5 Algebraic Expressions

By studying this lesson, you will be able to;

- find the value of simple algebraic expressions by substituting directed numbers,
- expand the product of two binomial expressions of the form $(x \pm a)(x \pm b)$,
- verify the expansion of the product of two binomial expressions by considering areas.

Algebraic expressions

Do the following exercise to review what you have learnt in grade 8, related to algebraic expressions.

Review Exercise

1. Expand the following expressions.

a. $5(x+2)$	**b.** $3(y+1)$	**c.** $4(2m+3)$
d. $3(x-1)$	**e.** $4(3-y)$	**f.** $2(3x-2y)$
g. $-2(y+3)$	**h.** $-3(2+x)$	**i.** $-5(2a+3b)$
j. $-4(m-2)$	**k.** $-(5-y)$	**l.** $-10(-3b-2c)$

2. Expand the following expressions.

a. $x(a+2)$	**b.** $y(2b-3)$	**c.** $a(2x+3y)$
d. $2a(x+5)$	**e.** $2b(y-2)$	**f.** $3p(2x-y)$
g. $(-3q)(p+8)$	**h.** $(-2x)(3-2y)$	**i.** $(-5m)(x-2y)$

3. Find the value of each of the following expressions when $x = 3$ and $y = -2$.

a. $x+y$	**b.** $x-y$	**c.** $3x-2y$
d. $-2x+y$	**e.** $2(x+y)$	**f.** $3(2x-y)$

4. Expand and simplify each of the following expressions.

a. $3(x+y)+2(x-y)$	**b.** $5(a+b)+4(a+c)$
c. $4(a+b)+3(2a-b)$	**d.** $2(a-b)+(2a-b)$
e. $5(m+n)+2(m+n)$	**f.** $3(m+n)-(m-n)$
g. $5(x-y)-3(2x+y)$	**h.** $2(3p-q)-3(p-q)$
i. $-4(m+n)+2(m+2)$	**j.** $-4(a-b)-2(a-b)$

5.1 Substitution

In grade 8, you learnt to find the value of an algebraic expression by substituting integers for the unknown terms. Let us now find out how to obtain the value of an algebraic expression by substituting directed numbers.

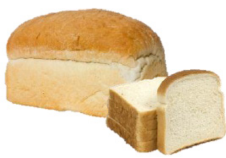
- 20 adults and 16 children went on a trip. Each adult was given x amount of bread and each child was given y amount of bread for breakfast.

Let us write the total amount of bread that was distributed as an algebraic expression.

Amount of bread given to the 20 adults $= 20x$
Amount of bread given to the 16 children $= 16y$
Total amount of bread that was distributed $= 20x + 16y$

Let us find out the total amount of bread that was distributed, if an adult was given half a loaf of bread and a child was given a quarter loaf of bread.

Then $x = \frac{1}{2}$ and $y = \frac{1}{4}$. To find out the total amount of bread that was distributed, $x = \frac{1}{2}$ and $y = \frac{1}{4}$ should be substituted in the expression $20x + 16y$.



Accordingly, the total number of loaves of bread that were distributed $= 20 \times \frac{1}{2} + 16 \times \frac{1}{4}$
$= 10 + 4$
$= 14$

Example 1

Find the value of each of the following algebraic expressions when $a = \frac{1}{2}$.

i. $2a + 3$

$$2a + 3 = 2 \times \frac{1}{2} + 3$$
$$= 1 + 3$$
$$= \underline{\underline{4}}$$

ii. $6 - 4a$

$$6 - 4a = 6 - 4 \times \frac{1}{2}$$
$$= 6 - 2$$
$$= \underline{\underline{4}}$$

iii. $3a - 1$

$$3a - 1 = 3 \times \frac{1}{2} - 1$$
$$= \frac{3}{2} - 1$$
$$= \frac{3 - 2}{2}$$
$$= \underline{\underline{\frac{1}{2}}}$$

Example 2

Find the value of each of the following algebraic expressions when $b = -\frac{2}{3}$.

i. $3b + 5$

$3b + 5$

$= 3 \times \frac{-2}{3} + 5$

$= (-2) + 5$

$= \underline{\underline{3}}$

ii. $5 - 6b$

$5 - 6b$

$= 5 - 6 \times (-\frac{2}{3})$

$= 5 + (-6) \times (-\frac{2}{3})$

$= 5 + 4$

$= \underline{\underline{9}}$

iii. $2b + \frac{1}{3}$

$2b + \frac{1}{3}$

$= 2 \times (\frac{-2}{3}) + \frac{1}{3}$

$= \frac{-4}{3} + \frac{1}{3}$

$= \frac{-3}{3}$

$= \underline{\underline{-1}}$

Example 3

Find the value of each of the following algebraic expressions when $x = \frac{1}{2}$ and $y = -\frac{1}{4}$.

