# Angles related to straight lines and parallel lines

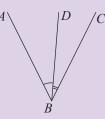
#### By studying this lesson, you will be able to;

- identify and verify the theorems related to the adjacent angles/vertically opposite angles formed by one straight line meeting or intersecting another straight line, and use them to solve problems,
- identify the angles formed when a transversal intersects two straight lines,
- identify and verify the theorems related to the angles formed when a transversal intersects two straight lines, and use them to solve problems.

# Introduction

Let us first recall the basic geometrical facts we learnt in previous grades.

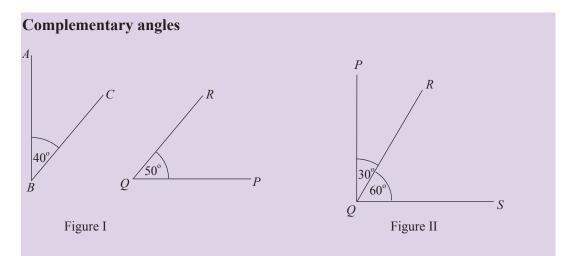
Adjacent angles



The angles  $A\hat{B}D$  and  $D\hat{B}C$  in the above figure have a common vertex. This common vertex is *B*. They also have a common arm *BD*. The pair of angles  $A\hat{B}D$  and  $D\hat{B}C$  lie on opposite sides of the common arm *BD*. Such a pair of angles is known as a pair of adjacent angles.

 $A\hat{B}D$  and  $D\hat{B}C$  are a pair of adjacent angles.

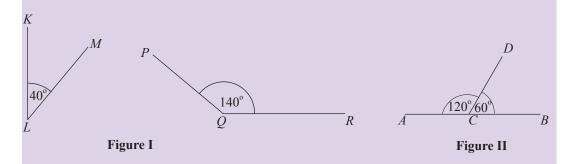
However,  $A\hat{B}D$  and  $A\hat{B}C$  are not a pair of adjacent angles. This is because, these two angles are not on opposite sides of the common arm AB.



In figure I, since  $ABC + PQR = 40^\circ + 50^\circ = 90^\circ$ , ABC and PQR are a pair of complementary angles.

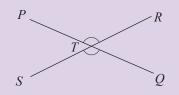
In figure II,  $P\hat{QR}$  and  $R\hat{QS}$  are a pair of adjacent angles. Furthermore, since  $P\hat{QR} + R\hat{QS} = 90^{\circ}$ , they are a pair of complementary angles too. Therefore,  $P\hat{QR}$  and  $R\hat{QS}$  are a pair of complementary adjacent angles.

Supplementary angles



In figure I, since  $KLM + PQR = 180^\circ$ , KLM and PQR are a pair of supplementary angles. In figure II, ACD and BCD are a pair of adjacent angles. Furthermore, since  $ACD + BCD = 180^\circ$ , they are a pair of supplementary angles too. Therefore, ACD and BCD are a pair of supplementary adjacent angles.

#### Vertically opposite angles



The pair of angles  $P\hat{T}R$  and  $S\hat{T}Q$ , formed by the intersection of the straight lines PQ and RS at the point T, are vertically opposite angles.

Similarly,  $P\hat{T}S$  and  $R\hat{T}Q$  are another pair of vertically opposite angles. Vertically opposite angles are equal in magnitude.

Therefore,  $P\hat{T}R = S\hat{T}Q$  and  $P\hat{T}S = R\hat{T}Q$ .

C

#### **Parallel lines**

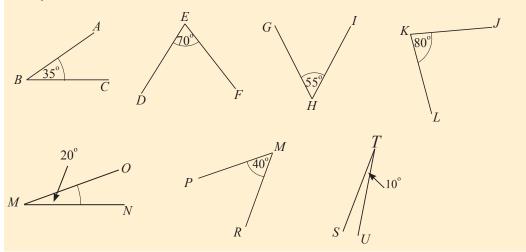
Two straight lines in a plane which do not intersect each other are called parallel straight lines. The gap between two parallel straight lines is a constant. A

As shown in the figure, parallel lines are indicated using arrow. We use the notation *AB*//*CD* to indicate that *AB* and *CD* are Parallel.

