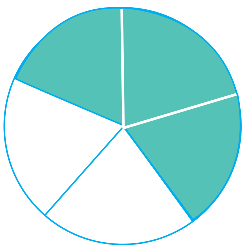
3 Fractions

By studying this lesson, you will be able to;

- simplify expressions of fractions which contain "of",
- simplify expressions of fractions which contain brackets,
- identify the BODMAS method and solve problems involving fractions.

Fractions

Let us recall the facts that were learnt about fractions in previous grades. The circle shown below is divided into five equal parts of which three are shaded.

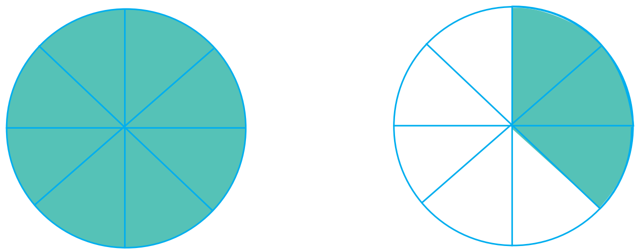


The shaded region can be expressed as $\frac{3}{5}$ of the whole region. We can express this in terms of the area of the circle too. That is, the shaded area is $\frac{3}{5}$ of the area of the whole figure. If the total area of the circle is taken as 1 unit, then the shaded area is $\frac{3}{5}$ units.

When an object is divided into equal portions, one portion or several portions can be expressed as a fraction. A portion of a collection can also be expressed as a fraction.
For example, if we consider a team consisting of three boys and two girls, then the boys in the team can be considered as $\frac{3}{5}$ of the team. Here, if the whole team is considered as a unit, then the boys in the team can be expressed as $\frac{3}{5}$.

You have learnt that fractions between zero and one such as $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{2}{3}$ are called proper fractions.

Let us now recall the facts that have been learnt previously about mixed numbers and improper fractions.
Two identical circles are given below. One is shaded completely and three parts of the other (which is divided into equal parts) are shaded.



If a circle is considered as one unit, the shaded fraction is $1 + \frac{3}{8}$. This is usually written as $1\frac{3}{8}$, which is called a mixed number ("mixed fractions" are most often called "mixed numbers"). This can also be written as $\frac{11}{8}$, which is called an improper fraction. It is important to remember here that the mixed number and the improper fraction are expressed by taking a circle as one unit.

Some other examples of mixed numbers are $1\frac{1}{2}$, $3\frac{2}{5}$, $2\frac{3}{7}$.

$\frac{3}{2}$, $\frac{8}{5}$, $\frac{11}{4}$ are examples of improper fractions. Fractions such as $\frac{3}{3}$, $\frac{5}{5}$, $\frac{1}{1}$ which are equal to 1 are also considered as improper fractions.

You have learnt to represent mixed numbers as improper fractions and improper fractions as mixed numbers.

Accordingly,

(i) $1\frac{1}{2} = \frac{3}{2}$ and

(ii) $\frac{5}{3} = 1\frac{2}{3}$.

We can obtain a fraction equivalent to a given fraction by multiplying or dividing both the denominator and the numerator by the same number (which is not zero).

For example,

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

The addition and subtraction of fractions can be performed easily if the denominators of the fractions are equal.
For example,

(i) $\frac{1}{5} + \frac{4}{5} - \frac{2}{5}$

$$\frac{1}{5} + \frac{4}{5} - \frac{2}{5} = \frac{1 + 4 - 2}{5}$$

$$= \frac{3}{5}$$

If the denominators of the fractions are unequal, then we convert the fractions into equivalent fractions with equal denominators.

For example,

(ii) $\frac{1}{4} + \frac{2}{3} - \frac{5}{6} = \frac{1 \times 3}{4 \times 3} + \frac{2 \times 4}{3 \times 4} - \frac{5 \times 2}{6 \times 2}$

$$= \frac{3}{12} + \frac{8}{12} - \frac{10}{12}$$

$$= \frac{3 + 8 - 10}{12}$$

$$= \frac{1}{12}$$

- When multiplying two fractions, the numerator of the product is obtained by multiplying the numerators of the two fractions and the denominator is obtained by multiplying the denominators of the two fractions.

