24 Sets

After studying this lesson you will acquire knowledge about the following :

- Use of Venn diagrams in explaining set operations
- To show the relevant regions of set operations on Venn diagram.
- Solving problems using Venn diagrams.

24.1 Introduction of sets

As an introduction to this lesson it is very important to recollect what you have learned about sets starting from the definition of a set. You have already learned about, set notation, elements, sub sets, universal set, union of sets, intersection of sets and the complement of a set and also about the venn diagrams. Consider the examples given below.

Example 1

(i) Consider the set

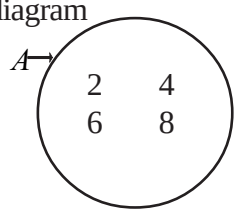
A = {Multiples of 2 between 1 and 10}

You know that this set can be represented in different ways.

i.e. (1) With elements

A = {2, 4, 6, 8}

(2) Using a Venn diagram



(3) Set generating method

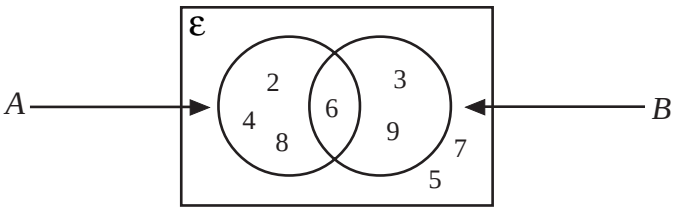
A = {x : 1< x <10 and x is a multiple of 2}

Consider the sets

B ={Multiples of 3 between 1 and 10}

and ε ={Integers between 1 and 10}

Now we will represent the sets A,B and ε in the same Venn diagram



Example 2

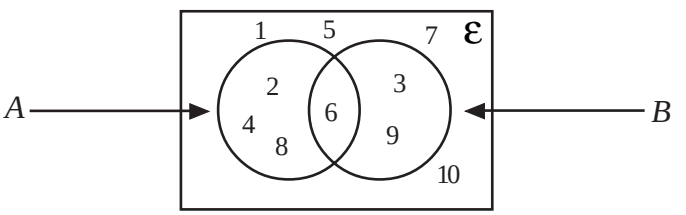
ε = {Integers from 1 to 10}
A = {Multiples of 2 between 1 and 10}
B = {Multiples of 3 between 1 and 10}

When the above sets are written with elements

ε = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {2, 4, 6, 8}
B = {3, 6, 9}

$A \cup B = \{2, 3, 4, 6, 8, 9\}$
$A \cap B = \{ 6 \}$

When these sets are represented in a Venn diagram



Also

$(A \cup B)' = \{1, 5, 7, 10\}$

$(A \cap B)' = \{1, 2, 3, 4, 5, 7, 8, 9, 10\}$

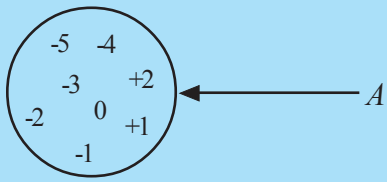
$A' = \{1, 3, 5, 7, 9, 10\}$

$B' = \{1, 2, 4, 5, 7, 8, 10\}$

Exercise 24.1

(1) Write the following sets in other notations

(i) X = $\{x : x$ is a prime number less than 15$\}$
(ii) Y = {3, 6, 10, 15, 21}
(iii) Z = {Even prime numbers}
(iv)



(2) Write all the subsets of the set S = { *p, q, r* } How many sub sets did do you get ?

(3) Write the universal set for each of the following sets.

(i) P = {Plantain , Papaw, Mango, Pineapple}
(ii) Q = {2, 4, 6, 8, 10, 12}
(iii) R = {Students who are studying combined maths for Advanced Level}
(iv) A = {1, 3, 5, 7, 9}, B = { 2 }, C= { 3, 9 }
(v) T = {Triangles}, N = {Quadrilaterals}, L = {Pentagons}
M = {Hexagons}

(4) ε = {Letters of the word 'combinatorics'}
A = {Letters of the word 'contrast'}
B = {Letters of the word 'ammonia'}

(i) Represent the above sets in a Venn diagram.
(ii) Using your Venn diagram find
n (A), n(B), n, $(A \cup B)$ and n $(A \cap B)$
(iii) Using your answers to (i) and (ii) verify the equation
n $(A \cup B)$= n (A) + n (B) - n $(A \cap B)$
'n' means the number of elements.