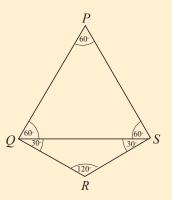
Do the following exercise, to strengthen your understanding of the above facts.

#### **Review Exercise**

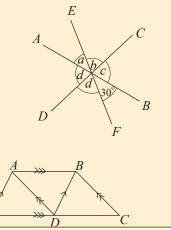
1. From the angles given below, select and write the pairs which are complementary.



- 2. Based on the magnitudes of the angles shown in the figure, write
  - i. four pairs of complementary angles,ii. two pairs of complementary adjacent angles,
- iii. two pairs of supplementary angles.



- **3.** The straight line segments *AB*, *CD* and *EF* shown in the figure intersect at a point. According to the information given in the figure,
  - **i.** find the value denoted by *a*.
  - ii. give reasons why b = d.
  - iii. find the value denoted by *d*.
  - iv. find the values denoted by *b* and *c*.
- 4. Name three pairs of parallel straight lines.

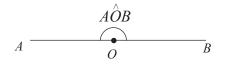


# 8.1 Angles related to straight lines

Let us assume that the point O is located on the straight line AB.

 $A \longrightarrow B$ 

Then AOB can be considered as an angle between the arms AO and OB. Such an angle is known as a straight angle.

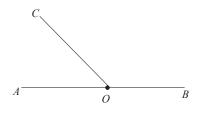


The unit "degree" which is used to measure angles has been selected such that a straight angle is equal in magnitude to 180°. Therefore, we can write  $A\hat{OB} = 180^{\circ}$ .

$$A \xrightarrow{\hat{O}B} = 180^{\circ}$$

Accordingly, the magnitude of a straight angle is180°.

In the following figure, two angles have been drawn at the point O which is located on the straight line AB.



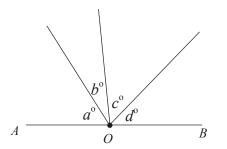
Here, AOC and BOC are a pair of adjacent angles. In a situation such as this, we say that the adjacent angles AOC and BOC are on the straight line AB. Furthermore, since  $AOB = 180^{\circ}$ , it is clear that,

$$A \stackrel{\wedge}{O}C + B \stackrel{\wedge}{O}C = 180^{\circ}.$$

Hence, the two angles AOC and BOC are a pair of supplementary adjacent angles. The facts we have discussed can be stated as a theorem as follows.

> Theorem: The sum of the adjacent angles formed by a straight line meeting another straight line is two right angles.

The facts discussed above can be generalized further. As an example, in the figure given below, four angles have been drawn at the point O on the straight line AB.



The values of these angles in degrees have been denoted by *a*, *b*, *c* and *d*. In a situation such as this too, we say that the angles are on the straight line *AB*. Furthermore, since  $AOB = 180^\circ$ , it is clear that,

a + b + c + d = 180.

It is also clear that this relationship holds true for any number of angles on a straight line. Therefore,

#### "The sum of the magnitudes of the angles on a straight line is 180°."

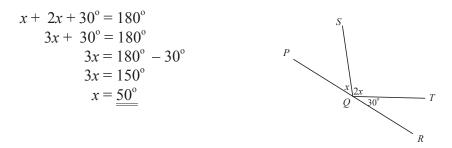
Now, by considering some examples, let us learn how to solve problems using these theorems.

#### Example 1

In each figure given below, if *PQR* is a straight line, then find the value denoted by *x*.  $P\dot{Q}D + D\dot{Q}R = 180^{\circ}$  (Angles on the straight line *PQR*)



 $P\hat{Q}S + S\hat{Q}T + T\hat{Q}R = 180^{\circ}$  (Angles on the straight line *PQR*)



### Example 2

In the given figure,  $A\hat{Q}R = 70^{\circ}$  and the bisector of  $P\hat{Q}A$  is QB. If PQR is a straight line, then find the magnitude of  $A\hat{Q}B$ .