Example 1

$\frac{2}{5} \times \frac{1}{3}$

$$\frac{2}{5} \times \frac{1}{3} = \frac{2 \times 1}{5 \times 3} = \underline{\underline{\frac{2}{15}}}$$

Example 2

$1\frac{1}{3} \times 1\frac{3}{4}$

$$1\frac{1}{3} \times 1\frac{3}{4} = \frac{\cancel{4}^{1}}{3} \times \frac{7}{\cancel{4}_{1}}$$ (converting the mixed numbers into improper fractions)

$$= \frac{7}{3}$$

$$= \underline{\underline{2\frac{1}{3}}}$$

- If the product of two numbers is 1, then each number is said to be the reciprocal of the other.

Accordingly, since $2 \times \frac{1}{2} = 1$,

2 is the reciprocal of $\frac{1}{2}$ and $\frac{1}{2}$ is the reciprocal of 2.

You have learnt that the reciprocal of a number can be obtained by interchanging the denominator and the numerator.

Hence, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ (In the same way, the reciprocal of $\frac{b}{a}$ is $\frac{a}{b}$.)

- In grade 8 you learnt that dividing a number by another number means multiplying the first number by the reciprocal of the second number.

Let us revise this by considering a couple of examples.

Example 3

$\frac{4}{3} \div 2$

$$\frac{4}{3} \div 2 = \frac{\cancel{4}^{2}}{3} \times \frac{1}{\cancel{2}_{1}}$$

$$= \underline{\underline{\frac{2}{3}}}$$

Example 4

$1\frac{2}{7} \div 1\frac{1}{2}$

$$1\frac{2}{7} \div 1\frac{1}{2} = \frac{9}{7} \div \frac{3}{2}$$

$$= \frac{\cancel{9}^{3}}{7} \times \frac{2}{\cancel{3}_{1}}$$

$$= \underline{\underline{\frac{6}{7}}}$$

Do the following review exercise to revise what you have learnt thus far about fractions.

Review Exercise

1. For each of the fractions given below, write two equivalent fractions.

i. $\frac{2}{3}$ ii. $\frac{4}{5}$ iii. $\frac{4}{8}$ iv. $\frac{16}{24}$

2. Express each mixed number given below as an improper fraction.

i. $1\frac{1}{2}$ ii. $2\frac{3}{4}$ iii. $3\frac{2}{5}$ iv. $5\frac{7}{10}$

3. Express each improper fraction given below as a mixed number.

i. $\frac{7}{3}$ ii. $\frac{19}{4}$ iii. $\frac{43}{4}$ iv. $\frac{36}{7}$

4. Find the value.

i. $\frac{3}{7}+\frac{2}{7}$ ii. $\frac{5}{6}-\frac{2}{3}$ iii. $\frac{7}{12}+\frac{3}{4}-\frac{2}{3}$

iv. $1\frac{1}{2}+2\frac{1}{4}$ v. $3\frac{5}{6}-1\frac{2}{3}$ vi. $1\frac{1}{2}+2\frac{1}{4}-1\frac{2}{3}$

5. Simplify.

i. $\frac{1}{2}\times\frac{4}{7}$ ii. $\frac{2}{3}\times\frac{5}{8}\times\frac{3}{10}$ iii. $1\frac{3}{5}\times 2\frac{1}{2}$ iv. $3\frac{3}{10}\times 2\frac{1}{3}\times 4\frac{2}{7}$

6. Write the reciprocal of each of the following.

i. $\frac{1}{3}$ ii. $\frac{1}{7}$ iii. $\frac{3}{8}$ iv. 5 v. $2\frac{3}{5}$

7. Simplify.

i. $\frac{6}{7}\div 3$ ii. $8\div\frac{4}{5}$ iii. $\frac{9}{28}\div\frac{3}{7}$ iv. $5\frac{1}{5}\div\frac{6}{7}$ v. $1\frac{1}{2}\div 2\frac{1}{4}$

3.1 Simplifying expressions of fractions containing "of"

We know that $\frac{1}{2}$ of 100 rupees is 50 rupees.

