

18 Trigonometry

After studying this chapter you will be able to acquire the knowledge and the applications about the following :

- finding the trigonometrical ratios using tables
- finding the lengths and heights by using the angle of elevation, angle of depression and bearing

The word trigonometry is formed by combining the Greek words for triangle and measurement. Now a days the use of trigonometry is widely spread among other fields such as physics, astronomy, surveys etc.

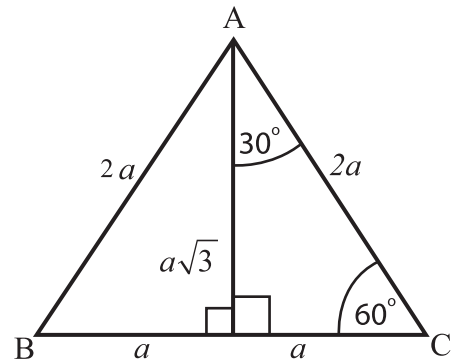
18.1 Trigonometrical ratios of 30° , 45° , and 60°

Do the following activities to refresh knowledge gained in grade 10 about these angles.

18.1 Activity

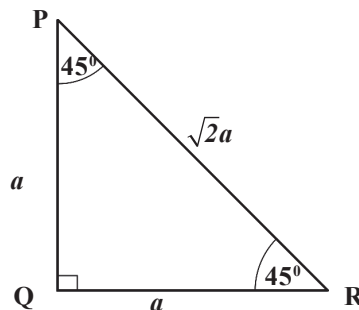
(i) Complete the table given below using the given diagrams

Angle θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
30°			
45°			
60°			



(ii) Comment on the following ratios?

- $\sin 45^\circ$ and $\cos 45^\circ$
- $\sin 30^\circ$ and $\cos 60^\circ$
- $\cos 30^\circ$ and $\sin 60^\circ$



(iii) Using the values shown in the table find the values of the following expressions

- a) $\sin 60^\circ \cos 30^\circ$
- b) $\cos 60^\circ \sin 30^\circ$
- c) $\sin 30^\circ \cos 60^\circ + \cos 60^\circ \sin 30^\circ$

Example 1

A road running along the slope of a hill is inclined at an angle of 30° to the horizontal. A child walks along this road a distance of 20 m starting from the foot of the hill. Find the vertical displacement has moved.

Let h be the vertical displacement, when the child complete the distance of 20 m travelled along the road.

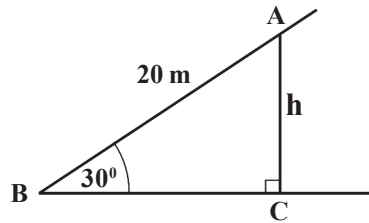
From $\triangle ABC$,

$$\frac{AC}{AB} = \sin 30^\circ$$

$$\frac{h}{20} = \sin 30^\circ$$

$$h = 20 \times \frac{1}{2} \text{ m}$$

$$\underline{\underline{h = 10 \text{ m}}}$$



\therefore The vertical displacement = 10 m

Exercise 18.1

(In this exercise simplification of the answers is not expected.)

Find the values of each of the following expressions

- (1) $\cos 45^\circ \sin 45^\circ + \sin 30^\circ$
- (2) $\sin 30^\circ \cos 30^\circ + \sin 60^\circ \cos 60^\circ$
- (3) $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$
- (4) $\tan 45^\circ \cos 45^\circ + \tan 45^\circ \sin 45^\circ$
- (5) $\cos 45^\circ \tan 30^\circ + \sin 45^\circ \tan 30^\circ$

Verify each of the following

(6) $\sin 60^\circ = 2\sin 30^\circ \cos 30^\circ$

(7) $\cos 30^\circ \sin 60^\circ + \tan 30^\circ \tan 60^\circ = \frac{7}{4}$

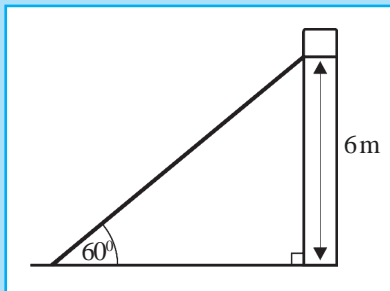
(8) $\cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ = 0$

(9) In $\triangle ABC$, $\hat{B} = 90^\circ$, $AB = 6$ cm and $BC = 8$ cm. Find the length of AC and hence find the trigonometrical ratios of:

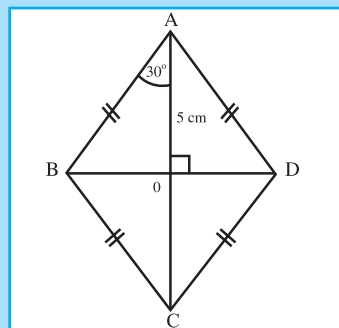
(i) $\sin \hat{A}$ (ii) $\cos \hat{A}$ (iii) $\tan \hat{A}$ (iv) $\sin \hat{C}$ (v) $\cos \hat{C}$ (vi) $\tan \hat{C}$

(10) A ladder is kept against a wall as shown in the figure

- Find the length of the ladder
- Find the distance from the foot of the ladder to the foot of the wall.



