## 25 Probability

## After studying this lesson you will acquire knowledge about the following :

- Introduction of the probability tree and to show the sample space on the probability tree
- Solving problems using probability tree

It is important to recollect what you have learnt about representing the occurance of an event in day to day life in different ways.

### 25.1 Sample space

## Example 1

(i) In a random experiment where an unbiased coin is tossed, the sample space,

$$
S=\{H, T\}
$$

Here, getting the tail is $T$ and the head is H
(ii) In a random experiment a fair dice with the faces numbered from 1 to 6 is thrown. Then the sample space,

$$
S=\{1,2,3,4,5,6\}
$$

(iii) In a random experiment to find out whether a new born child is a boy or a girl, the sample space

$$
S=\{M, F\}
$$

Here $M$ is for a boy and $F$ is for a girl

### 25.2 Events

An event is a subset of the sample space.

## Example 2

(i) When a coin is tossed, getting a head is an event. If we name this event as A.
then $A=\{H\}$
This is a subset of the sample space $\mathrm{S}=\{\mathrm{H}, \mathrm{T}\}$
(ii) When a dice is thrown, getting an odd number is an event. If we denote this event by B .
then $B=\{1,3,5\}$
This is a subset of $S=\{1,2,3,4,5,6\}$

### 25.3 Simple events

If an event cannot be divided further into events it is called a simple event. Hence when an event is a simple event, it contains only one element.

## Example 3

(i) When a coin is tossed, getting a head is a simple event. It has only H as an element.
(ii) In the random experiment of throwing a dice the sample space is $S=\{1,2,3,4,5,6\}$ Simple events of this can be written as
$\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}$
Any event can be expressed as a union of simple events.

Even the sample space is an event. From the above example (ii)

$$
S=\{1\} \cup\{2\} \cup\{3\} \cup\{4\} \cup\{5\} \cup\{6\}
$$

### 25.4 Compound events

If an event can be further divided into two or more events it is called a compound event.

## Example 4

(i) When a dice is thrown, getting an odd number is a compound event.
$\mathrm{A}=\{$ Getting an odd number when a dice is thrown. $\}$
$\mathrm{A}=\{1,3,5\}$

This can be divided into a number of events such as

$$
\{1\},\{3\},\{5\},\{1,3\},\{1,5\},\{3,5\}
$$

### 25.5 Mutually exclusive events

If there are no common simple events in two or more events in a sample space then the events are considered as mutually exclusive events. For any two events A and B ,
$\mathrm{A} \cap \mathrm{B}=\varnothing$, then A and B are mutually exclusive events.

## Example 5

In the random event of throwing a dice
A $=$ \{Getting an odd number $\}$
$B=\{$ Getting an even number $\}$
$\mathrm{A}=\{1,3,5\}$ and $\mathrm{B}=\{2,4,6\}$
then $\mathrm{A} \cap \mathrm{B}=\varnothing$,
(Getting an odd number and getting an even number cannot occur at the same time)
$\therefore \mathrm{A}$ and B are mutually exclusive events.

### 25.6 Probability

Recollect what you have learnt so far about probability. Probability of an event is the possibilities of an event occuring.

This is originally defined for events which are equally likely.

## Example 6

Consider the random event of throwing a dice.

$$
S=\{1,2,3,4,5,6\}
$$

Given below are the simple events corresponding to this sample space

$$
\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}
$$

If $\mathrm{A}=\{2\}$, probability of $A$ is given as $\mathrm{P}(\mathrm{A})$. You will remember that $P(A)$ is defined as

$$
P(A)=\frac{n(A)}{n(s)}=\frac{1}{6}
$$

Here $n(\mathrm{~S})$ denotes the number of elements in the sample space $S$.
If all the simple events in the sample space has the same probability as above, these events are called equally likely events.

If all the simple events are mutually exclusive and also if they are equally likely, then the sum of their probabilities is equal to one.

Now consider a compound event, which consists of equally likely simple events

$$
\begin{aligned}
& X=\{\text { Getting an odd number }\} \\
& X=\{1,3,5\}
\end{aligned}
$$

The probability of a compound event as above is

$$
\mathrm{P}(\mathrm{X})=\frac{\mathrm{n}(\mathrm{x})}{\mathrm{n}(\mathrm{~S})}=\frac{3}{6}=\frac{1}{2}
$$

Consider the sample space of a random experiment consisting of equally likely simple events as $S$ and any compound even in it as $M$.

$$
\text { If } \begin{aligned}
\mathrm{n}(\mathrm{~S}) & =\mathrm{N} \text { and } \mathrm{n}(\mathrm{M})=\mathrm{m} \\
\mathrm{P}(\mathrm{M}) & =\frac{\mathrm{n}(\mathrm{M})}{\mathrm{n}(\mathrm{~S})}=\frac{\mathrm{m}}{\mathrm{~N}}
\end{aligned}
$$

Here $0 \leq \mathrm{P}(\mathrm{M}) \leq 1$

Also if $A$ and $B$ are any two events,
then $\mathrm{P}(\mathrm{A} \cup \mathrm{B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A} \cap \mathrm{B})$
and If $A$ and $B$ are mutually exclusive events,
then $\mathrm{P}(\mathrm{A} \cup \mathrm{B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})$.

## Example 7

A survey was done among 75 customers and the following information was collected. 12 buy new washing machines, 18 buy new refrigerators, 24 buy new television sets, 6 buy a washing machines and a refrigerator, 4 buy a washing machines and a television set and 10 buy a television set and a refrigerator. One person buys all three appliances.
(i) Show the above information in a Venn diagram.
(ii) If one of these customers is selected at random,

Find the probability that the person selected
(a) does not buy any of the commodities
(b) buys only washing machines
(c) buys only a television set
(d) buys only a refrigerator
(e) at least one of the two commodities, a television and a refrigerator

M - Washing machines
T-Television
R - Refrigerator

(ii) (a) Probability of being a person who does not buy any of the commodities

$$
\begin{aligned}
& =P(M \cup R \cup T)^{\prime} \\
& =\frac{n(M \cup R \cup T)^{\prime}}{n(S)}=\frac{40}{75}=\frac{8}{15}
\end{aligned}
$$

(b) Probability of being a person who buys only a washing machine

$$
\mathrm{P}\left[\mathrm{M} \cap(\mathrm{R} \cup \mathrm{~T})^{\prime}\right]=\frac{3}{75}=\frac{1}{25}
$$

(c) Probability of being a person who buys only a television set

$$
\mathrm{P}\left[\mathrm{~T} \cap(\mathrm{M} \cup \mathrm{R})^{\prime}\right]=\frac{11}{75}
$$