24.2 To shade the relevent regions in Venn diagrams in set operations.

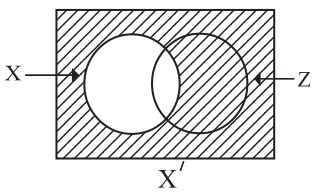
Any two non empty sets A and B can be shown in a Venn diagram by shading the relevant regions.

Example 3

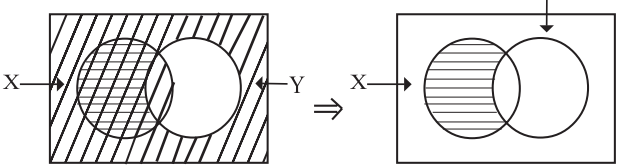
If X and Y are two non empty sets, represent the following using Venn diagrams by shading the relevant regions.

(i) X' (ii) $X \cap Y'$ (iii) $X \cup Y$ (iv) $X \cap Y$
(v) $(X \cap Y)'$ (vi) $(X \cup Y)'$

(i)



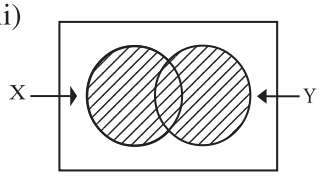
(ii)



Here $X \cap Y'$ is the region shaded as

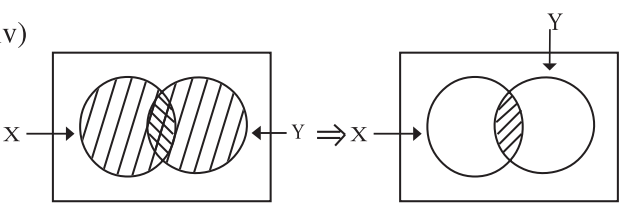
$X \cap Y'$

(iii)



$(X \cup Y)$

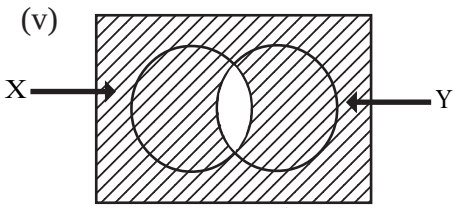
(iv)



Here $X \cap Y$ is the region shaded as

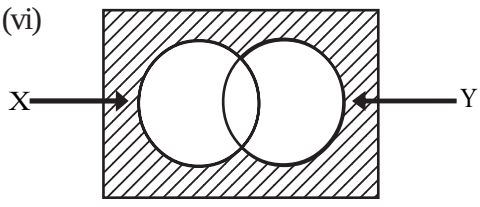
$X \cap Y$

(v)



$(X \cap Y)'$

(vi)



$(X \cup Y)'$

24.1 Activity

(i) $\mathcal{E}$= {Letters of the word 'extraordinary'}
A = {Letters of the word 'train'}
B = {Letters of the word 'entity'}

Write the elements of the sets

A', B', $A \cup B$, $A \cap B$, $A' \cap B'$, $A' \cup B'$

Hence show that

$(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

(ii) For any non empty sets P and Q show that $P' \cap Q' = (P \cup Q)'$ and $P' \cup Q' = (P \cap Q)'$ using Venn diagrams

Exercise 24.2

(1) Verify the laws given below, related to set operations using Venn diagrams

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

(iii) $(A \cup B)' = A' \cap B'$

(iv) $(A \cap B)' = A' \cup B'$

(iv) $A \cup A' = \mathcal{E}$

(vi) $A \cap A' = \varnothing$

(2) Verify using Venn diagrams

(i) $A \cap (A \cup B) = A$

(ii) $A \cup B = (A \cap B') \cup (A \cap B) \cup (A' \cap B)$

(iii) $A = (A \cap B) \cup (A \cap B')$

(iv) $B = (A \cap B) \cup (A' \cap B)$

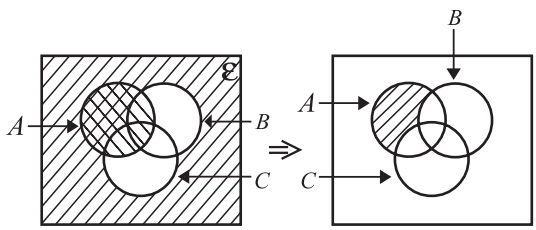
24.3 Shading the relevant regions in set operations can also be applied to three sets

Example 4

If A, B and C are non empty sets, the way they intersect each other can be shown using a Venn diagram

(i) If the set X denotes the region shaded by using two types of lines then X contains the elements belonging only to A and elements not belonging to B and C