(11) ABCD is a rhombus. The diagonals AC and BD intersect at O. $AO = 5$ cm. $\hat{BAO} = 30^\circ$. Find the length of a side and the lengths of the diagonals of the rhombus.



18.2 Activity

$$\sin^2 \theta = \sin \theta \sin \theta$$

Verify that

$$\sin^2 60^\circ + \cos^2 60^\circ = 1$$

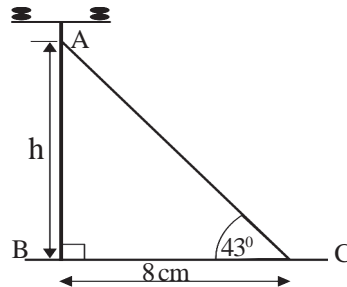
See whether this is true for $0^\circ, 30^\circ$ and 45°

18.2 Trigonometric Tables

So far we did calculations using trigonometrical ratios of angle an 30° , 45° and 60° . Look at the problem given below.

A lamp post AB is fixed vertically and is supported by a wire, AC as shown in the diagram. Using the information, find the height (h) of the lamp post AB.

We can find the height h by drawing a scale diagram according to the given data, For this you need to be accurate in drawing and measuring. The geometrical instruments that we use may not be perfect. And also it is not that easy to draw different angles in the scale drawing.



- From triangle ABC,

$$\frac{AB}{BC} = \tan 43^\circ$$

$$\text{i.e. } \frac{h}{8} = \tan 43^\circ$$

$$\therefore AB = h = 8 \times \tan 43^\circ$$

- To find the height using trigonometrical ratios we need to find the value of $\tan 43^\circ$
- For this trigonometric tables should be used
- Trigonometrical tables too can be used, to find the ratios of angles from 0° to 90°

18.3 Measuring angles in degrees.

We know that angles are measured in degrees. To be more accurate, a degree is divided into 60 equal parts called **minutes**. One minute is written as ($1'$).

Accordingly $60' = 1^\circ$

The trigonometric table is prepared for angles from 0° to 90° for all the ratios sine, cosine and tangents.

18.4 Table of Tangents of angles

The table is given at the end of the book.

- Study the table carefully
You will note that $\tan 43^\circ = 0.9325$

Accordingly $\frac{h}{8} = \tan 43$

$$\frac{h}{8} = 0.9325$$

$$h = 8 \times 0.9325$$

$$h = 7.4600 \quad (\text{to the nearest first decimal place})$$

$$\underline{\underline{h = 7.5 \text{ m}}}$$

Observe the tangent table carefully and compare your observations with the following:

- $\tan 0^\circ = 0$
- $\tan 45^\circ = 1$
- As the angle increases, the tangent value too increases
- For any angle less than 45° , the tangent value of angle increases gradually from 0 to 1
- For angles greater than 45° , the tangent value is greater than 1
- The value for $\tan 90^\circ$ tends to infinite

Example 1

- Find the value of $\tan 22^\circ 30'$
- Find the value of $\tan 54^\circ 27'$

Note: Observe the relevant part of the table is given here.

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								மேலவை ஒன்றை இடை வித்தியாசங்கள் Mean Differences									
	0'	10'	20'	30'	40'	50'	60'	1'	2'	3'	4'	5'	6'	7'	8'	9'	
20	0.3640	0.3673	0.3706	0.3739	0.3772	0.3805	0.3839	69	3	7	10	13	17	20	24	27	30
21	.3839	.3872	.3906	.3939	.3973	.4006	.4040	68	3	7	10	13	17	20	24	27	30
22	.4040	.4074	.4108	.4142	.4176	.4210	.4245	67	3	7	10	14	17	20	24	27	31
23	.4245	.4279	.4314	.4348	.4383	.4417	.4452	66	3	7	10	14	17	21	24	28	31
24	.4452	.4487	.4522	.4557	.4592	.4628	.4663	65	4	7	11	14	18	21	25	28	32
50	1.1918	1.1988	1.2059	1.2131	1.2203	1.2276	1.2349	39	7	14	22	29	36	43	50	58	65
51	.2349	.2423	.2497	.2572	.2647	.2723	.2799	38	8	15	23	30	38	45	53	60	68
52	.2799	.2876	.2954	.3032	.3111	.3190	.3270	37	8	16	24	31	39	47	55	63	71
53	.3270	.3351	.3432	.3514	.3597	.3680	.3764	36	8	16	25	33	41	49	57	66	74
54	.3764	.3848	.3934	.4019	.4106	.4193	.4281	35	9	17	26	35	43	52	60	69	78