(d) Probability of being a person who buys only a refrigerator

$$
\mathrm{P}\left[\mathrm{R} \cap(\mathrm{M} \cup \mathrm{~T})^{\prime}\right]=\frac{3}{75}=\frac{1}{25}
$$

(e) Probability of being a person who buys at least one of the two appliances television set and a refrigerator.

$$
\begin{aligned}
& =P(C \cup R)=\frac{n(C \cup R)}{n(S)} \\
& =\frac{32}{\underline{75}}
\end{aligned}
$$

### 25.1 Activity

Using the above example (7) show that

$$
\begin{aligned}
\mathrm{P}(\mathrm{M} \cup \mathrm{R} \cup \mathrm{C})= & \mathrm{P}(\mathrm{M})+\mathrm{P}(\mathrm{R})+\mathrm{P}(\mathrm{C})-\mathrm{P}(\mathrm{M} \cap \mathrm{R})-\mathrm{P}(\mathrm{M} \cap \mathrm{C})-\mathrm{P}(\mathrm{R} \cap \mathrm{C}) \\
& +\mathrm{P}(\mathrm{M} \cap \mathrm{R} \cap \mathrm{C})
\end{aligned}
$$

## Exercise 25.1

(1) 46 students were asked as to which one of the force, Army, Navy and Air force they would like to join. There were 21 who liked to join Army 14 to join Air force, 23 for Navy, 5 who liked to join all three, 7 to join Army and Airfoce. 8 to join Army and Navy only, and 4 who liked Air force only.

- Show this information in a Venn diagram
- When a student is chosen at random, find the probability that
(a) he likes only to join the Army
(b) he likes only to join the Navy
(c) he does not like to join any one of these
(2) In a bag there are 10 identical beads, of which 5 are white, 3 are red and 2 are black. A bead is taken out at random from the bag and without replacing it another bead is taken.
- Show the sample space for this random experiment in a Cartesian plane.
- Find the probability that
(a) both beads are white
(b) both are red
(c) both are black
(d) one is white and the other is red
(e) they are of different colours.


### 25.7 Independent events

This is an important concept in probability.
It can be explained from the examples given below.

## Example 8

Consider a group of students consisting of 36 children.
$A=\{$ A person with blue eyes $\}$
$B=\{$ Being a boy $\}$
The table below given information regarding them.

|  | A | $\mathrm{A}^{\prime}$ | Sum |
| :---: | :---: | :---: | :---: |
| B | 6 | 6 | 12 |
| $\mathrm{~B}^{\prime}$ | 12 | 12 | 24 |
| Sum | 18 | 18 | 36 |

Here $\quad A^{\prime}=\{$ A person without blue eyes $\}$

$$
B^{\prime}=\{\text { Not a boy }\}
$$

When a child is selected at random, the following probabilities can be found.

$$
\begin{array}{r}
\mathrm{P}(\mathrm{~A})=\frac{18}{36}=\frac{1}{2} \\
\mathrm{P}(\mathrm{~B})=\frac{12}{36}=\frac{1}{3} \\
\mathrm{P}(\mathrm{~A} \cap \mathrm{~B})=\frac{6}{36}=\frac{1}{6}
\end{array}
$$

Also $\mathrm{P}(\mathrm{A}) \times \mathrm{P}(\mathrm{B})=\frac{1}{2} \times \frac{1}{3}$

$$
\begin{aligned}
& =\frac{1}{6} \\
& =\mathrm{P}(\mathrm{~A} \cap \mathrm{~B}) \\
\therefore \mathrm{P}(\mathrm{~A} \cap \mathrm{~B}) & =\mathrm{P}(\mathrm{~A}) \cdot \mathrm{P}(\mathrm{~B})
\end{aligned}
$$

In the above events A and $\mathrm{B}, \mathrm{A}$ does not affect the occurrence of B , ie. if a child is selected from the group, that child being a boy does not affect him having blue eyes or not

Events of this nature are called independent events.
If $A$ and $B$ are independent events,

$$
\text { then } \quad \mathrm{P}(\mathrm{~A} \cap \mathrm{~B})=\mathrm{P}(\mathrm{~A}) \cdot \mathrm{P}(\mathrm{~B})
$$

The converse is also true
ie. for the events A and B , if

$$
\mathrm{P}(\mathrm{~A} \cap \mathrm{~B})=\mathrm{P}(\mathrm{~A}) \cdot \mathrm{P}(\mathrm{~B}) \text { then }
$$

$A$ and $B$ are independent events.

## Example 9

(1) If two identical coins are tossed at once, then S = \{HH, HT, TH, TT $\}$
$\mathrm{A}=$ Getting a Head in the first coin
$\mathrm{B}=$ Getting a Head in the second coin
$\mathrm{C}=$ Getting a Head in only one coin
$\therefore A=\{\mathrm{HH}, \mathrm{HT}\}, B=\{\mathrm{HH}, \mathrm{TH}\}, C=\{\mathrm{HT}, \mathrm{TH}\}$

$$
\begin{aligned}
& \therefore \mathrm{P}(\mathrm{~A})=\mathrm{P}(\mathrm{~B})=\mathrm{P}(\mathrm{C})=\frac{2}{4}=\frac{1}{2} \\
& \mathrm{P}(\mathrm{~A} \cap \mathrm{C})=\frac{1}{4}, \mathrm{P}(\mathrm{~A} \cap \mathrm{~B})=\frac{1}{4}, \mathrm{P}(\mathrm{~B} \cap \mathrm{C})=\frac{1}{4} \\
& \therefore \mathrm{P}(\mathrm{~A} \cap \mathrm{C})=\mathrm{P}(\mathrm{~A}) \mathrm{P}(\mathrm{~A}) \\
& , \mathrm{P}(\mathrm{~A} \cap \mathrm{~B})=\mathrm{P}(\mathrm{~A}) \cdot \mathrm{P}(\mathrm{~B}) \\
& \mathrm{P}(\mathrm{~B} \cap \mathrm{C})=\mathrm{P}(\mathrm{~B}) \cdot \mathrm{P}(\mathrm{C})
\end{aligned}
$$

$\therefore$ the pairs A and $\mathrm{C}, \mathrm{A}$ and B , and B and C are independent events
Discuss whether
$\mathrm{P}(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})=\mathrm{P}(\mathrm{A}) . \mathrm{P}(\mathrm{B}) . \mathrm{P}(\mathrm{C})$ is true in the above example.