We also know that this is one half of 100 rupees and that its value can be obtained by dividing 100 by 2.

This can be written as $100 \div 2$.

That is, $100 \times \frac{1}{2}$ (multiplying by the reciprocal).

Accordingly, $\frac{1}{2}$ of 100 $= 100 \times \frac{1}{2} = \frac{1}{\cancel{2}} \times \cancel{100} = 50$.

According to the above facts, $\frac{1}{2}$ of 100 can be written as $\frac{1}{2} \times 100$.

Let us similarly determine how much $\frac{1}{5}$ of 20 kilogrammes is.

This amount can be considered as one part from 5 equal parts into which 20 kilogrammes is divided.

We can write this as $20 \div 5$.

That is, $20 \times \frac{1}{5}$ (multiplying by the reciprocal).

$20 \times \frac{1}{5} = \frac{1}{\cancel{5}} \times \cancel{20}^{4} = 4$.

According to the above facts, $\frac{1}{5}$ of 20 can be written as $\frac{1}{5} \times 20$.

It can be seen from the above instances that we can replace the term "of" by the operation "×".

$\frac{1}{2}$ of 100 rupees $= \frac{1}{2} \times 100$ rupees

$\frac{1}{5}$ of 20 kilogrammes $= \frac{1}{5} \times 20$ kilogrammes

Now let us find the value of $\frac{1}{2}$ of $\frac{1}{3}$. Let us illustrate this using figures.

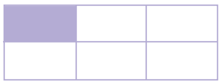
When a unit is divided into three equal parts, one part is $\frac{1}{3}$.

If this figure is taken as one unit, $\frac{1}{3}$ of it is shown below.

$\frac{1}{3}$

Let us separate out $\frac{1}{2}$ of the shaded region.

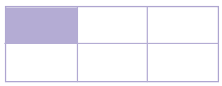
$\frac{1}{2}$



Accordingly,

$\frac{1}{3}$

$\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$



According to the figure, it is clear that $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$.

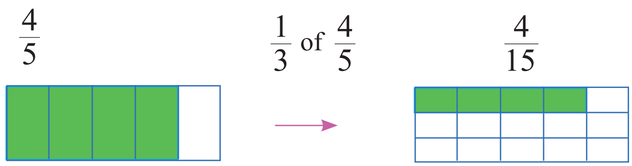
More accurately, if $\frac{1}{3}$ of a unit is taken and then $\frac{1}{2}$ of that $\frac{1}{3}$ is separated out, the portion we get is $\frac{1}{6}$ of the original unit.

Moreover, based on what we have learnt regarding multiplying fractions, we obtain $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Accordingly, we can express $\frac{1}{2}$ of $\frac{1}{3}$ as $\frac{1}{2} \times \frac{1}{3}$.

Let us consider another example to verify this. Let us simplify $\frac{1}{3}$ of $\frac{4}{5}$.

Let us consider the rectangle given below as one unit.



According to the figure, it is clear that $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$.

Moreover, $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$.

Therefore we can write, $\frac{1}{3}$ of $\frac{4}{5} = \frac{1}{3} \times \frac{4}{5}$.

It is clear that , we can replace “of” by the mathematical operation “multiplication” in the expressions $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{4}{5}$.

Example 1

Find the value of $\frac{1}{2}$ of $\frac{2}{3}$.

$\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{2} \times \frac{2}{3}$ (writing × for "of")

$= \underline{\underline{\frac{1}{3}}}$

Example 2

How much is $\frac{2}{3}$ of $1\frac{4}{5}$?

$\frac{2}{3}$ of $1\frac{4}{5} = \frac{2}{3} \times \frac{9}{5}$

$= \frac{6}{5}$

$= \underline{\underline{1\frac{1}{5}}}$

Example 3

How much is $\frac{3}{5}$ of 500 metres?