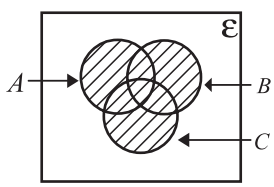
$\therefore X = A \cap (B \cup C)'$



$A \cap (B \cup C)'$

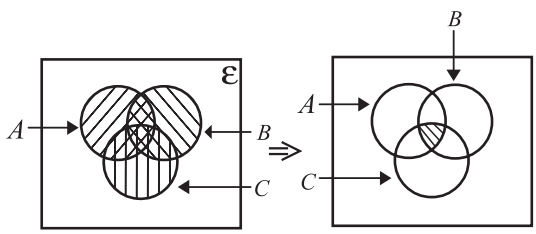
(ii) If the shaded region denotes the set Y then Y contains all the elements in the sets A, B and C

$\therefore Y = A \cup B \cup C$



(iii) If the set $\mathbb{Z}$ denotes the region shaded using three types of lines, then $\mathbb{Z}$ contains the elements belonging only to all three sets A, B and C

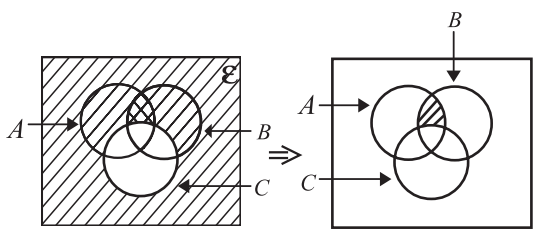
$\therefore Z = A \cap B \cap C$



$A \cap B \cap C$

(iv) If the set P denotes the region shaded using two types of lines, then P contains the elements not belonging to C and elements belonging to both A and B only

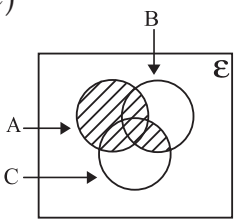
$\therefore P = (A \cap B) \cap C'$



$(A \cap B) \cap C'$

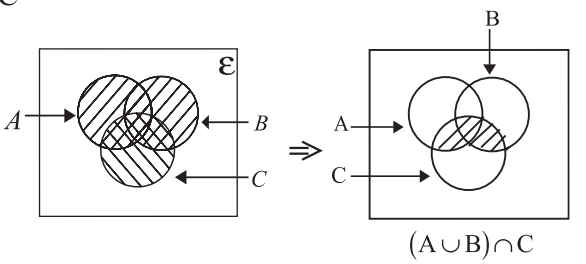
(v) Shaded region denotes the set

$A \cup (B \cap C)$



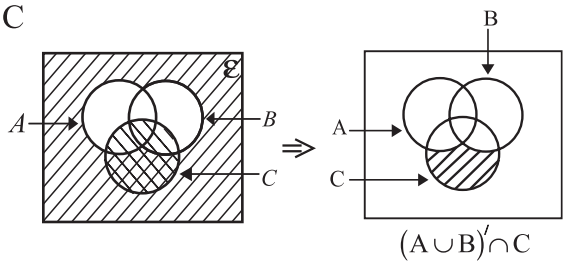
(vi) The region shaded by using two types of lines denotes the set

$(A \cup B) \cap C$



(vii) The region shaded by using two types of lines denotes the set

$(A \cup B)' \cap C$



Laws related to three sets are as follows

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

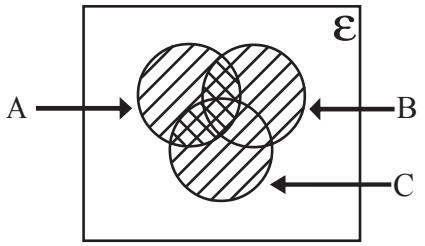
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

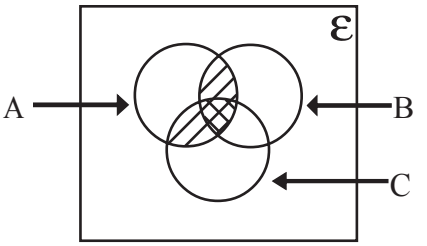
(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

These laws too can be proved by using Venn diagrams. Here the proof of (iii) only is given. The rest can be done as an exercise.