## Exercise 25.2

(1) Two dice, one red and the other green with their sides numbered from 1 to 6 are rolled. Show the sample space in a graph.
If $\mathrm{A}=\{$ Getting a 5 in the red dice $\}$
$B=\{$ Getting a 6 in the green dice $\}$
Show that A and B are independent events.
(2) An unbiased coin is tossed thrice.
(i) $\mathrm{A}=$ \{Getting a Tail in the first toss $\}$
$\mathrm{B}=$ \{Getting a Tail in the second toss $\}$
$\mathrm{C}=\{$ Getting only two heads in all three tosses. $\}$
Which of the events $\mathrm{A}, \mathrm{B}$ and C are independent?
(3) The boxes $A$ and $B$ contain a certain item,

The box $A$ has 8 , of which 3 are faulty and box $B$ has 5 , of which 2 are faulty.
If one item from each box is taken at random find the probability that
(a) Both items are faulty ones
(b) One is faulty and the other is good
(c) Getting a faulty item from A
(d) Getting faulty item from $B$
(4) In an event of shooting at target, of the two people $A$ and $B$, the probability of A, shooting at a target is $\frac{1}{4}$ and for B it is $\frac{1}{3}$.
What is the probability that
(i) both shoot at the target?
(ii) At least one of them shoot at the target?
(5) Three horses namely $\mathrm{A}, \mathrm{B}$ and C are to complete in a race.

The probability of winning the race for each horse is given as

$$
\mathrm{P}(\mathrm{~A})=\frac{1}{2}, \mathrm{P}(\mathrm{~B})=\frac{1}{3} \text { and } \mathrm{P}(\mathrm{C})=\frac{1}{6}
$$

If they had the competition twice, write the set of all the possible results.

If $A B$ indicates that $A$ wins the first race and $B$ wins the second, then find the following probabilities.

$$
\begin{aligned}
& \mathrm{P}(\mathrm{AB}), \mathrm{P}(\mathrm{AC}), \mathrm{P}(\mathrm{BA}), \mathrm{P}(\mathrm{BB}), \mathrm{P}(\mathrm{BC}), \\
& \mathrm{P}(\mathrm{AA}), \mathrm{P}(\mathrm{CA}), \mathrm{P}(\mathrm{CB}), \mathrm{P}(\mathrm{CC})
\end{aligned}
$$

## Example 10

Two people A and B are competing in a tennis match. The person who wins two rounds will be the champion.


The sample space can be shown as

$$
\begin{aligned}
\mathrm{S} & =\{(\mathrm{A}, \mathrm{~A}),(\mathrm{A}, \mathrm{~B}),(\mathrm{B}, \mathrm{~A}),(\mathrm{B}, \mathrm{~B})\} \\
\mathrm{n}(\mathrm{~S}) & =4
\end{aligned}
$$

The way they are shown in the example given above, the sample space can be clearly shown by these diagrams which are called Tree diagrams.

This is very useful in random experiments where there are more than one step. And it is very easy to find the probability of such an event using the tree diagram.

## Example 11

When two unbiased coins, a two rupee coin and an one rupee coin are tossed show all the possible results in a tree diagram
(i) Find the probability that both coins will show the heads
(ii) Find the probability that at least one will show the head.

Consider H as the head and T as the tail then,

| $\frac{\text { Two rupee }}{\frac{\text { coin rupee }}{\text { coin }}}$ | $\underline{\text { Result }}$ |
| :--- | :--- |
| H, $\mathrm{H} \rightarrow \frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$ |  |
| $\mathrm{H}, \mathrm{T} \rightarrow \frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$ |  |
| $\mathrm{~S}=\{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$ | $\mathrm{T}, \mathrm{H} \rightarrow \frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$ |
| $\mathrm{~T}, \mathrm{~T} \rightarrow \frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$ |  |

Probability of getting a head on both coins $=P(H H)$

$$
\mathrm{P}(\mathrm{HH})=\mathrm{P}(\mathrm{H}) \times \mathrm{P}(\mathrm{H})=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4}
$$

Getting a tail in the one rupee coin and getting a tail in the two rupee coin are independent events.
[Note:- In the tree diagram, at the end of each branch, the relevant probability is written]

Probability of getting a tail at least in one coin

$$
\begin{aligned}
& =\mathrm{P}(\mathrm{~T}, \mathrm{~T})+\mathrm{P}(\mathrm{H}, \mathrm{~T})+\mathrm{P}(\mathrm{~T}, \mathrm{H}) \\
& =\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}
\end{aligned}
$$

This also can be calculated as follows

$$
\begin{aligned}
\mathrm{S} & =\{(\mathrm{H}, \mathrm{H}),(\mathrm{H}, \mathrm{~T}),(\mathrm{T}, \mathrm{H}),(\mathrm{T}, \mathrm{~T})\} \\
\mathrm{P}(\mathrm{~S}) & =1
\end{aligned}
$$

Getting a tail in at least one coin

$$
=1-\mathrm{P}(\mathrm{H}, \mathrm{H})=1-\frac{1}{2} \times \frac{1}{2}=1-\frac{1}{4}=\frac{3}{\underline{4}}
$$

Note :-

- Tossing two unbiased coins together
or - Tossing one coin after the other
or - Tossing the same coin twice, are experiments which will give the same sample space.


### 25.9 Random experiments with replacement

## Example 12

There are 9 identical balls in a bag, of which 5 are red and 4 are blue. A ball is taken out from the bag at random, the colour is noted down and then the ball is replaced. Then a second ball is taken out at random.

Find the following probabilities using a tree diagram

1. Both balls will be red
2. Both balls will be blue
3. One ball will be blue and the other red
4. At least one ball will be red
5. First ball taken out will be red

(i) $\mathrm{P}(\mathrm{R}, \mathrm{R})=\frac{25}{81}$
(ii) $\mathrm{P}(\mathrm{B}, \mathrm{B})=\frac{16}{81}$
(iii) $P(R, B)+P(B, R)=\frac{20}{81}+\frac{20}{81}=\frac{40}{81}$
(iv) $\mathrm{P}(\mathrm{R}, \mathrm{R})+\mathrm{P}(\mathrm{B}, \mathrm{R})+\mathrm{P}(\mathrm{R}, \mathrm{B})=\frac{25}{81}+\frac{20}{81}+\frac{20}{81}=\frac{65}{81}$
or

$$
1-P(B, B)=1-\frac{16}{81}=\frac{65}{81}
$$

(v) $P(R, R)+P(R, B)=\frac{25}{81}+\frac{20}{81}=\frac{45}{81}=\frac{5}{9}$

## Exercise 25.3

(1) Out of 5 identical marbles in a bag, 4 are green and other is yellow in colour. A marble is taken out at random, the colour is noted and then replaced. A second marble is then taken out at random.
(i) Show the sample space for the above experiment in a tree diagram.
(ii) Find the probability of the following events
(a) Both are green
(b) Both are yellow
(c) Both are of the same colour
(2) In a competition of shooting at a target, the probability of shooting at a target for a particular person is $\frac{4}{7}$
(i) If he shoots twice at the target, show the sample space in a tree diagram.
(ii) Find the probability of the following events.
(a) He succeeds in both attempts
(b) He does not succeed in any of the two attempts
(c) He succeeds in at least one attempt
(d) He succeeds only in one attempt
(3) An institute where electric bulbs are manufactured states that the probability of the life time of a bulb to be less than 1000 hours is $\frac{1}{100}$.