$\frac{3}{5}$ of $500 = \frac{3}{5} \times 500$

$= \underline{\underline{300 \text{ m}}}$

Exercise 3.1

1. Simplify the following expressions.

i. $\frac{2}{3}$ of $\frac{4}{5}$ **ii.** $\frac{6}{7}$ of $\frac{1}{3}$ **iii.** $\frac{2}{5}$ of $\frac{5}{8}$ **iv.** $\frac{5}{6}$ of $\frac{9}{11}$

v. $\frac{2}{7}$ of $1\frac{3}{4}$ **vi.** $1\frac{1}{3}$ of $2\frac{5}{8}$ **vii.** $1\frac{3}{11}$ of $5\frac{1}{2}$ **viii.** $\frac{5}{9}$ of $1\frac{4}{5}$

2. Find the value.

i. How much is $\frac{3}{4}$ of 64 rupees?

ii. How many grammes is $\frac{2}{5}$ of 400 g?

iii. How many hectares is $\frac{1}{3}$ of 6 ha?

iv. How many metres is $\frac{1}{8}$ of 1 km?

3. A person who owns $\frac{3}{5}$ of a land, gives $\frac{1}{3}$ of it to his daughter. What is the portion received by the daughter as a fraction of the whole land?

4. Nimal's monthly income is 40 000 rupees. He spends $\frac{1}{8}$ of this on travelling. How much does he spend on travelling?

3.2 Simplifying expressions with brackets according to the BODMAS rule

A numerical expression (or algebraic expression) may involve several of the operations addition, subtraction, division, multiplication and raising to the power of. There should be agreement on the order in which these operations should be performed and a set of rules describing it. In previous grades we learnt these rules to some extent. In this section we will discuss the "BODMAS" rule that is used when simplifying fractions.

The acronym "BODMAS" stands for brackets, orders/of, division, multiplication, addition and subtraction. When simplify numerical expressions, priority is given according to the BODMAS order. However, some operations have the same priority. Multiplication and division have equal priority and so do addition and subtraction. Accordingly, expressions should be simplified as follows.

1. First, simplify all expressions within brackets.

2. Then simplify powers and roots (expressions with indices) and the expressions with "of" .

* Simplifying expressions with powers and roots is not included in the syllabus.

3. Next, perform divisions and multiplications. These have equal priority and hence if both these operations are involved, priority is given from left to right.
4. Finally perform addition and subtraction. Since these too have equal priority, precedence is given from left to right, as in 3 above.

The BODMAS rule can be used to simplify expressions with fractions too. In some expressions of fractions the term "of" is used.
For example,

$$\frac{5}{12} \text{ of } \frac{6}{25}$$

As learnt in the previous section, this means $\frac{5}{12} \times \frac{6}{25}$.

A consensus is needed on how a fairly complex expression such as $\frac{2}{3} \div \frac{6}{25} \text{ of } \frac{5}{12} \times \frac{1}{2}$ is to be simplified. Here, precedence is given to "of" over ÷ and ×.

Note: Since "of" and "raising to the power of" have the same priority, the "O" in BODMAS is considered to stand for both "of" and "order". However in this syllabus only "of" is considered.

Now, let us consider how the BODMAS rule is used in simplifying the expression $\frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \frac{4}{3} \text{ of } \frac{3}{2}$.

$\frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \frac{4}{3} \text{ of } \frac{3}{2} = \frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div \left(\frac{4}{3} \times \frac{3}{2}\right)$ (inserting brackets after replacing "of" by "×" , to indicate that this operation should be performed first)

$$= \frac{1}{4} + \frac{5}{6} \times \frac{1}{2} \div 2$$

$= \frac{1}{4} + \left(\frac{5}{6} \times \frac{1}{2}\right) \div 2$ (inserting brackets to indicate the mathematical operation which is to be performed next)

$= \frac{1}{4} + \frac{5}{12} \times \frac{1}{2}$ (multiplying by $\frac{1}{2}$ instead of dividing by 2)

$= \frac{1}{4} + \left(\frac{5}{12} \times \frac{1}{2}\right)$ (inserting brackets to indicate the mathematical operation which is to be performed next)

$= \frac{1}{4} + \frac{5}{24}$

$= \frac{6}{24} + \frac{5}{24}$ (writing both fractions with a common denominator)

$= \frac{11}{24}$

Note: The order in which the mathematical operations in an expression should be performed can be indicated very easily using brackets.