i. $2x + 4y$

$2x + 4y = 2 \times \frac{1}{2} + 4 \times \left(\frac{-1}{4}\right)$

$= 1 - 1$

$= \underline{\underline{0}}$

ii. $2x - 2y$

$2x - 2y = 2 \times \frac{1}{2} - 2 \times \left(\frac{-1}{4}\right)$

$= 1 + \frac{1}{2}$

$= \underline{\underline{1\frac{1}{2}}}$

iii. $4xy$

$4xy = 4 \times \frac{1}{2} \times \left(\frac{-1}{4}\right)$

$= \underline{\underline{\frac{-1}{2}}}$

iv. $-2xy$

$-2xy = -2 \times \left(\frac{1}{2}\right) \times \left(\frac{-1}{4}\right)$

$= \underline{\underline{\frac{1}{4}}}$

Exercise 5.1

1. Find the value of each of the following algebraic expressions when $x = \frac{1}{4}$.

i. $4x$ **ii.** $2x$ **iii.** $3x$ **iv.** $-8x$

2. Find the value of each of the following algebraic expressions when $y = \frac{-1}{3}$.

i. $3y$ **ii.** $2y$ **iii.** $-6y$ **iv.** $-4y$

3. Find the value of each of the following algebraic expressions when $a = -2$ and $b = \frac{1}{2}$.

i. $a + 2b$ **ii.** $4b - a$ **iii.** $3a + b$

4. Find the value of each of the following algebraic expressions when $x = \frac{2}{3}$ and $y = \frac{3}{4}$.

i. $3x + 4y$ **ii.** $3x - 2y$ **iii.** $8y - 6x$

5. Find the value of each of the following algebraic expressions when $p = -\frac{1}{2}$ and $q = -3$.

i. $2p + q$ **ii.** $4p - q$ **iii.** $6pq - 2$

5.2 The product of two binomial expressions

Let us first recall what is meant by algebraic symbols, algebraic terms, algebraic expressions and binomial expressions. The letters x, y, z, a, b, c, …are considered as algebraic symbols.

Algebraic symbols such as x, y and z are also considered as algebraic terms.
When an algebraic symbol is multiplied or divided by a number, as for example, $2x$, $5y$, $-2a$ and $\frac{x}{3}$, it too is considered as an algebraic term.
Similarly, when an algebraic symbol is multiplied or divided by another algebraic symbol, as for example, xy, ay and $\frac{b}{z}$, it is also called an **algebraic term**. The products and quotients of algebraic symbols and numbers such as $2xy$, $-3zab$ and $\frac{2}{5}xy$ are also called **algebraic terms**.

Algebraic terms can also be considered as algebraic expressions (expressions with one term).

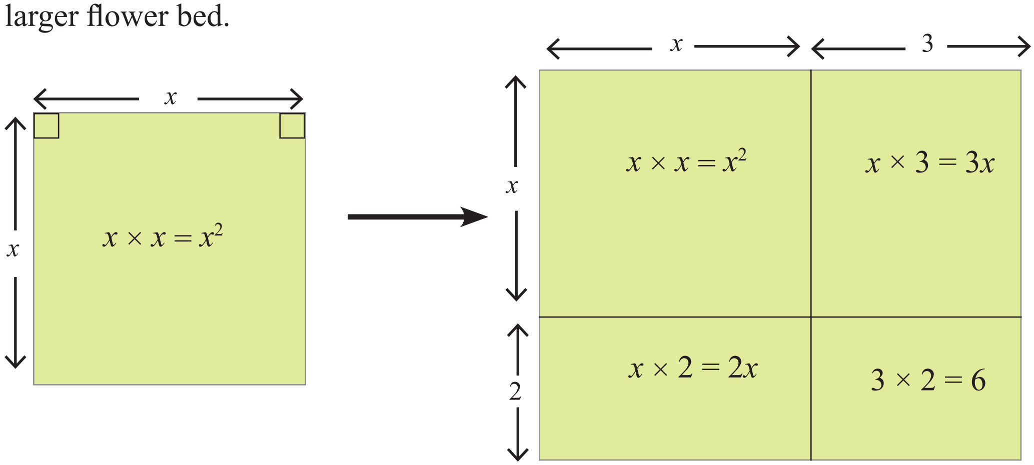
A sum or a difference of algebraic terms is called an **algebraic expression**. For example, $x + y$, $2a + xyz$, $4xy^2 - yz$ and $-2x + 3xy$ are algebraic expressions. Similarly, when a number is added to or subtracted from an algebraic term, it is also called an algebraic expression. For example, $4 + x$ and $1 - 3ab$ are **algebraic expressions**.