Draw a tree diagram to show the life time of two bulbs, a customer has purchased.
Find the probability
(i) that both bulbs will be worn out before 1000 hours
(ii) that both bulbs will have a life time more than 1000 hours
(iii)At least one of the two bulbs will last for more than 1000 hours
(4)


The figure shows a circle divided into 5 equal sectors which are numbered from 1 to 5 . A pointer attached to the centre is free to spin.

When the pointer stops, it is noted whether it stops at an odd number or an even number. When this is done by spinning the pointer twice,
(i) Show the result on a tree diagram
(ii) Find the probability that at both instances it will be an odd number.
(iii) Find the probability that it will stop at an even number in the first instance
(5) The table given below shows how 35 students in grade 11 followed the subjects Art and Music.

|  | Art | Music |
| :---: | :---: | :---: |
| Boys | 3 | 12 |
| Girls | 7 | 13 |

(i) Draw a tree diagram to show the above information
(ii) If a student from this class is selected at random,
(a) find the probability that, this student is a boy
(b) find the probability that, this student studies Art
(c) find the probability that, this student studies Music

### 25.9 Random experiment without replacement

## Example 13

In a bag containing identical balls, there are 5 red balls and 3 blue balls. One ball is taken out at random and without replacing it another ball is taken out at random.

By drawing a probability tree,
(i) find the probability that both are red
(ii) find the probability that only one ball is red

(i) $P(R, R)=\frac{20}{56}$
(ii) $P(R, B)+P(B, R)=\frac{15}{56}+\frac{15}{56}=\frac{30}{56}=\frac{15}{28}$

## Exercise 25.4

(1) A box contains identical ball point pens of which 6 are black and 3 are red. Two pens are taken out in succession and not replaced.
Show the sample space in a tree diagram.
Find the probability that
(i) both pens are black
(ii) both pens are red
(iii) One is red and the other is black
(2) There are 10 identical balls in a paper bag. 6 are white, 3 are red and 1 is black. Two balls are taken out in succession without replacing. Show the sample space in a tree diagram.
Find the probability that
(i) both balls are white
(ii) both balls are red
(iii) one is white and the other is red
(iv) they are of different colours
(3) In an office, the staff consists of 7 males and 4 females. Two of them left the office on short leave. Show the sample space for this event in a tree diagram.

Hence find the probability that the two who left the office are
(i) males
(ii) females
(iii) at least one of them is a female
(4) In a bus all the seats are occupied by passengers. 6 men and a woman were standing. At a certain bus halt, two of the passengers who were seated, got down. Two of those who were standing took the seats thus vacated.

Show the relevant sample space in a tree diagram.
Find the probability that out of the two who took the vacant seats,
(i) both are men
(ii) at least one of them is a man

## Example 14

The bags ' A ' and ' B ' contain identical beads. Bag ' A ' has 5 black beads and 3 white beads. The bag $B$ has 3 black beads and 2 white beads. A bead is taken out at random, from bag ' $A$ ' and put into bag B and then a bead is taken out at random from bag B . Draw a tree diagram relevant to this experiment and hence find the probability that the bead taken from $B$ is a black bead.

Bag A
Bag B


BagA


Bag B

(B, B)
$\frac{20}{48}$
(B, W)
$\frac{10}{48}$


(W, B)
(W, W)

Probability of getting a black bead from bag ' B ' is

$$
\mathrm{P}(\mathrm{~B}, \mathrm{~B})+\mathrm{P}(\mathrm{~W}, \mathrm{~B})=\frac{20}{48}+\frac{9}{48}=\frac{29}{48}
$$

## Exercise 25.5

(1) A blue box contains 5 black beads and 3 white beads. A yellow box contains 2 black beads and 3 white beads. All the beads are identical. A bead is taken from blue box, colour noted and put into the yellow box. Then a bead is taken from yellow box.

Show the relevant sample space in a tree diagram.
Hence find the probability that
(i) both beads taken out are black.
(ii) the bead taken from the yellow box is yellow
(2) A box contains ball point pens of which 5 are red and 2 are blue. Another box contains 2 blue ball point pens of the same type. A pen is taken out at random from the first box and put into the second box. Then a pen is taken out at random from the second box. Show the relevant sample space in a tree diagram.

Find the probability that
(i) both pens taken out are blue
(ii) the pen taken out from the second box is red
(3) In farm ' $A$ ' there are 6 male animals and 5 female animals. In farm ' $B$ ' there are 7 male animals and 2 female animals. A animal is randomly selected from farm ' $A$ ' and is sent to farm ' $B$ '. Then an animal is randomly selected from farm ' $B$ ' and is sent to farm ' $A$ '.
(i) Draw the relevant tree diagram to represent the sample space.
(ii) Find the probability that the animal taken from farm ' B ' is a female.