Consider the following expression.

$$\frac{5}{4} \times \frac{3}{4} - \frac{1}{3} \text{ of } \frac{1}{5} \div \frac{2}{3} \div \frac{8}{9}$$

How this should be simplified according to the BODMAS rule can be expressed using brackets as follows.

$$\left(\frac{5}{4} \times \frac{3}{4}\right) - \left(\left(\left(\frac{1}{3} \text{ of } \frac{1}{5}\right) \div \frac{2}{3}\right) \div \frac{8}{9}\right)$$

There are disadvantages in using brackets too. When brackets are used, the expression will seem long and complex. Moreover, when simplifying an expression with the aid of a calculator, we have to insert brackets carefully because there is a greater chance of making an error. Therefore, it is important to decide on a convention to simplify expressions without using brackets. Such a convention is necessary especially when writing software for computers and calculators. However, a common convention accepted worldwide has not been agreed upon yet. There are several conventions which are accepted by different countries. Similarly, manufacturers of computers and calculators also have their own conventions.

Now let us consider some examples of expressions with fractions which are simplified using the BODMAS convention.

Example 1

Simplify the expression $\frac{4}{10}$ of $\left(\frac{1}{6}+\frac{1}{4}\right)$ and write the answer in the simplest form.

$$\frac{4}{10} \text{ of } \left(\frac{1}{6}+\frac{1}{4}\right) = \frac{4}{10} \times \left(\frac{2}{12}+\frac{3}{12}\right)$$

$$= \frac{4}{10} \times \frac{5}{12} = \underline{\underline{\frac{1}{6}}}$$

Example 2

Simplify $\left(1\frac{2}{5} \div 2\frac{1}{3}\right)$ of $\left(\frac{2}{3}-\frac{1}{2}\right)$.

$$\left(1\frac{2}{5} \div 2\frac{1}{3}\right) \text{ of } \left(\frac{2}{3}-\frac{1}{2}\right) = \left(\frac{7}{5} \div \frac{7}{3}\right) \text{ of } \left(\frac{4}{6}-\frac{3}{6}\right)$$

$$= \left(\frac{7}{5} \times \frac{3}{7}\right) \text{ of } \frac{1}{6}$$

$$= \frac{3}{5} \times \frac{1}{6}$$

$$= \underline{\underline{\frac{1}{10}}}$$

Exercise 3.2

1. Simplify and write the answer in the simplest form.

i. $\frac{1}{2}+\frac{2}{3}\times\frac{5}{6}$ **ii.** $\frac{1}{4}$ of $3\frac{1}{3}\div 2\frac{1}{6}$ **iii.** $\frac{3}{5}\times\left(\frac{1}{3}+\frac{1}{2}\right)$

iv. $\frac{1}{4}$ of $\left(3\frac{1}{3}\div 2\frac{1}{6}\right)$ **v.** $3\frac{3}{4}\div\left(2\frac{1}{2}+3\frac{1}{4}\right)$ **vi.** $\left(1\frac{2}{3}\times\frac{3}{5}\right)+\left(\frac{3}{4}+\frac{1}{2}\right)$

vii. $2\frac{2}{3}\times\left(1\frac{1}{4}-\frac{1}{12}\right)\div 2\frac{1}{3}$ **viii.** $\frac{5}{6}\div\frac{7}{18}$ of $\frac{2}{3}\times\frac{3}{4}$

2. A person puts aside $\frac{1}{4}$ of his income for food and $\frac{1}{2}$ for his business and saves the remaining amount. What fraction of his income does he save?

3. Kumuduni walked $\frac{1}{8}$ of a journey, travelled $\frac{2}{3}$ of it by train and travelled the remaining distance by bus.