- (i) $\tan 22^{\circ} 30' = 0.4142$
(ii) value of $\tan 54^{\circ} 27'$ should take a value more than $\tan 54^{\circ} 20'$
 $\tan 54^{\circ} 20' = 1.3934$
 $\therefore \tan 54^{\circ} 27' = 1.3934 + 0.0060$ (the value 0.0060 is taken from the mean difference)
 $\therefore \tan 54^{\circ} 27' = 1.3994$

To Finding the corresponding angle when the tangent of the angle is given

Example 3

- (iii) Find the angle whose tangent is 0.4568

Note : Observe the relevant part of the table in the other page.

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	0' 10' 20' 30' 40' 50' 60'							மெய்யை அளவையு இடை வித்தியாசங்கள் Mean Differences									
	0'	10'	20'	30'	40'	50'	60'	1'	2'	3'	4'	5'	6'	7'	8'	9'	
20'	0.3640	.3673	.3706	0.3739	0.3772	0.3805	0.3839	69	3	7	10	13	17	20	23	27	30
21	.3839	.3872	.3906	.3939	.3973	.4006	.4040	68	3	7	10	13	17	20	24	27	30
22	.4040	.4074	.4108	.4142	.4176	.4210	.4245	67	3	7	10	14	17	20	24	27	31
23	.4245	.4279	.4314	.4348	.4383	.4417	.4452	66	3	7	10	14	17	21	24	28	31
24	.4452	.4487	.4522	0.4557	.4592	.4628	.4663	65	4	11	14	18	21	25	28	32	

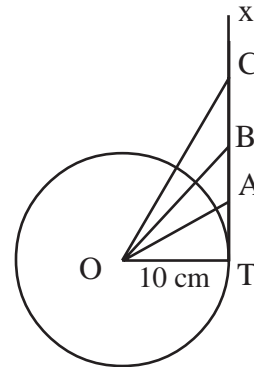
Note that $0.4568 = 0.4557 + 0.0011$

$$\begin{array}{r} 24^{\circ} 30' \\ \underline{\quad 3'} \\ 24^{\circ} 33' \end{array}$$

The angle whose tangent is 0.4568 is $24^{\circ}33'$

Activity 18.3

- Draw a circle with radius 10 cm.
- Mark a point T on the circle and draw the radius OT. Draw a perpendicular to OT at T. Name it as TX.
- Draw OA, OB and OC, such that $\hat{A}OT = 20^{\circ}$, $\hat{B}OT = 40^{\circ}$ and $\hat{C}OT = 50^{\circ}$ where points A, B, and C on TX.
- Measure the distance AT and divide this value by 10
- Compare the answer with the value of $\tan 20^{\circ}$ in the table
- Do the same activity for the distances BT and CT
- Repeat this activity for a few other angles



Exercise 18.2

(1) Find the tangent value of the following angles

- (i) $25^{\circ}30'$ (ii) $23^{\circ}40'$ (iii) $30^{\circ}30'$ (iv) $42^{\circ}58'$ (v) $45^{\circ}58'$
(vi) $60^{\circ}27'$ (vii) $46^{\circ}36'$ (viii) $70^{\circ}50'$ (ix) $10^{\circ}54'$ (x) $40^{\circ}47'$

(2) Write the angles corresponding to each tangent value given below

- (i) 0.5216 (ii) 1.7321 (iii) 1.1785 (iv) 7.3000 (v) 0.5116
(vi) 8.0281 (vii) 57.29 (viii) 20.45 (ix) 2.0794 (x) 2.1759

18.5 Table of sines of angles

Natural sines

- Observe the natural sine table given at the end of the text book
- Compare your observations with the following
- $\sin 0^{\circ} = 0$
- $\sin 45^{\circ} = 0.7071$
- $\sin 90^{\circ} = 1.0000$
- For the values from 0° to 90° the value of sine will gradually increase from 0 to 1.0000

Example 3

- (i) Find the value of $\sin 25^{\circ}40'$
(ii) Find the value of $\sin 55^{\circ}56'$

Note: Observe

- Relevant part of the table in the other page

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							மெழை ஏர்நெய இடை வித்தியாசங்கள் Mean Differences										
							1'	2'	3'	4'	5'	6'	7'	8'	9'		
	0'	10'	20'	30'	40'	50'	60'										
25	0.4226	0.4253	0.4279	0.4305	0.4331	0.4358	0.4384	64	3	5	8	10	13	6	18	21	24
26	.4348	.4410	.4436	.4462	.4488	.4514	.4540	63	3	5	8	10	13	6	18	21	23
55	0.8192	0.8208	0.8225	0.8241	0.8258	0.8274	0.8290	34	2	3	5	7	8	10	12	13	15
56	.8290	.8307	.8323	.8339	.8355	.8371	.8387	33	2	3	5	6	8	10	11	13	14