## C8C0 $0^{51}$ Uடகळைぁள் <br> LOGARITHMS

|  |  |  |  |  |  |  |  |  |  |  |  <br> @eac ax\$ownendedr <br> Mean Differences |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1034 | 1072 | 1106 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 | 5 | 8 | 11 | 13 | 16 | 18 | 21 | 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 | 4 | 6 | 8 | 11 |  | . 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 | 4 | 5 | 7 | 9 | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 | 3 | 5 | 7 | 9 | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 | 3 | 5 | 7 | 8 | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | I | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 |
| 36 | 5563 | 5575 | 55871 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 |  | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 44 | 5435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 50 | 6990 | 6898 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



NATURAL SINES

|  | $0^{\prime}$ | $10^{\circ}$ | $20^{\circ}$ | $30^{\circ}$ | $40^{\circ}$ | $50^{\circ}$ | $60^{\circ}$ |  |  <br>  <br> Mean Differences |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  | $1 \cdot$ | 2 | 3. | $4^{*}$ | 5 | $6^{\circ}$ | $7{ }^{\prime}$ | 8 | 9 - |
| $0^{\circ}$ | 0.0000 | 0.0029 | 0-0058 | 0.0087 | 0.0116 | 0.0145 | 0.0175 | $89^{\circ}$ | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | 26 |
| 1 | . 0175 | -0204 | . 0233 | . 0262 | . 0291 | . 0320 | . 0349 | 88 | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | 26 |
| 2 | . 0349 | . 0378 | . 0407 | . 0436 | . 0465 | . 0494 | . 0523 | 87 | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | 26 |
| 3 | . 0523 | . 0552 | . 0581 | . 0610 | . 0640 | . 0669 | . 0698 | 86 | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | 26 |
| 4 | . 0698 | . 0727 | . 0756 | . 0785 | . 0814 | . 0843 | . 0872 | 85 | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | . 26 |
| 5 | 0.0872 | 0.0901 | 0.0929 | 0.0958 | 0.0987 | 0-1016 | 0.1045 | 84 | 3 | 6 | 9 | 12 | 14 | 17 | 20 | 23 | 26 |
| 6 | . 1045 | . 1074 | . 1103 | . 1132 | . 1161 | . 1190 | . 1219 | 83 | 3 | 6 | 9 | 12 | 14 | 17 | 20 | 23 | 26 |
| 7 | . 1219 | . 1248 | . 1276 | . 1305 | . 1334 | . 1363 | . 1392 | 82 | 3 | 6 | 9 | 12 | 14 | 17 | 20 | 23 | 26 |
| 8 | . 1392 | -1421 | . 1449 | . 1478 | . 1507 | . 1536 | . 1564 | 81 | 3 | 6 | 9 | 11 | 14 | 17 | 20 | 23 | 26 |
| 9 | . 1564 | . 1593 | . 1622 | . 1650 | $\cdot 1679$ | . 1708 | . 1736 | $80^{\circ}$ | 3 | 6 | 9 | 11 | 14 | 17 | 20 | 23 | 26 |
| 10 | 0.1736 | 0.1765 | 0.1794 | 0.1822 | 0.1851 | 0.1880 | 0.1908 | 79 | 3 | 6 | 9 | 11 | 14 | 17 | 20 | 23 | 26 |
| 11 | . 1908 | . 1937 | . 1965 | . 1984 | - 2022 | -2051 | - 2079 | 78 | 3 | 6 | 9 | 11 | 14 | 17 | 20 | 23 | 26 |
| 12 | . 2079 | . 2108 | . 2136 | . 2164 | . 2193 | . 21212 | . 2250 | 77 | 3 | 6 | 9 | 11 | 14 | 17 | 20 | 23 | 26 |
| 13 | . 2250 | . 2278 | . 2306 | . 2334 | . 2363 | . 2391 | . 2419 | 76 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 23 | 25 |
| 14 | -2419 | . 2447 | . 2476 | . 2504 | . 2532 | . 2560 | . 2588 | 75 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 23 | 25 |
| 15 | 0.2588 | 0.2616 | 0.2644 | 0.2672 | 0.2700 | 0.2728 | 0.2756 | 74 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | . 2756 | . 2784 | . 2812 | . 2840 | . 2868 | . 2896 | . 2924 | 73 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| 17 | . 2924 | . 2952 | . 2979 | . 3007 | . 3035 | . 3062 | - 3090 | 72 | 3 | 6 | 8 | 11 | 14 | 17 | 19 | 22 | 25 |