R

P

 $\overline{O}$ 

Since *PQR* is a straight line,

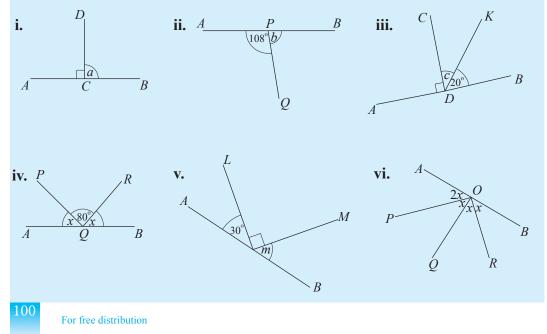
 $P\dot{Q}A + A\dot{Q}R = 180^{\circ} \text{ (angles on the straight line } PQR\text{)}$  $P\dot{Q}A + 70^{\circ} = 180^{\circ}$  $\therefore P\dot{Q}A = 180^{\circ} - 70^{\circ}$  $= 110^{\circ}$ 

Since BQ is the bisector of PQA,

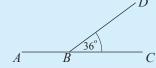
$$P\hat{Q}B = A\hat{Q}B = \frac{1}{2}P\hat{Q}A$$
  
$$\therefore \quad A\hat{Q}B = \frac{110^{\circ}}{2}$$
$$= \underline{55^{\circ}}$$

+2 Exercise 8.1

**1.** In each of the following figures, *AB* is a straight line. Based on the information in each figure, find the value of the angle denoted by the lower case letter.



**2.** In the figure, *ABC* is a straight line. If  $D\hat{B}C = 36^{\circ}$ , show that the magnitude of  $A\hat{B}D$  is four times the magnitude of  $D\hat{B}C$ .



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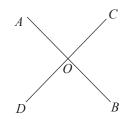
0

R

**3.** In the figure, *ABC* is a straight line. Based on the information given in the figure, show that  $P\hat{B}R$  is a right angle.

4. In the figure, ABC is a straight line. Moreover,  $P\hat{B}C = C\hat{B}Q$ . Show that  $A\hat{B}P = A\hat{B}Q$ .





In the figure, the straight lines AB and CD intersect each other at O.

The vertex *O* is common to both the angles AOC and DOB. Furthermore, they are on opposite sides of *O*.

The pair of angles AOC and DOB are known as a pair of vertically opposite angles.

Similarly, AOD and BOC lie on opposite sides of O, which is the common vertex of these two angles.



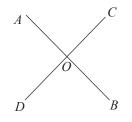
Therefore AOD and BOC are also a pair of vertically opposite angles.

Accordingly, it is clear that two pairs of vertically opposite angles are formed by the intersection of two straight lines.

Let us now consider a theorem related to vertically opposite angles.

#### Theorem: The vertically opposite angles formed by the intersection of two straight lines are equal.

By considering the figure, it can clearly be seen that 'vertically opposite angles are equal'. However, by using the fact we learnt earlier, that 'the sum of the angles on a straight line is 180', and also the axioms learnt in the previous lesson, this theorem can be proved as follows.



Data: The straight lines *AB* and *CD* intersect each other at *O*. To Prove:  $A \stackrel{\wedge}{OC} = B \stackrel{\wedge}{OD} and$ 

$$A \stackrel{\wedge}{OD} = B \stackrel{\wedge}{OC} C$$

Proof:

Since *AB* is a straight line,

 $AOC + BOC = 180^{\circ}$  (angles on the straight line *AB*) Similarly, since *CD* is a straight line,

> $BOC + BOD = 180^{\circ}$  (angles on the straight line *CD*)  $\therefore AOC + BOC = BOC + BOD$  (axiom)

Subtracting BOC from both sides of the equation,

$$\hat{AOC} + \hat{BOC} - \hat{BOC} = \hat{BOC} - \hat{BOC} + \hat{BOD}$$
  
$$\therefore \hat{AOC} = \hat{BOD}$$

Similarly,  $A \stackrel{\wedge}{OD} + A \stackrel{\wedge}{OC} = 180^{\circ}$  (angles on the straight line *CD*)

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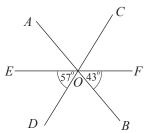
 $AOC + BOC = 180^{\circ}$  (since *AB* is a straight line)  $\therefore AOD + AOC = AOC + BOC$  (axiom) Subtracting AOC from both sides of the equation we obtain, AOD = BOC.