(iii) To show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



$A \cap (B \cup C)$



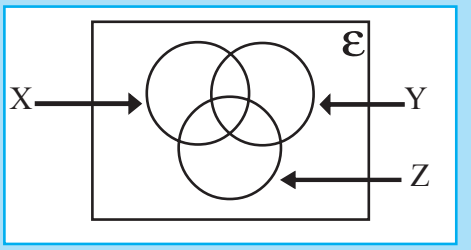
$(A \cap B) \cup (A \cap C)$

$\therefore$ According to the two diagrams

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Exercise 24.3

(1) Copy the given Venn diagram, and show each set given, by shading the relevant region,
Recognise the set in each region



(i) $(X \cup Y) \cup Z$

(ii) $(X \cup Y) \cap Z$

(iii) $X \cap (Y \cup Z)$

(iv) $X \cup (Y \cap Z)$

(v) $X \cap (Y \cap Z)$

(vi) $(X \cap U) \cup (X \cap Z)$

(2) $\varepsilon = \{1,2,3,4,5,6,7,8,9\}$
$P = \{2,3,5,8\}$
$Q = \{1,2,3,5,6\}$
$R = \{1,3,5,7,9\}$

Write the elements of the following sets and hence write down the equal sets

(i) $(P \cup Q) \cup R$

(ii) $P \cup (Q \cup R)$

(iii) $(P \cap Q) \cap R$

(iv) $P \cap (Q \cap R)$

(v) $P \cap (Q \cup R)$

(vi) $(P \cap Q) \cup (P \cap R)$

(vii) $P \cup (Q \cap R)$

(viii) $(P \cup Q) \cap (P \cup R)$

(3) $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7\}$, $C = \{5,6,7,8,9\}$
Write the number of elements in each set given below.

$n(A)$, $n(B)$, $n(C)$, $n(A \cap B)$, $n(A \cap C)$,
$n(B \cap C)$, $n(A \cap B \cap C)$, $n(A \cup B \cup C)$

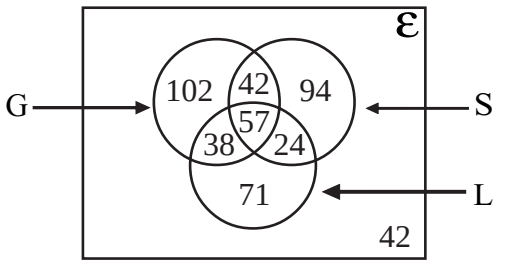
(4) Hence verify the equations given below

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Example 5

ε = {Students in a mixed school}
G = {Girls in the school}
S = {Students who can swim}
L = {Left handed students}

These sets are represented in the Venn diagram given below. Write down the number of elements in each region.



(a) How many left handed students are there ?
(b) What is the number of girls who cannot swim?
(c) What is the number of boys who can swim?
(d) What is the number of left handed girls?
(e) What is the number of left handed boys?
(f) Out of the left handed girls, how many can swim ?
(g) What is the total number of boys in the school.

Solutions

(a) 38 + 57 + 24 + 71 = 190

(b) 102 + 38 = 140

(c) 94 + 24 = 118

(d) 38 + 57 = 95

(e) 71 + 24 = 95

(f) 57

(g) 42 + 71 + 24 + 94 = 231

Example 6

ε = {Girls in a School}
A = {Girls who play basket ball}
B = {Girls who play net ball}
C = {Girls who play hockey}

Write below statements, using set notation.

(1) Girls playing all three games
(2) Girls playing net ball and basket ball but not hockey.
(3) Girls playing only hockey
(4) Girls playing net ball and hockey
(5) Girls playing net ball and hockey or basket ball
(6) Girls playing net ball and hockey or net ball and basket ball.

Solutions

(1) $A \cap B \cap C$
(2) $C' \cap (A \cup B)$
(3) $C \cap (A \cup B)'$
(4) $B \cap C$
(5) $(B \cap C) \cup A$
(6) $(B \cap C) \cup (A \cap B)$

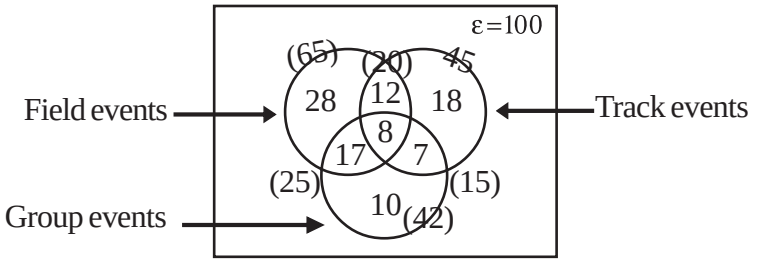
Example 7

(1) 100 Students in a school participated in the school interhouse athletic meet. Details of participation are given below

65 students took part in track events, 45 took part in field events, 42 took part in group events 20 took part in track events and field events, and 25 took part in track events and group events while 15 took part in field events and group events. 8 students took part in all three events.

