can be found using the information given in the problem.

Since the price of 3 exercise books is Rs 75, when $x = 3$, $y = 75$.

By substituting these values in the equation we obtain, $75 = k \times 3$.
By solving this we obtain $k = 25$.

By substituting this value of k in the first equation, we obtain the relationship between x and y as $y = 25x$.

Now, using this equation, for any value of x the corresponding value of y and for any value of y the corresponding value of x can be found.
In this problem, since we need the price of 5 exercise books, y needs to be found when $x = 5$.

By substituting $x = 5$ in the equation $y = 25x$ we get,

$y = 25 \times 5$
$= 125$

Accordingly, the price of 5 exercise books is Rs 125.

Example 2

If a vendor sells an item he bought for Rs 500 such that he earns a profit of 20%, determine the selling price of the item.

Taking the purchase price of the item as x and selling price as y we can write $\frac{y}{x} = k$.

Since the selling price is Rs 120 when the purchase price is Rs 100, we obtain
$\frac{120}{100} = k$.

Let us assume that the selling price of an item bought for Rs 500 is y. Then we obtain the equation $\frac{y}{500} = k$.

Since k is a constant, we can write, $\frac{y}{500} = \frac{120}{100}$.

Therefore, $y = \frac{120 \times 500}{100}$.

$y = 600$.

$\therefore$ The selling price of the item is Rs 600.

Exercise 10.4

Do the problems in this exercise, using the algebraic equation method.

1. If the price of 3 shirts is Rs 1200, find the price of 5 shirts.

2. If the daily wage of 8 labourers who are paid equal wages is Rs 7200, find the daily wage of 3 labourers.

3. If a distance of 25 m is represented by 5 cm on a map drawn to scale, find the actual distance represented by 8 cm on this map.

4. If a machine in a factory produces 7500 drink bottles in 5 hours, find the number of drink bottles it produces in 7 hours.

5. A bookstore offers a discount of 8% on every book that is purchased. Find the amount a person has to pay if he purchases books worth Rs 1 200.

10.5 Foreign currency

We know that every country has its own currency unit and that the rate of conversion of the currency of one country to that of another country varies depending on the countries. The rate at which one country exchanges its currency with that of another country is called the **exchange rate**. This rate is not a constant value; it increases and decreases daily due to various reasons.

The currency units used by certain countries and their exchange rates with respect to the Sri Lankan rupee on a particular day, is given below.

Here the exchange rate given is the value of one foreign currency unit in Sri Lankan rupees.

Country/Union	Foreign currency unit	Exchange rate (Rs)
United States of America	American Dollar	151.20
England	Sterling Pound	185.90
European Union	Euro	160.60
Japan	Yen	1.33
India	Indian Rupee	2.26
Saudi Arabia	Saudi Riyal	40.32
Singapore	Singapore Dollar	107.30

(From the internet on 2017-03-05)

Now let us consider how to solve problems related to exchange rates using proportions.

Example 1

On a day that the exchange rate is Rs 151 for an American dollar, how many Sri Lankan rupees will a person who converts 50 American dollars receive?

Value of 1 American dollar = Rs 151
Value of 50 American dollars = Rs 151×50
= Rs $\underline{\underline{7550}}$

Therefore, the person will receive Rs 7550.

Example 2

A person visiting England, converted Rs 74 000 into sterling pounds on a day when the exchange rate was Rs 185 for a sterling pound. How many sterling pounds did he receive?

The value of 185 Sri Lankan rupees = 1 sterling pound

The value of 1 Sri Lankan rupee = $\frac{1}{185}$ sterling pounds

The value of 74 000 Sri Lankan rupees = $\frac{1}{185} \times 74\,000$ sterling pounds

= 400 sterling pounds

(It is easy to simplify this if we keep $\frac{1}{185}$ as a fraction without converting it into a decimal number). Therefore, the amount of sterling pounds he received is 400.

Exercise 10.5

Do the following exercise by using the exchange rate table given earlier.

1. If the monthly salary of a person working in a foreign country is 1500 American dollars, what is his salary in Sri Lankan rupees?

2. If the price of a television set imported from Japan is 12 500 yen, what is its value in Sri Lankan rupees?

3. A monthly allowance of 2500 sterling pounds is given to a scholarship student engaged in further studies in Great Britain. How much is this amount in Sri Lankan rupees?

4. A sports equipment in a duty free shop is worth 750 euros. How many Sri Lankan rupees have to be paid to purchase it?

5. A pilgrim who travels to India, converts 56 000 Sri Lankan rupees into Indian rupees. How many Indian rupees does he receive?

6. How many Singapore dollars are received when readymade garments worth Rs 600 880 are exported from Sri Lanka to Singapore?