Let us consider the following examples to learn how this theorem is used to solve problems.

# Example 1

Based on the information given in the figure, giving reasons, determine the following.

- (i) The magnitude of DOB
- (ii) The magnitude of AOC.



(i) Since *EOF* is a straight line,

 $E\hat{O}D + D\hat{O}B + B\hat{O}F = 180^{\circ}$  (sum of the magnitudes of the angles on a straight line)  $57^{\circ} + D\hat{O}B + 43^{\circ} = 180^{\circ}$ 

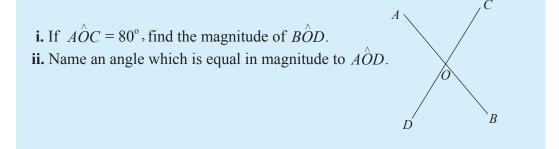
$$DOB = 180^{\circ} - (57^{\circ} + 43^{\circ})$$
  
 $DOB = 80^{\circ}$ 

(ii) 
$$A \stackrel{\frown}{O} C = D \stackrel{\frown}{O} B$$
 (vertically opposite angles)  
 $D \stackrel{\frown}{O} B = 80^{\circ}$  (proved above)  
 $\therefore A \stackrel{\frown}{O} C = \underline{80^{\circ}}$ 

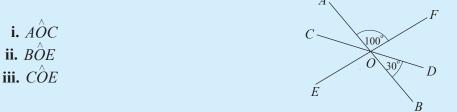
 $\frac{2}{+2}$  Exercise 8.2

· · .

1. In the figure, the straight lines *AB* and *CD* intersect each other at *O*.

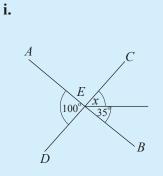


2. In the figure, the straight lines AB, CD and EF intersect at O. Based on the information provided in the figure, find the magnitude of each of the following angles.



3. Based on the information given in each of the figures shown below, find the value of each English letter representing an angle.

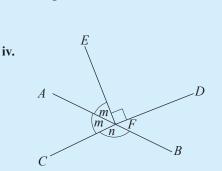
ii.



Р S R 38 0

straight lines.

In the figure AB and CD are straight lines.



In the figure, RS and PQ are

In the figure, AB and CD are straight lines.

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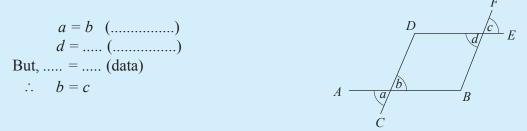
F



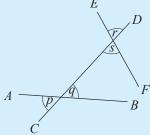
D

R In the figure, the straight lines AB, CD and EF intersect at G.

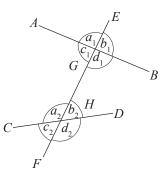
4. In the figure, *AB*, *CD*, *DE* and *BF* are straight lines. Moreover, a = d. Fill in the blanks given below to prove that b = c.



5. In the figure, *AB*, *CD* and *EF* are straight lines. Moreover p = r. Prove that s = q.



8.3 Corresponding angles, alternate angles and allied angles



In the above figure, the two straight line AB and CD are intersected by the straight line EF at the points G and H respectively. The line EF is known as a transversal.

A line intersecting two or more straight lines is known as a transversal.

In the above figure, there are four angles around the point G and four angles around the point H. According to where these angles are located, they are given special names in pairs.



# **Corresponding angles**

Consider the four pairs of angles given below.