Represent this information in a Venn diagram and answer the following questions.

(a) How many students participated only in group events?
(b) How many participated in one class of events only?



(a) $\underline{\underline{10}}$

(b) $28+10+18=\underline{\underline{56}}$

Exercise 24.4

(1) A vendor who sells oranges, apples and grapes gives below, the details of his customar on a particular day.

The number of people who bought only apples is 12 while 15 bought apples and grapes. The number of people who bought grapes and oranges is 9. Four of them did not buy apples. 31 bought grapes while 25 bought oranges. 7 bought only oranges. Everyone who came to the shop bought least one fruit.

(i) Show this information in a Venn diagram
(ii) How many bought only grapes?
(iii) How many bought apples and oranges but not grapes
(iv) What is the total number of customers who came to the shop.

(2) 500 electrical appliances, after manufacture were tested at three different places namely P, Q and R for their quality Out of them, at P, 38 appliances and at Q, 29 appliances and at R , 30 appliances did not pass the quality test. Also at P and R 8, P and Q 10, Q and R 12 Appliances and in all three tests 3 appliances did not pass the test.

(i) How many did pass all three tests
(ii) How many did pass in only one test
(iii) How many did not pass only in two tests

(3) $\mathcal{E}$ = {Triangles}
I = {Isosceles triangles}
R = {Right angled triangles}
A = {Triangles with their area equal to $32m^2$}

(i) Draw a Venn diagram to show the relationship between A,I,R
(ii) Write the characteristics of the triangles in the set $I \cap R$
(iii) Find the lengths of the sides of the triangles in the set $I \cap A \cap R$

(4) Out of 75 farmers in a village, 45 grow chillies while 25 grow brinjals. Out of 20 farmers who grow chillies and ladies fingers, 15 do not grow brinjals. 10 grow brinjals and chillies while 15 grow ladies fingers and brinjals. All the farmers grow at least one of the these vegetables. Answer the follwing questions.

(i) How many farmers cultivated all three types?
(ii) How many farmers cultivated only chillies?
(iii) How many farmers cultivated only chillies and brinjals?

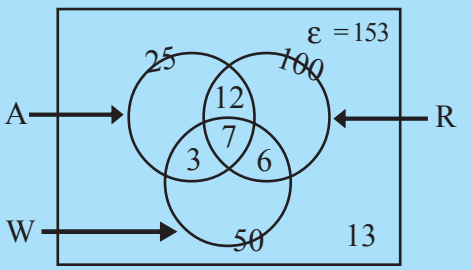
(5) 60 different vitamin pills contain at least one of the vitamins A, B or C. 12 of the pills contain only vitamin A, 7 contain only B and 11 contain only C. Equal number of pills contain the vitamin A and B, A and C and B and C. number of pills contain the vitamin A, B and C is 3.
How many pills contain vitamin A ?

(6) A college of Education received 172 applications. Out of them 45 have applied for Maths and 38 for English and 21 for Information technology. 18 have applied for Maths and English, 9 have applied for Maths and Information technology and 4 have applied for English and Information technology. 4 have applied for all three subjects.

(i) How many have applied only for Maths ?
(ii) How many have applied for other subjects ?
(iii) How many have applied only for English and Information technology.

(7) The Venn diagram given below shows the details of the sale of motor vehicles in an establishment during a certain period.

ε = {Motor vehicles sold}
A = {Vehicles with air conditioners}
R = {Vehicles with radios}
W = {Vehicles with electronic doors}



(i) How many vehicles are there with only electronic doors ?
(ii) How many vehicles are there with only air conditioners ?
(iii) How many vehicles are there with only a radio ?
(iv) How many vehicles are there with a radio and electronic door but not an air conditioner ?

(8) The following information was gathered about reading news papers which are T,M,G in a street where there are 150 houses.

- 40 houses buy the type T
- 35 houses buy the type G
- 60 houses buy the type M
- 7 houses buy the type T and G
- 10 houses buy the types G and M while 8 of them do not buy T
- 4 houses buy the types T and M
- 34 houses do not buy any of these papers.

Find the number of houses buying all three types of papers.

(9) The following information was collected by questioning 60 television viewers.

25 watch Sinhala channels 26 watch Tamil channels. 9 watch Sinhala and English channels and 11 watch Sinhala and Tamil while 8 watch Tamil and English channels. 8 do not watch any of these channels. 10 watch only Tamil channels.

Represent this infomation in a Venn diagram and answer the following questions.

(i) How many watch all three channels?
(ii) How many watch only one of three channels?
(iii) How many watch only Sinhala channels?