All the algebraic expressions we have considered thus far have consisted of two terms. A "binomial algebraic expression" (or simply a "binomial expression) is an expression which is a sum or difference of two terms.

However, there can be any number of terms in an algebraic expression.
$3 + ax - 2xyz + xy$ is an algebraic expression with four terms. It has three algebraic terms and a number (constant term).

In this lesson we will be studying binomial expressions. Now let us consider the product of two binomial expressions.

Let us take the length of a side of the square shaped flower bed shown in the figure below as x units. If a larger rectangular flower bed is made by increasing the length of one side by 3 units and the length of the adjacent side by 2 units, let us consider how an algebraic expression can be constructed in terms of x, for the area of the larger flower bed.



The length of the larger flower bed $= x + 3$ units
The breadth of the larger flower bed $= x + 2$ units

According to the figure,
the area of the larger flower bed = length × breadth= $(x + 3)(x + 2)$ square units ——(1)

Observe that $(x + 3)\ (x + 2)$ is a product of two binomial expressions.

The area of the larger flower bed can also be found by using a different method, that is, by adding the areas of the four smaller sections of which it is composed. The four sections are, the initial square shaped section and the three smaller rectangular sections in the figure.

Accordingly,

the area of the larger flower bed = the sum of the areas of the four smaller sections

$= x^2 + 2x + 3x + 6$ square units

$= x^2 + 5x + 6$ square units ———— (2)

Irrespective of the method used to find the area, the expressions obtained for the area should be equal to each other. Therefore, from (1) and (2) the following equality is established.

$$(x + 3)\ (x + 2) = x^2 + 5x + 6$$

Let us now consider how this equality can be obtained without the aid of a figure.

Let us multiply the terms within the second pair of brackets by the two terms within the first pair of brackets.

$$\begin{aligned}(x + 3)\ (x + 2) &= (x + 3)\ (x + 2)\\ &= x\ (x + 2) + 3\ (x + 2)\\ &= x^2 + 2x + 3x + 6\\ &= x^2 + 5x + 6\end{aligned}$$

Accordingly, the product of two binomial expressions can be obtained in the above manner without the aid of a figure.

Let us consider another activity similar to the one above.

Activity 1

Fill in the blanks using the given information.

A square shaped metal sheet of side length x centimetres is shown in Figure I. Figure II illustrates how two strips of width 2 centimetres and 3 centimetres respectively have been cut off from the two sides of the sheet.

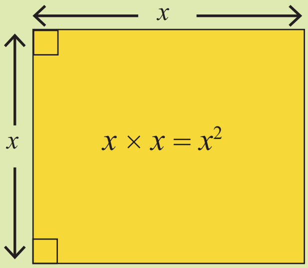


Figure I

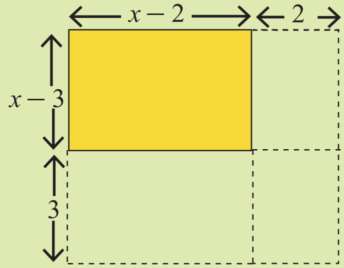


Figure II

The area of the remaining rectangular sheet $= (x-2)(x-3)$

According to Figure II,

the area of the remaining rectangular sheet = the area of the square − the area of the three rectangular parts—②

$$= x^2 - 2\,(\ldots\ldots\ldots) - \ldots\,(x-2) - 2\times 3$$

Accordingly, $(x-2)(x-3) = x^2 - 2(\ldots\ldots\ldots) - \ldots(x-2) - 2\times 3$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

Let us consider a few examples to develop a better understanding of how the product of two binomial expressions is obtained.

Example 1

$(x+5)(x+3)$

$$\begin{aligned}(x+5)(x+3) &= x(x+3) + 5(x+3)\\ &= x^2 + 3x + 5x + 15\\ &= \underline{\underline{x^2 + 8x + 15}}\end{aligned}$$

Example 2

$(x+5)(x-3)$

$$\begin{aligned}(x+5)(x-3) &= x(x-3) + 5(x-3)\\ &= x^2 - 3x + 5x - 15\\ &= \underline{\underline{x^2 + 2x - 15}}\end{aligned}$$

Example 3

$(x-5)(x+3)$

$$\begin{aligned}(x-5)(x+3) &= x(x+3) - 5(x+3)\\ &= x^2 + 3x - 5x - 15\\ &= \underline{\underline{x^2 - 2x - 15}}\end{aligned}$$

Example 4

$(x-5)(x-3)$

$$\begin{aligned}(x-5)(x-3) &= x(x-3) - 5(x-3)\\ &= x^2 - 3x - 5x + 15\\ &= \underline{\underline{x^2 - 8x + 15}}\end{aligned}$$

Example 5

Show that $(x + 8)(x - 3) = x^2 + 5x - 24$ when $x = 5$.