(i)  $a_1$  and  $a_2$  (ii)  $b_1$  and  $b_2$  (iii)  $c_1$  and  $c_2$  (iv)  $d_1$  and  $d_2$ 

Each of these pairs of angles is a pair of corresponding angles.

To be a pair of corresponding angles, the following characteristics should be there in the two angles.

1. Both angles should lie on the same side of the transversal.

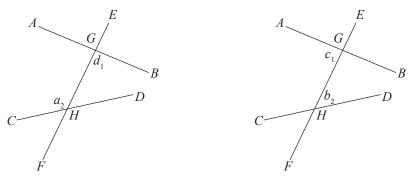
According to the above figure, both the angles  $a_1$  and  $a_2$  are on the left side of the transversal. Similarly, both the angles  $b_1$  and  $b_2$  are on the right side of the transversal. Also, the two angles  $c_1$  and  $c_2$  are on the left side of the transversal and the two angles  $d_1$  and  $d_2$  are on the right side of the transversal.

2. Both angles should be in the same direction with respect to the two straight lines.

According to the given figure, the angles  $a_1$  and  $a_2$  lie above the lines *AB* and *CD* respectively. The angles  $b_1$  and  $b_2$  also lie above the lines *AB* and *CD* respectively. Similarly, the angles  $c_1$  and  $c_2$  lie below the lines *AB* and *CD* respectively, and the angles  $d_1$  and  $d_2$  also lie below the lines *AB* and *CD* respectively.

In the given figure, the pairs of angles  $A\hat{G}E$  and  $C\hat{H}G$ ,  $B\hat{G}E$  and  $D\hat{H}G$ ,  $A\hat{G}H$  and  $C\hat{H}F$ ,  $B\hat{G}H$  and  $D\hat{H}F$  are pairs of corresponding angles.

Alternate angles



In the figures, the two pairs of angles given below are pairs of alternate angles. (i)  $a_2$  and  $d_1$ (ii)  $c_1$  and  $b_2$ 



The characteristics common to these pairs of angles that can be used to identify pairs of alternate angles are the following.

1. The two angles should be on opposite sides of the transversal.

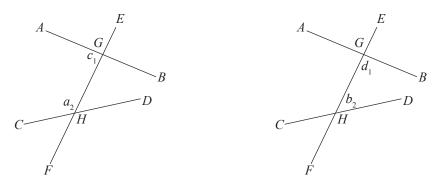
According to the above figure, the angles  $a_2$  and  $d_1$  lie on opposite sides of the transversal. Similarly,  $c_1$  and  $b_2$  also lie on opposite sides of the transversal.

2. The line segment of the transversal, which lies between the two straight lines, should be a common arm of the two angles.

According to the given figure, the line segment GH is a common arm of the angles  $a_2$  and  $d_1$  and also of the angles  $c_1$  and  $b_2$ .

In the given figure, the pair of angles  $B\hat{G}H$  and  $G\hat{H}C$  and the pair of angles  $A\hat{G}H$  and  $G\hat{H}D$  are pairs of alternate angles.

**Allied Angles** 



In the figures, the two pairs of angles given below are pairs of allied angles.

(i)  $a_2$  and  $c_1$ (ii)  $d_1$  and  $b_2$ 

In the figure, the two straight lines are intersected by a transversal. The pairs of angles on the same side of the transversal segment GH, between the straight lines AB and CD are

(i) the pair  $A\hat{G}H$  and  $C\hat{H}G$ (ii) the pair  $B\hat{G}H$  and  $D\hat{H}G$ 

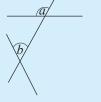


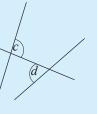
For all four of these angles, *GH* is a common arm. A pair of angles on the same side of the common arm *GH* and between the straight lines *AB* and *CD* is called a pair of allied angles, Accordingly,

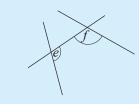
while the pair of angles  $A\hat{G}H$  and  $C\hat{H}G$  is a pair of allied angles, the pair of angles  $B\hat{G}H$  and  $D\hat{H}G$  is also a pair of allied angles.

