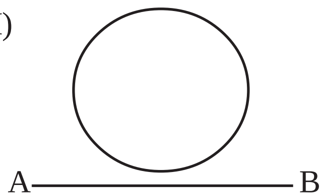
23 Constructions

After studying this lesson you will acquire knowledge about the following :

- Construction of a tangent at a point on a circle
- Construction of a tangent to a circle from an external point
- To divide a given straight line into a number of equal parts

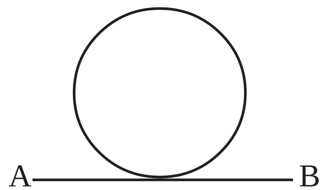
The behaviour of a given straight line and a circle can be shown in three possible ways.

(I)



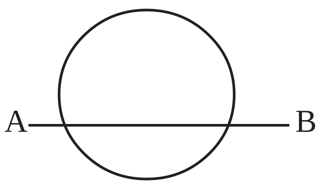
Straight line AB can be outside the circle.

(II)



Straight line AB may touch the circle.

(III)



Straight line AB may cut the circle.

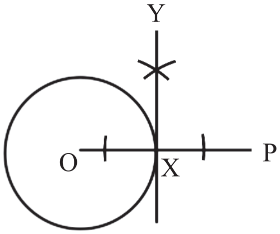
When the line touches the circle as shown in figure (II), " AB is a tangent to the circle."

Draw a circle with centre O, using a pair of compasses. Using a ruler, draw line AB to touch the circle. Mark the point of contact as P. Join OP. Measure the angles . $O\hat{P}A$ and $O\hat{P}B$

You already know that the angle between the tangent and the radius drawn at the point of contact is 90^0 .

23.1 Construction

Construction of a tangent to a circle at a point on the circle.
X is a point on the circle with centre O. The tangent XY to the circle at the point X is to be constructed.



Step

(1) Draw OX and produce it to P.
(2) Using a pair of compasses draw the perpendicular XY to the line OXP at X.

Proof : Since the radius OX and the line XY are perpendicular to each other, XY is the tangent drawn to the circle at X.

XY is the tangent drawn at the point X on the circumference of the circle.

Two useful theorems related to tangents are given below for your references.

- **A line drawn through a point on the circumference of a circle, perpen dicular to the radius at that point is a tangent to the circle.**
- **A tangent to a circle and the radius at the point of contact are perpen dicular to each other.**

Exercise 23.1

(1) Draw a circle of radius 3.5 cm. Mark point P on the circle. Construct the tangent PQ of length 4.5 cm.

(2) Mark any point Y on the circle with centre X and radius 4.5 cm. Construct a tangent to the circle at Y and mark a point Z on the tangent such that XZ = 7.5 cm.

(i) Measure and write the length of YZ.
(ii) Calculate the length of YZ using Pythagoras theorem and verify your answer of part *(i)*.
(iii) Mark a point W on the circle, such that ZY = ZW and join WX.
(iv) Prove that Δ XYZ and Δ XWZare congruent.
(v) Show that WZ is a tangent drawn to the circle at W.

(3) P is a point on the straight line AB. Q is a point outside AB.
(i) Construct the circle which touches the line AB at P and passes through the point Q.
(ii) State the two theorems that you used for this construction.

Activity 23.1

Draw a circle with centre O and mark a point X outside the circle. Draw the two tangents XA and XB using a ruler. Join the radii OA and OB.

(a) By measuring the values verify the following statements.

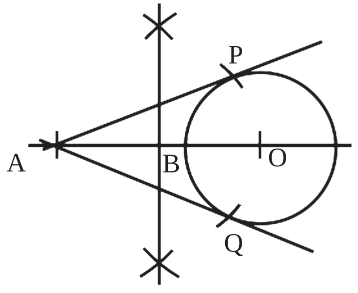
(i) XA = XB

(ii) $X\hat{A}O = X\hat{B}O$

(iii) $X\hat{O}A = X\hat{O}B$

(iv) Is XAOΔ and XBOΔ congruent

23.2 Construction of tangent to a circle from an exterior point



A is a point outside the circle with centre O.

Step :
(1) Draw line OA and draw its perpendicular bisector using a pair of compasses.
(2) Mark the point of intersection of OA and the perpendicular bisector as B.

(3) Take the measurement BO to the pair of compasses and mark P and Q on the circle such that BO = BP = BQ.
(4) Draw the lines AP and AQ.
(5) AP and AQ are the tangents to the circle drawn from the point A.

Proof :

$$BA = BO \quad (\text{B is the mid point of AO})$$

but $BO = BP$

$BA = BO = BP$

B is the centre of the circle through the vertices of triangle AOP (circumcircle) AO is a diameter to this circle.

$\therefore\ A\hat{P}O = 90^0$ (Angle subtended in a semicircle)

AP is a tangent to the circle with centre O

Similarly it can be proved that $A\hat{Q}O = 90^0$

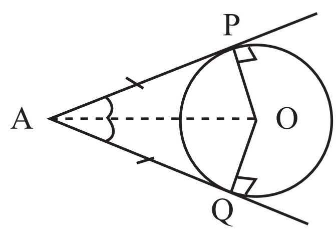
Hence AQ too is a tangent to the circle.