- (i) $\sin 25^{\circ}40' = 0.4331$
- (ii) $\sin 55^{\circ}56' = \sin (55^{\circ}50' + 6') = 0.8274 + 0.0010$
/ $\sin 55^{\circ}56' = \underline{0.8284}$

Finding the corresponding angle when the sin value of the angle is given

Example 5

Find the angle whose sin is 0.8864

Note: • Only the relevant part of the table is shown here

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							மெழை ஏர்நெய இடை வித்தியாசங்கள் Mean Differences										
							1'	2'	3'	4'	5'	6'	7'	8'	9'		
	0'	10'	20'	30'	40'	50'	60'										
57	.8387	.8403	.8418	.8434	.8450	.8465	.8480	32	2	3	5	6	8	9	11	13	14
58	.8480	.8496	.8511	.8526	.8542	.8557	.8572	31	2	3	5	6	8	9	11	12	14
59	.8572	.8587	.8601	.8616	.8631	.8646	.8660	30	1	3	4	6	7	9	10	12	13
60	0.8660	0.8675	0.8689	0.8704	0.8718	0.8732	0.8746	29	1	3	4	6	7	9	10	11	13
61	.8746	.8760	.8774	.8788	.8802	.8816	.8829	28	1	3	4	6	7	8	10	11	12
62	.8829	.8843	.8857	.8870	.8884	.8897	.8910	27	1	3	4	5	7	8	9	11	12
63	.8910	.8923	.8936	.8949	.8962	.8975	.8988	26	1	3	4	5	6	8	9	10	12
64	.8988	.9001	.9013	.9026	.9038	.9051	.9063	25	1	3	4	5	6	8	9	10	11

∴ The angle whose sin value is 0.8864 is $62^{\circ} 25'$

Exercise 18.3

(1) Find the sine value of each of the following angles

$26^{\circ} 36'$	$42^{\circ} 23'$	$30^{\circ} 54'$	$52^{\circ} 42'$	$58^{\circ} 45'$
$62^{\circ} 7'$	$42^{\circ} 37'$	$72^{\circ} 50'$	$12^{\circ} 55'$	$80^{\circ} 11'$

(2) Find the angle corresponding to each of the sin values given below.

(i) 0.3717	(ii) 0.9643	(iii) 0.8225	(iv) 0.5555	(v) 0.2372
(vi) 0.7232	(vii) 0.5897	(viii) 0.3227	(ix) 0.4325	(x) 0.6767

18.6 Table of Cosines of angles

Natural Cosines

Observe the natural cosine table given at the end of the text book.

Compare your observations with the following

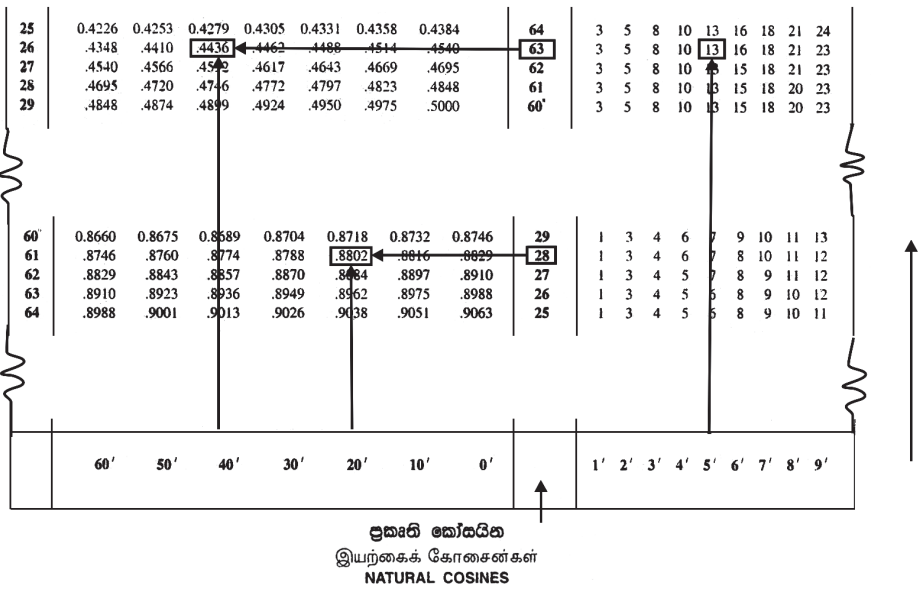
- The value of $\cos 0^{\circ}$ is 1.0000
- The value of $\cos 45^{\circ}$ is 0.7071 (compare with the value of $\sin 45^{\circ}$)
- The value of $\cos 90^{\circ}$ is 0.0000
- When the angle increases from 0° to 90° , the values of cosines gradually decrease from 1.0000 to 0.
- As the cosine value decreases when the angle increases, the mean difference should be subtracted.
- Find the cosine value for a few other angles.