# <u>+2</u> Exercise 8.3

1. Consider the figures given below of two straight lines intersected by a transversal.







#### Figure 1

Figure 2

Figure 3

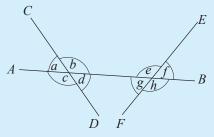
By considering the angles represented by the lower case English letters in the given figures, fill in the blanks.

(i) In figure 1, <i>a</i> and <i>b</i> form a pair o	of angles.
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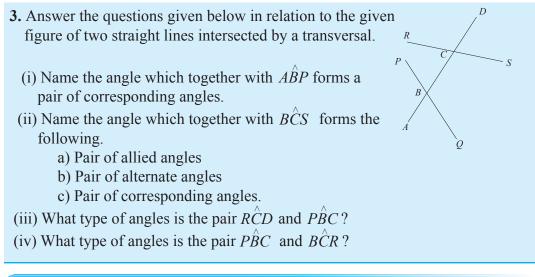
(ii) In	figure 2	2, <i>c</i> and <i>d</i>	form a p	air of	angles.

(iii) In figure 3, *e* and *f* form a pair of ..... angles.

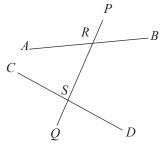
**2.** Consider the figure given below of two straight lines intersected by a transversal. Its angles are indicated by lowercase English letters.



- i. Name the line which can be considered as the transversal.
- ii. Name the two straight lines which are intersected by the transversal.
- iii. One pair of corresponding angles is the pair of angles *a* and *e*. Write the other three pairs of corresponding angles in a similar manner.
- iv. Write the two pairs of allied angles in terms of the lower case English letters.
- v. Write the two pairs of alternate angles in terms of the lower case English letters.



# 8.4 Angles related to parallel lines



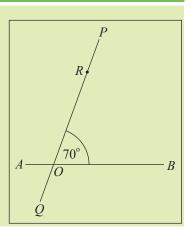
As indicated in the figure, the transversal PQ intersects the two straight lines AB and CD at R and S respectively. Now let us consider how the two lines AB and CD are positioned in each of the following cases.

- $\star$  When a pair of corresponding angles are equal
- \* When a pair of alternate angles are equal
- $\star$  When the sum of a pair of allied angles is 180°

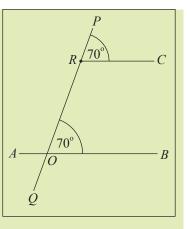
Do the following activity to identify this.



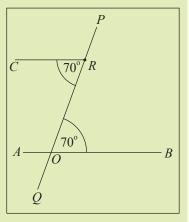
**Step 1:** On a sheet of A4 paper, draw two straight lines *AB* and *PQ* such that they intersect at *O* and such that  $POB = 70^\circ$ , as shown in the figure. Mark a point *R* on *OP*.



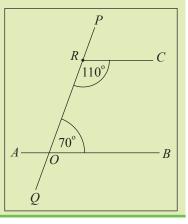
Step 2: Using the protractor, as shown in the figure, draw  $P\hat{R}C$  at R such that its magnitude is 70°. Observe that  $P\hat{O}B$  and  $P\hat{R}C$  are a pair of corresponding angles (considering PQ as a transversal which intersects the straight lines RC and AB)



- **Step 3:** Using a set square and a straight edge, examine whether the lines *AB* and *RC* are parallel.
- **Step 4:** Selecting different values for POB, repeat the above three steps and in each case examine whether the lines *AB* and *RC* are parallel.
- **Step 5:** Carry out steps 1 to 3 which were performed for corresponding angles, for alternate angles too. When completing these steps you will obtain a figure similar to the one shown here.



**Step 6:** Carry out the steps which were performed for corresponding angles, for allied angles too. In this case, the line drawn in step 2 above should be drawn as shown in the figure, such that  $C\hat{R}O = 180^\circ - 70^\circ = 110^\circ$ .