| 18 | . 3090 | . 3118 | . 3145 | .3173 | . 3201 | . 3228 | . 3256 | 71 | 3 | 6 | 8 | 11 | 14 | 17 | 19 | 22 | 25 |
| 19 | . 3256 | . 3283 | . 3311 | . 3338 | . 3365 | .3393 | . 3420 | $70^{\circ}$ | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 20 | 0.3420 | 0.3448 | 0.3475 | 0.3502 | 0.3529 | 0.3557 | 0.3584 | 69 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 21 | . 3584 | . 3611 | . 3638 | . 3665 | . 3692 | . 3719 | . 3746 | 68 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 24 |
| 22 | . 3746 | . 3773 | . 3800 | . 3827 | . 3854 | . 3881 | . 3907 | 67 | 3 | 5 | 8 | 11 | 13 | 16 | 19 | 21 | 24 |
| 23 | . 3907 | . 3934 | . 3961 | . 3987 | . 4014 | . 4041 | . 4067 | 66 | 3 | 5 | 8 | 11 | 13 | 16 | 19 | 21 | 24 |
| 24 | . 4067 | . 4094 | . 4120 | . 4148 | . 4173 | . 4200 | . 4226 | 65 | 3 | 5 | 8 | 11 | 13 | 16 | 19 | 21 | 24 |
| 25 | 0.4226 | 0.4253 | 0.4279 | 0.4305 | 0.4331 | 0.4358 | 0.4384 | 64 | 3 | 5 | 8 | 10 | 13 | 16 | 18 | 21 | 24 |
| 26 | . 4384 | . 4410 | . 4436 | . 4462 | . 4488 | . 4514 | . 4540 | 63 | 3 | 5 | 8 | 10 | 13 | 16 | 18 | 21 | 23 |
| 27 | . 4540 | . 4566 | . 4592 | . 4617 | . 4643 | . 4669 | . 4695 | 62 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 21 | 23 |
| 28 | . 4695 | 4720 | . 4746 | . 4772 | . 4797 | . 4823 | . 4848 | 61 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 29 | . 4848 | . 4874 | . 4899 | . 4924 | .4950 | . 4975 | . 5000 | $60^{\circ}$ | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| $30^{\circ}$ | 0.5000 | 0.5025 | 0.5050 | 0.5075 | 0.5100 | 0.5125 | 0.5150 | 59 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 31 | . 5150 | . 5175 | . 5200 | . 5225 | . 5250 | . 5275 | . 5299 | 58 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 32 | . 5299 | . 5324 | . 5348 | . 5373 | . 5398 | . 5422 | . 5446 | 57 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 33 | . 5446 | . 5471 | . 5495 | . 5519 | . 5544 | . 5568 | . 5592 | 56 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 19 | 22 |
| 34 | . 5592 | . 5616 | . 5640 | . 5664 | . 5688 | . 5712 | . 5736 | 55 | 2 | 5 | 7 | 10 | 12 | 14 | 17 | 19 | 22 |
| 35 | 0.5736 | 0.5760 | 0.5783 | 0.5807 | 0.5831 | 0.5854 | 0.5878 | 54 | 2 | 5 | 7 | 9 | 12 | 14 | 17 | 19 | 21 |
| 36 | . 5878 | . 5901 | . 5925 | . 5948 | . 5972 | . 5995 | . 6018 | 53 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 37 | . 6018 | . 6041 | . 6065 | . 6088 | . 6111 | . 6134 | . 6157 | 52 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 18 | 21 |
| 38 | . 6157 | . 6180 | . 6102 | . 6225 | .6249 | . 6271 | . 6293 | 51 | 2 | 5 | 7 | 9 | 11 | 14 | 16 | 18 | 20 |
| 39 | . 6293 | . 6316 | . 6338 | . 6361 | . 6383 | . 6406 | 6428 | $50^{\circ}$ | 2 | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |
| $40^{\circ}$ | 0.6428 | 0.6450 | 0.6472 | 0.6494 | 0.6517 | 0.6539 | 0.6561 | 49 | 2 | 4 | 7 | 9 | 11 | 13 | 15 | 18 | 20 |
| 41 | . 6561 | . 6583 | . 6604 | . 6626 | . 6648 | . 6670 | . 6691 | 48 | 2 | 4 | 7 | 9 | 11 | 13 | 15 | 17 | 20 |
| 42 | .6691 | . 6713 | . 6734 | . 6756 | . 6777 | . 6799 | . 6820 | 47 | 2 | 4 | 6 | 9 | 11 | 13 | 15 | 17 | 19 |
| 43 | . 6820 | . 6841 | . 6862 | . 6884 | . 6905 | . 6926 | . 6947 | 46 | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| 44 | . 6947 | . 6967 | . 6988 | . 7009 | . 7030 | . 7050 | .7071 | 45 | 2 | 4 | 6 | 8 | 10 | 12 | 15 | 17 | 19 |
|  | $60^{\circ}$ | $50^{\circ}$ | $40^{\circ}$ | $30^{\circ}$ | $20^{\circ}$ | $10^{\circ}$ | $0 \cdot$ |  | $1{ }^{\circ}$ | $2{ }^{\circ}$ | 3 | 4' | 5 | $6^{\circ}$ | 7* | 8 | $9{ }^{\prime}$ |