Example 6

- Find the cosine value of $28^{\circ} 20'$
- Find the cosine value of $63^{\circ} 45'$ (Subtract the mean difference)

Note:

- Observe relevant part of the table in the other page



(i) $\cos 28^{\circ} 20' = 0.8802$
(ii) $\cos 63^{\circ} 45' = 0.4436 - 0.0013$
 $\therefore \cos 63^{\circ} 45' = \underline{\underline{0.4423}}$

- Finding the corresponding angle when the cosine value is given

Example 7

Find the angle whose cosine value is 0.5698

- relevant part of the is shown table here
- observe how the value is obtained

33	.5446	.5471	.5495	.5519	.5544	.5568	.5592	56	2	5	7	10	12	15	17	19	22
34	.5592	.5616	.5640	.5664	.5688	.5712	.5736	55	2	5	7	10	12	14	17	19	22
35	0.5736	0.5760	0.5783	0.5807	0.5831	0.5854	0.5878	54	2	5	7	9	12	14	17	19	21
36	.5878	.5901	.5925	.5948	.5972	.5995	.6018	53	2	5	7	9	12	14	16	19	21

	60'	50'	40'	30'	20'	10'	0'		1'	2'	3'	4'	5'	6'	7'	8'	9'
--	-----	-----	-----	-----	-----	-----	----	--	----	----	----	----	----	----	----	----	----

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The approximate value to 0.5698 in the table is 0.5688
 Subtracting the value relevant to the mean difference
 hence the angle of which the cosine is 0.5698 is $55^{\circ} 20' - 4'$
 $= \underline{\underline{55^{\circ} 16'}}$

Exercise 18.4

(1) Find the cosine of the following angles

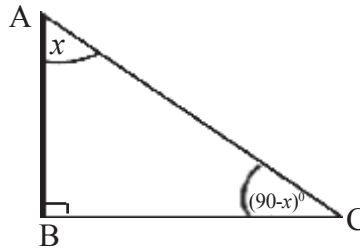
- (i) 36° (ii) $43^{\circ} 12'$ (iii) $74^{\circ} 42'$ (iv) $84^{\circ} 57'$ (v) $44^{\circ} 29'$
 (vi) $55^{\circ} 18'$ (vii) $64^{\circ} 45'$ (viii) $37^{\circ} 27'$ (ix) $22^{\circ} 35'$ (x) $75^{\circ} 35'$

(2) Find the angle of which the cosine is given below

- (i) 0.1435 (ii) 0.2346 (iii) 0.6172 (iv) 0.8457 (v) 0.4428
 (vi) 0.5586 (vii) 0.9928 (viii) 0.4391 (ix) 0.2251 (x) 0.7743

• **Finding the cosine value of an angle using the natural sine table**

In the right angled triangle ABC,



$$\begin{aligned} \text{If } \hat{A} &= x, & \hat{C} &= (90 - x)^\circ \\ \therefore \cos x &= \frac{AB}{AC} & \text{and } \sin (90 - x)^\circ &= \frac{AB}{AC} \\ \therefore \boxed{\cos x = \sin (90 - x)^\circ} \end{aligned}$$

Example 8

When $x = 40^\circ$

$$\cos 40^\circ = \sin (90^\circ - 40^\circ) = \sin 50^\circ$$

When $x = 60^\circ$

$$\cos 60^\circ = \sin (90^\circ - 60^\circ) = \sin 30^\circ$$

Activity 18.4

Find the cosine value of any three acute angles using,

- Natural cosine tables
- Natural sine tables
- Compare the above answers
- If there are differences find the reasons for them
- Discuss with your friends how the cosine value is obtained using the sine table.

Exercise 18.5

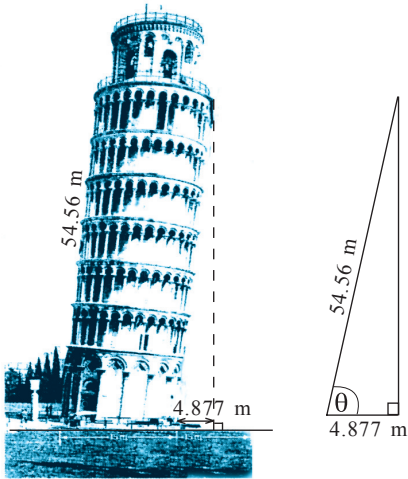
(1) Find the angles whose cosine values are given below:

a) Using cosine table

b) Using sine table

- | | | | |
|------------|-------------|--------------|---------------|
| (i) 0.3145 | (ii) 0.6127 | (iii) 0.4795 | (iv) 0.8527 |
| (v) 0.5555 | (vi) 0.9987 | (vii) 0.3149 | (viii) 0.1272 |

18.7 Using trigonometrical tables



A picture of the famous tower of Pisa is shown here. The measurements taken at a certain time are given; can't we find the inclination of the tower to the ground?