In doing the above activity you would have observed that, when

(i) a pair of corresponding angles are equal or

- (ii) a pair of alternate angles are equal or
- (iii) the sum of a pair of allied angles is equal to 180°,

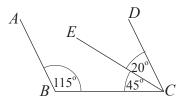
then the straight lines AB and RC are parallel.

This result which is true in general, can be expressed as a theorem as follows.

Theorem : When two straight lines are intersected by a transversal, if

- (i) a pair of corresponding angles are equal or
- (ii) a pair of alternate angles are equal or
- (iii) the sum of a pair of allied angles is 180°, then the two straight lines are parallel to each other.

Example 1



Based on the information given in the figure, show that AB and DC are parallel. The two angles ABC and BCD formed by the transversal BC meeting the two straight lines AB and DC, are a pair of allied angles.

$$A\hat{B}C = 115^{\circ} B\hat{C}D = B\hat{C}E + E\hat{C}D = 45^{\circ} + 20^{\circ} = 65^{\circ} A\hat{B}C + B\hat{C}D = 115^{\circ} + 65^{\circ} = 180^{\circ}$$

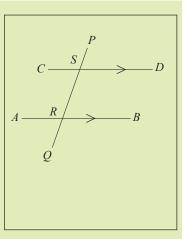
Since the sum of the pair of allied angles  $A\hat{B}C$  and  $B\hat{C}D$  is 180°, AB and DC are parallel.

Now let us consider another theorem which is related to parallel lines.



# Activity 2

Step 1: On a sheet of A4 paper, draw two straight lines AB and CD parallel to each other (the parallel lines can be drawn using a set square and a straight edge), and a transversal PQ such that it intersects the lines AB and CD at R and S respectively, as shown in the figure.



**Step 2:** Use a protractor to measure the magnitudes of the below given angles.

- (i) Measure the magnitudes of the pair of corresponding angles SRB and PSD and check whether they are equal. Similarly, measure the magnitudes of the other pairs of corresponding angles and check whether they too are equal.
- (ii) Measure the magnitudes of the pair of alternate angles  $C\hat{S}R$  and  $S\hat{R}B$  and check whether they are equal. Similarly, measure the magnitudes of the other pair of alternate angles and check whether they too are equal.
- (iii) Measure the magnitudes of the pair of allied angles DSR and SRB and check whether they are supplementary. Similarly, measure the magnitudes of the other pair of allied angles and check whether they too are supplementary.
- **Step 3:** Change the inclination of the transversal *PQ* and repeat the above two steps.

You would have observed in the above activity that when two parallel lines are intersected by a transversal,

- (i) each pair of corresponding angles is equal,
- (ii) each pair of alternate angles is equal,
- (iii) each pair of allied angles is supplementary.

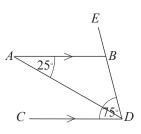
This result is true in general and can be expressed as a theorem as follows.

Theorem: When a transversal intersects a pair of parallel lines,

- i. the corresponding angles formed are equal,
- ii. the alternate angles formed are equal,
- iii. the sum of each pair of allied angles formed equals two right angles.

Observe that the above theorem is the converse of the theorem learnt earlier.





In the above figure, the straight lines *AB* and *CD* are parallel (this is denoted by *AB//CD*). Moreover,  $BDC = 75^{\circ}$  and  $BAD = 25^{\circ}$ .

(i) Giving reasons, determine the magnitude of  $A\hat{B}E$ .

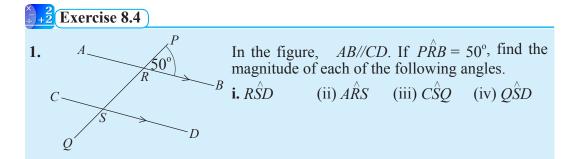
(ii) Giving reasons, determine the magnitude of ADB.