|  | இயற்கிக்் ๓ร์ க் கள் NATURAL SINES |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & 50200 \\ & \text { ait, } \\ & \operatorname{can} \text { D } \end{aligned}$ |  |  |  |  |  |
|  | 0 | $10^{\circ}$ | $20^{\circ}$ | $30^{\prime}$ | $40^{\circ}$ | $50^{\circ}$ | $60^{\circ}$ |  | $1{ }^{\prime}$ | $2{ }^{\prime}$ | $3^{\prime}$ | 4 | 5 | $6^{\circ}$ | $7{ }^{\circ}$ | $8{ }^{\circ}$ | 9 |
| $45^{\circ}$ | 0.7071 | 0.7092 | 0.7112 | 0.7133 | 0.7153 | 0.7173 | 0.7193 | $44^{\circ}$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 46 | . 7193 | . 7214 | . 7234 | . 7254 | . 7274 | . 7294 | . 7314 | 43 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 47 | . 7314 | . 7333 | . 7353 | . 7373 | . 7392 | . 7412 | . 7431 | 42 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 48 | .7431 | . 7451 | . 7470 | . 7490 | . 7509 | . 7528 | . 7547 | 41 | 2 | 4 | 6 | 8 | 10 | 12. | 13 | 15 | 17 |
| 49 | . 7547 | . 7566 | . 7585 | . 7604 | .7623 | . 7642 | . 7660 | 40 | 2 | 4 | 6 | 8 | 9 | 11 | 13 | 15 | 17 |
| $50^{\circ}$ | 0.7660 | 0.7679 | 0.7698 | 0.7716 | 0.7735 | 0.7753 | 0.7771 | 39 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 51 | . 7771 | . 7790 | . 7808 | . 7826 | . 7844 | . 7862 | . 7880 | 38 | 2 | 4 | 5 | 7 | 9 | 11 | 13 | 14 | 16 |
| 52 | . 7880 | . 7898 | . 7916 | . 7934 | . 7951 | . 7969 | . 7986 | 37 | 2 | 4 | 5 | 7 | 9 | 11 | 12 | 14 | 16 |
| 53 | . 7986 | . 8004 | . 8021 | . 8039. | . 8056 | . 8073 | . 8090 | 36 | 2 | 3 | 5 | 7 | 9 | 10 | 12 | 14 | 16 |
| 54 | . 8090 | . 8107 | . 8124 | . 8141 | . 8158 | . 8175 | . 8192 | 35 | 2 | 3 | 5 | 7 | 8 | 10 | 12 | 14 | 15 |
| $55^{\circ}$ | 0.8192 | 0.8208 | 0.8225 | 0.8241 | 0.8258 | 0.8274 | 0.8290 | 34 | 2 | 3 | 5 | 7 | 8 | 10 | 12 | 13 | 15 |
| 56 | . 8290 | . 8307 | . 8323 | . 8339 | . 8355 | . 8371 | . 8387 | 33 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 57 | . 8387 | . 8403 | . 8418 | . 8434 | . 8450 | . 8465 | . 8480 | 32 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 13 | 14 |
| 58 | . 8480 | . 8496 | . 8511 | . 8526 | . 8542 | . 8555 | . 8572 | 31 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 59 | .8572 | .8587 | .8601 | . 8616 | . 8631 | . 8646 | . 8660 | $30^{\circ}$ | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 |
| $60^{\circ}$ | 0.8660 | 0.8675 | 0.8689 | 0.8704 | 0.8718 | 0.8732 | 0.8746 | 29 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 13 |
| 61 | . 8746 | . 8760 | . 8774 | . 8788 | . 8802 | . 8816 | . 8829 | 28 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 12 |
| 62 | . 8829 | . 8843 | . 8857 | . 8870 | . 8884 | . 8897 | . 8910 | 27 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 63 | . 8910 | . 8923 | . 8936 | . 8949 | . 8962 | . 8975 | . 8988 | 26 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 12 |
| 64 | . 8988 | .9001 | . 9013 | . 9026 | . 9038 | . 9051 | . 9063 | 25 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| $65^{\circ}$ | 0.9063 | 0.9075 | 0.9088 | 0.9100 | 0.9112 | 0.9124 | 0.9135 | 24 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 66 | . 9135 | . 9147 | . 9159 | . 9171 | . 9182 | . 9194 | . 9205 | 23 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 67 | . 9205 | . 9216 | . 9228 | . 9239 | . 9250 | . 9261 | . 9272 | 22 | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9 | 10 |
| 68 | . 9272 | . 9283 | . 9293 | . 9304 | . 9315 | . 9325 | . 9336 | 21 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 |
| 69 | . 9336 | . 93.46 | . 9356 | . 9367 | . 9377 | . 9387 | . 9397 | $20^{\circ}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| $70^{\circ}$ | 0.9397 | 0.9407 | 0.9417 | 0.9426 | 0.9436 | 0.9446 | 0.9455 | 19 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 71 | . 9455 | . 9465 | . 9474 | . 9483 | . 9492 | . 9502 | . 9511 | 18 | 1 | 2 | 3 | 4 | 5 | 6 | 6 | 7 | 。 |
| 72 | . 9511 | . 9520 | . 9528 | . 9537 | . 9546 | . 9555 | . 9563 | 17 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 73 | . 9563 | . 9572 | . 9580 | . 9588 | . 9596 | . 9605 | . 9613 | 16 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 |  |
| 74 | . 9613 | . 9621 | . 9628 | . 9636 | . 9644 | . 9652 | .9659 | 15 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| $75^{\circ}$ | 0.9659 | 0.9667 | 0.9674 | 0.9681 | 0.9689 | 0.9696 | 0.9703 | 14 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 76 | . 9703 | . 9710 | . 9717 | . 9724 | . 9730 | . 9737 | . 9744 | 13 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 77 | . 9744 | . 9750 | . 9757 | . 9763 | . 9769 | . 9775 | . 9781 | 12 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 78 | . 9781 | . 9787 | . 9793 | . 9799 | . 9805 | . 9811 | . 9816 | 11 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 79 | . 9816 | . 9822 | .982i | . 9833 | . 9838 | . 9843 | . 9848 | $10^{\circ}$ | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| $80^{\circ}$ | 0.9848 | 0.9853 | 0.9858 | 0.9863 | 0.9868 | 0.9872 | 0.9877 | 9 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 81 | . 9877 | . 9881 | . 9886 | . 9890 | . 9894 | . 9899 | . 9903 | 8 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |
| 82 | . 9903 | . 9907 | . 9911 | . 9914 | . 9918 | . 9922 | . 9925 | 7 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 |
| 83 | . 9925 | . 9929 | . 9932 | . 9936 | . 9939 | . 9942 | . 9945 | 6 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 84 | . 9945 | . 9948 | . 9951 | . 9954 | . 9957 | . 9959 | . 9962 | 5 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| $85^{\circ}$ | 0.9962 | 0.9964 | 0.9967 | 0.9969 | 0.9971 | 0.9974 | 0.9976 | 4 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 86 | . 9976 | . 9978 | . 9980 | . 9981 | . 9983 | . 9985 | . 9986 | 3 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| 87 | . 9986 | . 9988 | . 9989 | . 9990 | . 9992 | . 9993 | . 9994 | 2 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |  |
| 88 | . 9994 | . 9995 | . 9996 | . 9997 | . 9997 | . 9998 | . 9998 | 1 |  |  |  |  |  |  |  |  |  |
| 89 | 0.9998 | 0.9999 | 0.9999 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0 |  |  |  |  |  |  |  |  |  |
|  | $60^{\circ}$ | $50^{\circ}$ | $40^{\circ}$ | $30^{\circ}$ | $20^{\circ}$ | $10^{\circ}$ | 01 |  | $1{ }^{\circ}$ | $2 \cdot$ | 3. | 4* | 5 | $6^{\circ}$ | $7{ }^{\circ}$ | 8 | $9{ }^{\circ}$ |