These measurements can be shown in a right angled triangle.

$$\text{As } \cos \theta = \frac{4.877}{54.56}$$

Using the tables

value of θ can be found (According to what is shown here, the inclination of the tower to the ground is about 85° , calculate and verify whether that answer is correct)

Trigonometric calculations can be used to find heights or distances of places that cannot be reached easily. Very often the distances that we have to find using scale diagrams, too can be found by simple calculations using trigonometry.

Example 9

In $\triangle ABC$, $\widehat{B} = 90^\circ$, $\widehat{C} = 37^\circ$, $BC = 6.5$ cm. Find the length of the sides AB and AC .

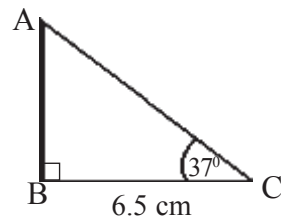
$$\frac{AB}{BC} = \tan 37^\circ$$

$$\frac{AB}{6.5 \text{ cm}} = 0.7536$$

$$AB = 0.7536 \times 6.5 \text{ cm}$$

$$AB = 4.8984$$

$$\underline{\underline{AB = 4.9 \text{ cm}}} \text{ (to the nearest first decimal place)}$$



In $\triangle ABC$

$$\frac{BC}{AC} = \cos 37^\circ$$

$$\frac{6.5}{AC} = 0.7986$$

$$AC = \frac{6.5}{0.7986} \quad (\text{can be found using logarithms})$$

$$= 8.1392$$

$$\underline{\underline{AC = 8.14 \text{ cm}}} \quad (\text{to the nearest second decimal place})$$

AC can be found by using sine table too.

$$\widehat{A} = 90^\circ - 37^\circ = 53^\circ$$

$$\sin 53^\circ = \frac{BC}{AC}$$

$$0.7986 = \frac{6.5}{AC}$$

$$AC = \frac{6.5}{0.7986} \text{ cm}$$

$$= 8.1392$$

$$\underline{\underline{AC = 8.14 \text{ cm}}} \quad (\text{to the nearest second decimal place})$$

Example 10

In triangle ABC, $AB = AC$, $AB = 20 \text{ cm}$, $\widehat{A} = 47^\circ 40'$
 Find the length of the perpendicular drawn from B to AC, Hence find the area of the triangle ABC, find also the length of side BC.

$$\triangle ABX, \widehat{AXB} = 90^\circ$$

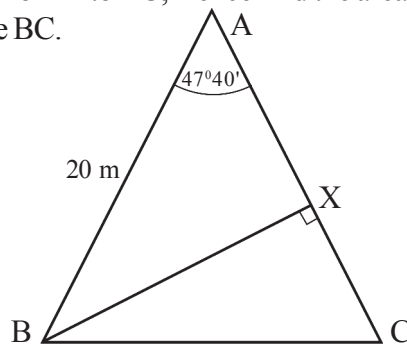
$$\frac{BX}{AB} = \sin 47^\circ 40'$$

$$\frac{BX}{20} = 0.7392$$

$$BX = 0.7392 \times 20 \text{ cm}$$

$$BX = 14.7840 \text{ cm}$$

$$\underline{\underline{BX = 15 \text{ cm}}} \quad (\text{to the nearest whole number})$$



$$\begin{aligned}
 \text{Area of } \triangle ABC &= \frac{1}{2} AC \times BX \\
 &= \frac{1}{2} \times 20 \times 14.784 \\
 &= 10 \times 14.786 \\
 &= 147.86 \\
 &= \underline{\underline{148 \text{ cm}^2}} \quad (\text{to the nearest whole number})
 \end{aligned}$$

$$\begin{aligned}
 \text{In } \triangle ABC \quad \hat{C} &= \frac{180^\circ - 47^\circ 40'}{2} = 66^\circ 10' \\
 \text{In } \triangle BXC, \widehat{BXC} &= 90^\circ \\
 \frac{BX}{BC} &= \sin 66^\circ 10' \\
 \frac{14.784}{BC} &= 0.9147 \\
 BC &= \frac{14.784}{0.9147} = 16.1626 \\
 BC &= \underline{\underline{16.16 \text{ cm}}} \quad (\text{to the nearest second decimal place})
 \end{aligned}$$

Some Greek letters used to name angles

Alpha α

Beta β

Gamma γ

Theta θ

Pi π

Example 11

The inclination of a straight road with a slope is $\frac{2}{7}$. Find the angle, that the road makes with the horizontal.