If AP and AQ are the tangents drawn to the circle with centre O from an outside point *A*. then,

(i) $AP = AQ$

(ii) $O\hat{A}P = O\hat{A}Q$

(iii) $A\hat{O}P = A\hat{O}Q$



This is a theorem we have already learned and it can be stated as follows.

Theorem

If two tangents are drawn to a circle from an outside point

(i) The two tangents are equal in length

(ii) The line joining the centre to the outside point, bisects the angle between the two tangents.

(iii) The angles subtended at the centre of the circle by each tangent are equal.

Exercise 23.2

(1) Draw a circle with centre O and radius 2.5 cm. Mark any point P on the circle and mark a point X on OP produced such that OX = 6.5 cm.
(i) Construct the two tangents XA and XB to the circle.
(ii) Measure the lengths of XA and XB.
(iii) By calculating the length of XA verify your answer to part (ii).

(2) Draw a circle with centre O and radius 4.0 cm. Mark a point *X* on the circle. Mark a point P on OX produced such that OP = 8 cm.
(i) Show that the perpendicular bisector of OP is a tangent to the circle.
(ii) Construct the tangent PQ to the circle from the point *P* and measure and the length of PQ.
(iii) Without measuring, show that $P\hat{Q}X = 30^0$
(iv) Use Pythagoras' theorem, to show that $PQ = 4\sqrt{3}$ cm.

(3) (i) Construct Δ ABC in which AB = 6 cm, BC = 7 cm and CA = 5 cm.
(ii) Construct the bisectors of angles $B\hat{A}C$ and $A\hat{B}C$ to meet at P.
(iii) Construct the perpendicular PX from P to BC.
(iv) Draw the circle with centre P and the radius PX.

(4) In Δ XYZ, the angle $X\hat{Y}Z$ is an obtuse angle. Construct the circle which touches the side YZ at Y while touching the side XZ.

(5) (i) Construct Δ ABC in which $A\hat{B}C = 90^0$, AB = 4.0 cm and BC = 6.5 cm.
(ii) Construct the circle which touches the side AB at A and also touches the side BC.

(6) (i) Construct Δ PQR in which PQ = 8 cm, $P\hat{Q}R = 90^0$ and QR = 4 cm.
(ii) Construct the bisector of $P\hat{R}Q$ to meet *PQ* at *S*.

(iii) Construct the perpendicular ST form *S* to PR.

(iv) Draw the circle with centre *S* and radius SQ.

(v) Prove that SQR and STR are congruent.

(vi) Show that PT is one of the tangents to the circle drawn from the point P.

(vii) Construct the other tangent PU to the circle from P.

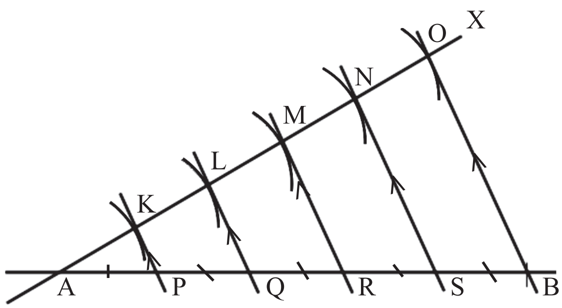
(viii) By applying Pythagoras' theorem write the length of PR in terms of $\sqrt{5}$.

(ix) Measure the length of PR and hence find the value of $\sqrt{5}$ to the first decimal place.

(x) Show that $\dfrac{\text{Area of PSU}}{\text{Area of QRS}} = \sqrt{5} - 1$.

23.2 Construction

To divide a given straight line into equal parts.



The given straight line AB is to be divided into 5 equal parts.

Step :

(1) Draw any straight line AX through A.

(2) Using a pair of compasses, mark the points K,L,M,N and O on AX such that AK = KL = KM = MN = NO

(3) Draw line OB and construct the straight lines *NS, MR, LQ, KP* parallel to OB.

Proof :

In ALQ,

AK = KL (Construction - Taking equal lengths)

KP // LQ (Construction)

P is the mid point of AQ (Mid point theorem)

AP = PQ

In AMR,

LQ // MR (Construction)

AL : LM = 2:1 (Construction)

AQ : QR = 2 : 1 (Line drawn parallel to one side of a triangle, divides the other two sides proportionally)

As AK = KL = LM,

AP = PQ = QR

Similarly it can be proved for the segments RS and SB.

Hence AP = PQ = QR = RS = SB

Mid pint theorem

The line joining the mid points of two sides of a trangle is paralled to the third side and is half the size of the third side

converse of mid point Theorem

In a trangle a line drawn through the mid point of one side, parallel to another side bisects the thirs side.

Exercise 23.3

(1) Draw line XY equal to 11 cm and divide it into 4 equal parts.

(2) Divide the line AB of length 13 cm into 5 equal parts

(3) (i) Construct PQR in which PQ = 9 cm, $P\hat{Q}R = 45^0$ and QR = 8 cm.

(ii) Mark the points A and B on PQ such that PA = AB = 3 cm using a pair of compasses.

(iii) Draw AX and BY parallel to PR such that PR is divided into three equal parts.

(iv) Measure the length of XY.

(v) Show that QAX and QBY are equiangular to PQR