L.H.S. $= (x + 8)(x - 3)$	R. H. S. $= x^2 + 5x - 24$
When $x = 5$	When $x = 5$
L.H.S. $= (5 + 8)(5 - 3)$	R. H. S. $= 25 + 25 - 24$
$= 13 \times 2$	$= 26$
$= 26$	

L.H.S. = R. H. S.

$\therefore (x + 8)(x - 3) = x^2 + 5x - 24$

Exercise 5.2

1. Expand and simplify each of the following products of binomial expressions.

a. $(x + 2)(x + 4)$ **b.** $(x + 1)(x + 3)$ **c.** $(a + 3)(a + 2)$
d. $(m + 3)(m + 5)$ **e.** $(p - 4)(p - 3)$ **f.** $(k - 3)(k - 3)$

2. Draw relevant rectangles for each product of binomial expressions in **a.**, **b.** and **e.** of **1.** above and verify the answers obtained in **1.** by calculating their areas.

3. Expand and simplify each of the following products of binomial expressions.

a. $(x + 2)(x - 5)$ **b.** $(x + 3)(x - 7)$ **c.** $(m + 6)(m - 1)$
d. $(x - 2)(x + 3)$ **e.** $(x - 5)(x + 5)$ **f.** $(m - 1)(m + 8)$
g. $(x - 3)(x - 4)$ **h.** $(y - 2)(y - 5)$ **i.** $(m - 8)(m - 2)$
j. $(x - 3)(2 - x)$ **k.** $(5 - x)(x - 4)$ **l.** $(2 - x)(3 - x)$

4. Join each of the expressions in column A, with the corresponding simplified expression in column B.

A	B
$(x + 2)(x + 1)$	$x^2 + 3x - 10$
$(x + 3)(x - 4)$	$x^2 - 25$
$(x + 5)(x - 2)$	$x^2 - 6x + 9$
$(x - 3)(x - 3)$	$x^2 + 3x + 2$
$(x - 5)(x + 5)$	$x^2 - x - 12$

5. Verify that $(x + 5)(x + 6) = x^2 + 11x + 30$ for each instance given below.

i. $x = 3$ **ii.** $x = -2$

6. Verify that $(x - 2)(x + 3) = x^2 + x - 6$, for each instance given below.

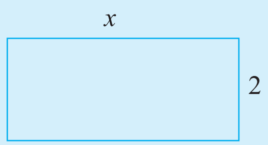
i. $x = 1$ **ii.** $x = 4$ **iii.** $x = 0$

7. Verify that $(2 - x)(4 - x) = x^2 - 6x + 8$, for each instance given below.

i. $x = 2$ **ii.** $x = 3$ **iii.** $x = -2$

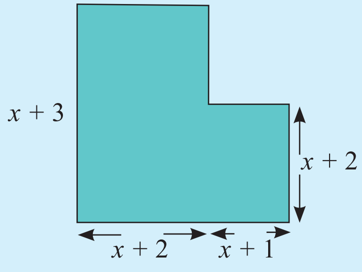
8. The length and breadth of a rectangular piece of decorative paper are 15 cm and 8 cm respectively. Two strips of breadth x cm each are cut off from the length and the breadth of this paper. Using a figure, obtain an expression for the area of the remaining portion. (Consider $x < 8$ cm).

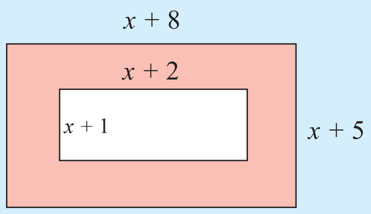
9. A rectangular flower bed of length x metres and breadth 2 metres is shown in the figure. Two metres are reduced from its length and x meters are added to its breadth. Construct an expression in terms of x for the area of the new flower bed by using a figure. (Consider that $x > 2$ m).



Miscellaneous Exercise

1. Write an expression for the shaded area in the given figure and simplify it.





2. If $(x + a)(x + 4) = x^2 + bx + 12$, find the values of a and b.