(i) 
$$B\hat{D}C = 75^{\circ}$$
 (data)  
 $B\hat{D}C = A\hat{B}E$  (corresponding angles,  $AB//CD$ )  
 $\therefore A\hat{B}E = \underline{75^{\circ}}$ 

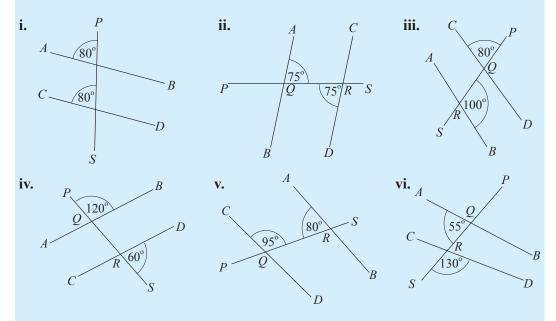
(ii)  $B\hat{A}D = 25^{\circ}$  (data)  $B\hat{A}D = A\hat{D}C$  (alternate angles, AB//CD)  $\therefore A\hat{D}C = 25^{\circ}$ 

But 
$$A\hat{D}B = B\hat{D}C - A\hat{D}C$$
  
= 75° - 25°  
= 50°

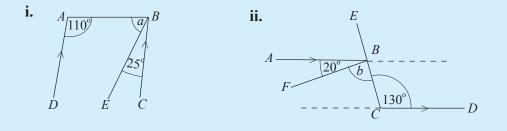


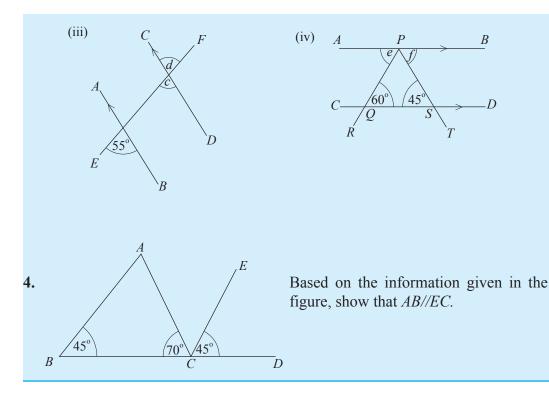


2. Based on the information in each of the following figures, giving reasons state whether *AB* and *CD* are parallel.



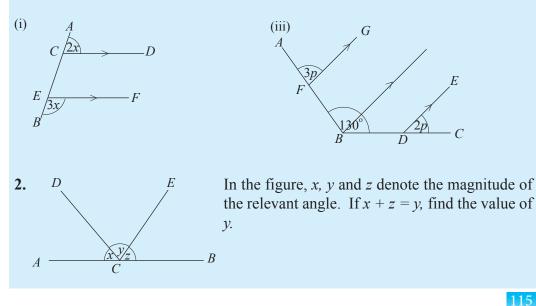
**3.** Find the value of each angle denoted by a lowercase English letter in the figures given below.

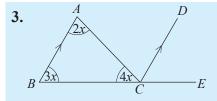




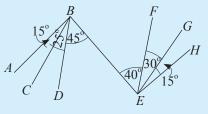
#### Miscellaneous Exercise

**1.** Find the magnitude of each of the angles denoted using lowercase English letters in the following figures.





- Based on the information in the figure,
  - (i) write the values of DCE and ACD in terms of x.
- (ii) find the value of *x*,
- (iii) find the magnitude of each angle in the triangle.
- **4.** Write all the pairs of parallel lines in the given figure, indicating the reasons for your selections.



**5.** In the figure,  $A\hat{B}C = p$  and  $C\hat{D}E = q$ . Show that p = q.



Summary

# Summary

- The sum of the adjacent angles formed by a straight line meeting another straight line is two right angles.
- The vertically opposite angles formed by the intersection of two straight lines are equal.
- When two straight lines are intersected by a transversal, if
  - i. a pair of corresponding angles are equal or
  - ii. a pair of alternate angles are equal or
  - iii. the sum of a pair of allied angles is 180°, then the two straight lines are parallel to each other.
- When a transversal intersects a pair of parallel lines,
  - i. the corresponding angles formed are equal,
  - ii. the alternate angles formed are equal,
  - iii. the sum of each pair of allied angles formed equals two right angles.