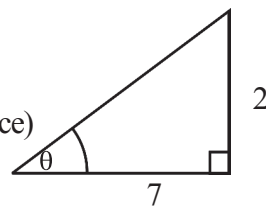
Let θ be the angle between the road and the horizontal

$$\text{Then } \tan \theta = \frac{2}{7} = 0.285714$$

$$\text{Tan } \theta = 0.2857 \text{ (nearest to fourth decimal place)}$$

From the tan table,

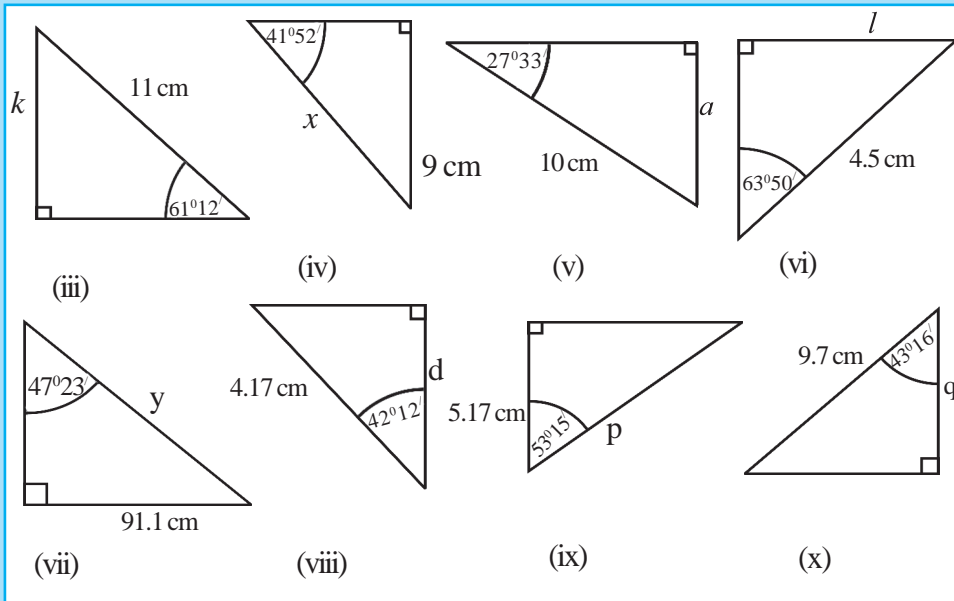
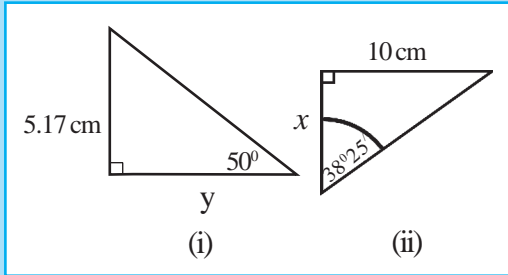
$$\theta = \underline{\underline{15^\circ 57'}}$$



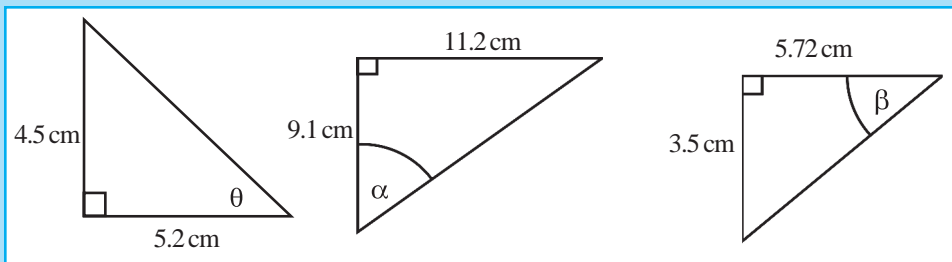
(Observe how the inclination is marked on the diagram)