|  <br>  NATURAL TANGENTS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  | $\begin{gathered} 00 \\ \text { Me } \end{gathered}$ | บセロT ais an |  |  |  |  |  |
|  | $0^{\circ}$ | $10^{\circ}$ | $20^{\circ}$ | $30^{\circ}$ | $40^{\circ}$ | $50^{\circ}$ | $60^{\circ}$ |  | 1 | $2 \cdot$ | 3 | 4 | 5 | $6^{\prime}$ | 7 | $8{ }^{\circ}$ | $9{ }^{-1}$ |
| $0{ }^{\circ}$ | 0.0000 | 00029 | 0.0058 | 0.0087 | 0.0116 | 0.0145 | 0.0175 | $89^{\circ}$ | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | 26 |
| 1 | . 0175 | . 0204 | . 0233 | . 0262 | . 0291 | . 0320 | . 0349 | 88 | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | 26 |
| 2 | . 0349 | . 0378 | . 0407 | . 0437 | . 0466 | . 0495 | . 0524 | 87 | 3 | 6 | 9 | 12 | 15 | 18 | 20 | 23 | 26 |
| 3 | . 0524 | . 0553 | . 0582 | . 0612 | . 0641 | . 0670 | . 0699 | 86 | 3 | 6 | 9 | 12 | 15 | 18 | 20 | 23 | 26 |
| 4 | . 0699 | . 0729 | . 0758 | . 0787 | . 0816 | . 0846 | . 0875 | 85 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 23 | 26 |
| 5 | 0.0875 | 0.0904 | 0.0934 | 0.0963 | 0.0992 | 0.1022 | 0.1051 | 84 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 26 |
| 6 | . 1051 | . 1080 | . 1110 | . 1139 | . 1169 | . 1198 | . 1228 | 83 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 7 | . 1228 | . 1257 | . 1287 | . 1317 | . 1346 | . 1376 | . 1405 | 82 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 8 | . 1405 | . 1435 | . 1465 | . 1495 | . 1524 | . 1554 | . 1584 | 81 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 9 | . 1584 | . 1614 | . 1644 | . 1673 | . 1703 | . 1733 | . 1763 | $80^{\circ}$ | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| $10^{\circ}$ | 0.1763 | 0.1793 | 0.1823 | 0.1853 | 0.1883 | 0.1914 | 0.1944 | 79 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 11 | . 1944 | - 1974 | . 2004 | . 2035 | . 2065 | . 2095 | . 2126 | 78 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 12 | . 2126 | . 2156 | . 2186 | . 2217 | . 2247 | -2278 | -2309 | 77 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 13 | - 2309 | . 2339 | . 2370 | . 2401 | . 2432 | . 2462 | . 2493 | 76 | 3 | 6 | 9 | 12 | 15 | 18 | 22 | 25 | 28 |
| 14 | . 2493 | . 2524 | . 2555 | . 2586 | . 2617 | - 2648 | -2679 | 75 | 3 | 6 | 9 | 12 | 15 | 15 | 22 | 25 | 28 |
| 15 | 0.2679 | 0.2711 | 0.2742 | 0.2773 | 0.2805 | 0.2836 | 0.2867 | 74 | 3 | 6 | 9 | 13 | 16 | 19 | 22 | 25 | 28 |
| 16 | . 2867 | . 2899 | . 2931 | . 2962 | . 2994 | . 3026 | -3057 | 73 | 3 | 6 | 9 | 13 | 16 | 19 | 22 | 25 | 28 |
| 17 | . 3057 | . 3089 | . 3121 | . 3153 | . 3185 | . 3217 | . 3249 | 72 | 3 | 6 | 10 | 13 | 16 | 19 | 22 | 26 | 29 |
| 18 | . 3249 | . 3281 | . 3314 | . 3346 | . 3378 | . 3411 | . 3443 | 71 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 19 | . 3443 | . 3476 | . 3508 | . 3541 | . 3574 | . 3607 | . 3640 | $70^{\circ}$ | 3 | 7 | 10 | 13 | 16 | 20 | 23 | 26 | 29 |
| $20^{\circ}$ | 0.3640 | 0.3673 | 0.3706 | 0.3739 | 0.3772 | 0.3805 | 0.3839 | 69 | 3 | 7 | 10 | 13 | 17 | 20 | 23 | 27 | 30 |
| 21 | . 3839 | . 3872 | . 3906 | . 3939 | . 3973 | . 4006 | 14040 | 68 | 3 | 7 | 10 | 13 | 17 | 20 | 24 | 27 | 30 |
| 22 | . 4040 | . 4074 | . 4108 | . 4142 | . 4176 | 4210 | .4245 | 67 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 23 | . 4245 | . 4279 | . 4214 | . 4348 | . 4383 | . 4417 | . 4452 | 66 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 24 | . 4452 | . 4487 | . 4522 | . 4557 | -4592 | . 4628 | . 4663 | 65 | 4 | 7 | 11 | 14 | 18 | 21 | 25 | 28 | 32 |
| 25 | 0.4663 | 0.4699 | 0.4734 | 0.4770 | 0.4806 | 0.4841 | 0.4877 | 64 | 4 | 7 | 11 | 14 | 18 | 21 | 25 | 29 | 32 |
| 26 | 4877 | . 4913 | . 4950 | . 4986 | . 5022 | . 5059 | . 5095 | 63 | 4 | 7 | 11 | 15 | 18 | 22 | 25 | 29 | 33 |
| 27 | . 5095 | . 5132 | . 5169 | . 5206 | . 5243 | . 5280 | . 5317 | 62 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 30 | 33 |
| 28 | . 5317 | . 5354 | . 5392 | . 5430 | . 5467 | . 5505 | . 5543 | 61 | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 29 | . 5543 | . 5581 | . 5619 | . 5658 | . 5696 | . $5735^{\circ}$ | . 5774 | $60^{\circ}$ | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| $30^{\circ}$ | 0.5774 | 0.5812 | 0.5851 | 0.5890 | 0.5930 | 0.5969 | 0.6009 | 59 | 4 | 8 | 12 | 16 | 20 | 24 | 27 | 31 | 35 |
| 31 | . 6009 | . 6048 | . 6088 | . 6128 | . 6168 | . 6208 | . 6249 | 58 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 32 | . 6249 | . 6289 | . 6330 | . 6371 | . 6412 | . 6453 | . 6494 | 57 | 4 | 8 | 12 | 16 | 20 | 25 | 29 | 33 | 37 |
| 33 | . 6494 | . 6536 | . 6577 | . 6619 | . 6661 | . 6703 | .6745 | 56 | 4 | 8 | 13 | 17 | 21 | 25 | 29 | 33 | 38 |
| 34 | . 6745 | . 6787 | . 6830 | . 6873 | . 6916 | . 6959 | . 7002 | 55 | 4 | 9 | 13 | 17 | 21 | 26 | 30 | 34 | 39 |
| 35 | 0.7002 | 0.7046 | 0.7089 | 0.7133 | 0.7177 | 0.7221 | 0.7265 | 54 | 4 | 9 | 13 | 18 | 22 | 26 | 31 | 35 | 40 |
| 36 | . 7265 | . 7310 | . 7335 | .7400 | . 7445 | . 7490 | . 7536 | 53 | 5 | 9 | 14 | 18 | 23 | 27 | 32 | 36 | 4 |
| 37 | . 7536 | . 7581 | . 7627 | . 7673 | . 7720 | . 7766 | . 7813 | 52 | 5 | 9 | 14 | 19 | 23 | 28 | 32 | 37 | 42 |
| 38 | . 7813 | . 7860 | . 7907 | . 7954 | . 8002 | . 8050 | . 8098 | 51 | 5 | 10 | 14 | 19 | 24 | 29 | 33 | 38 | 43 |
| 39 | . 8098 | . 8146 | . 8195 | . 8243 | . 8292 | . 8342 | . 8391 | $50^{\circ}$ | 5 | 10 | 15 | 20 | 24 | 29 | 34 | 39 | 44 |
| $40^{\circ}$ | 0.8391 | 0.8441 | 0.8491 | 0.8541 | 0.8591 | 0.8642 | 0.8693 | 49 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 41 | . 8693 | . 8744 | . 8796 | . 8847 | . 8899 | . 8952 | . 9004 | 48 | 5 | 10 | 16 | 21 | 26 | 31 | 36 | 41 | 47 |
| 42 | . 9004 | . 9057 | . 9110 | . 9163 | . 9217 | . 9271 | . 9325 | 47 | 5 | 11 | 16 | 21 | 27 | 32 | 37 | 43 | 48 |
| 43 | . 9325 | . 9380 | . 9435 | . 9490 | . 9545 | . 9601 | . 9657 | 46 | 6 | 11 | 17 | 22 | 28 | 33 | 39 | 44 | 50 |
| 44 | . 9657 | . 9713 | . 9770 | . 9827 | . 9884 | . 9942 | 1.000 | 45 | 6 | 11 | 17 | 23 | 29 | 34 | 40 | 46 | 51 |
|  | $60^{\circ}$ | $50^{\circ}$ | $40^{\circ}$ | $30^{\circ}$ | $20^{\prime}$ | 10' | 01 |  | $1{ }^{\circ}$ | $2{ }^{\prime}$ | $3{ }^{\circ}$ | $4{ }^{\prime}$ | 5' | 6 | 7 | $8{ }^{\circ}$ | $9 *$ |



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