(i). Express the distance she travelled by foot and by train as a fraction of the total distance.

(ii). Express the distance she travelled by bus as a fraction of the total distance.

4. A father gave $\frac{1}{2}$ of his land to his son and $\frac{1}{3}$ to his daughter. The son donated $\frac{1}{5}$ of his portion and the daughter $\frac{2}{5}$ of her portion to a charitable foundation. The foundation decided to construct a building on half the land it received. On what fraction of the total land was the building constructed?

For further knowledge

This is only for your knowledge and will not be checked in exams.
Let us consider the expression

$$8 - 3 \times (4 + 1) + 12 \div 3 \times 3^2 \div 4 \text{ as an example.}$$

How the above expression is simplified using the BODMAS rule is described below.

- First the expression 4 +1 which is within brackets is simplified. This is equal to 5. Therefore we obtain

$$8 - 3 \times 5 + 12 \div 3 \times 3^2 \div 4$$

- Next the power 3^2 is simplified. This is equal to 9. Hence we obtain

$$8 - 3 \times 5 + 12 \div 3 \times 9 \div 4$$

- Next the multiplications and divisions are performed from left to right. Therefore, 3×5 is simplified first. This is equal to 15. Therefore we obtain

$$8 - 15 + 12 \div 3 \times 9 \div 4$$

- Next, $12 \div 3$ is simplified. This is equal to 4. Hence we obtain

$$8 - 15 + 4 \times 9 \div 4$$

- Next, 4×9 is simplified. This is equal to 36. Now the expression is

$$8 - 15 + 36 \div 4$$

- Then $36 \div 4$ is simplified. This is equal to 9. Therefore we obtain

  $8 - 15 + 9$

- Now, since addition and subtraction have equal priority, simplification is done from left to right. Therefore we obtain

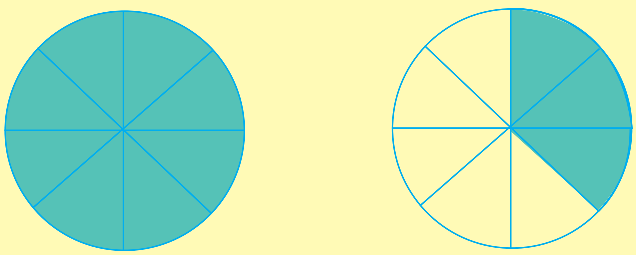
  $-7 + 9$

- Finally we get $-7 + 9 = 2$ as the answer. According to the BODMAS order,

  $8 - 3 \times (4 + 1) + 12 \div 3 \times 3^2 \div 4 = 2.$

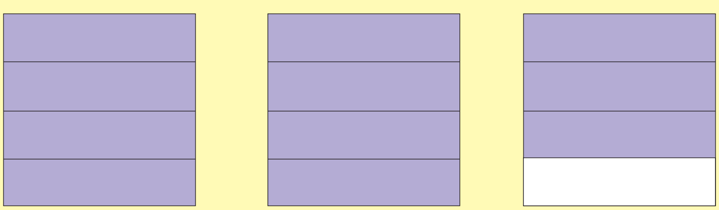
For further knowledge

This is only for your knowledge and will not be checked in exams.
Recall the figure on page 28.



We know that the shaded fraction is $1\frac{3}{8}$, if a circle is considered as one unit. This can be written as $\frac{11}{8}$.

If both these circles are considered as one unit, then the shaded fraction is $\frac{11}{16}$.



In the above diagram, if one square is considered as one unit, the shaded portion is

$2\frac{3}{4}$; That is, $\frac{11}{4}$.

a. What is the shaded fraction if all three squares together are considered as one unit?

b. What is the shaded fraction if half a square is considered as one unit?

Answers

a. $\frac{11}{12}$ **b.** $5\frac{1}{2}$

Summary

Summary

The order in which the mathematical operations are manipulated when simplifying fractions, is as follows:

- B - Brackets
- O - Of
- D - Division
- M - Multiplication
- A - Addition
- S - Subtraction