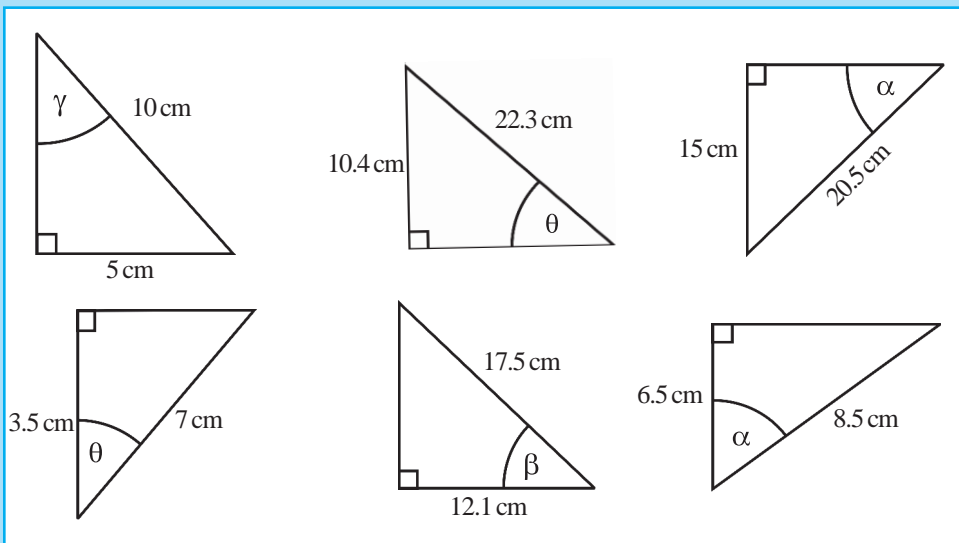
Exercise 18.6

- (1) Find the length of the sides denoted in English letters in each of the triangle given below



- (2) Find the angle denote by Greek letters in each of the trangle given below

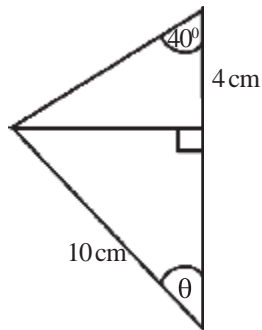




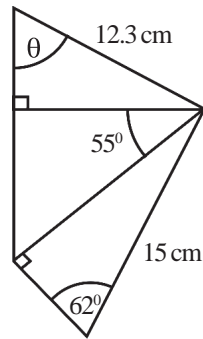
- (3) In ΔABC , $\widehat{B} = 90^\circ$, $BC = 4$ cm, and $\widehat{C} = 40^\circ$ Find AB
- (4) In ΔDEF , $\widehat{E} = 90^\circ$, $\widehat{F} = 35^\circ 30'$ $DE = 7$ cm, Find EF
- (5) ΔGHI , $\widehat{H} = 90^\circ$, $\widehat{I} = 70^\circ$ and $GI = 12$ cm, Find IH
- (6) ΔJKC , $\widehat{K} = 90^\circ$, $\widehat{C} = 55^\circ 20'$ and $KC = 8.21$ cm, Find JC
- (7) ΔMNO , $\widehat{N} = 90^\circ$, $\widehat{M} = 42^\circ 24'$ and $MO = 14.3$ cm, Find ON
- (8) ΔPQR , $\widehat{Q} = 90^\circ$, $\widehat{P} = 28^\circ 35'$ and $PR = 8.23$ cm, Find QR
- (9) ΔRST , $\widehat{S} = 90^\circ$, $ST = 4.5$ cm and $RT = 7.1$ cm Find \widehat{SRT}
- (10) ΔUVW , $\widehat{V} = 90^\circ$, $VW = 5.5$ cm and $UW = 10$ cm Find \widehat{UWV}
- (11) ΔXYZ , $\widehat{Y} = 90^\circ$, $XY = 9$ cm and $YZ = 10$ cm Find \widehat{XYZ}
- (12) ΔLMN , $\widehat{M} = 90^\circ$, $LM = 3.4$ cm and $LN = 15.2$ cm Find \widehat{MLN}

In each of the follow triangle, Find the angle denoted by θ

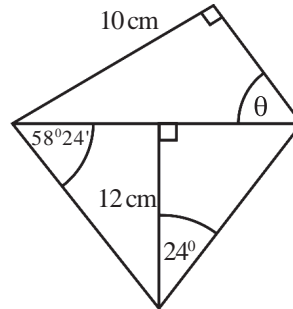
(13)



(14)



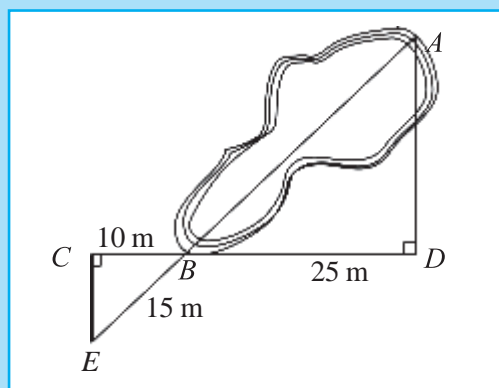
(15)



(16) In an isosceles triangle, the interior angles are 66° , 66° and 48° . The length of the shortest side of the triangle is 8.4 cm. Find the length of the other side and the area of the triangle.

(17) AB and DC are the parallel sides of the trapezium ABCD. $AB = 14$ cm, $DC = 20$ cm and $AD = BC = 6$ cm. Find the value of \hat{ADC} and the area of the trapezium.

(18) Given below are the measurements obtained by a surveyor to find the length of a pond. Using the given information, find the length AB of the pond.

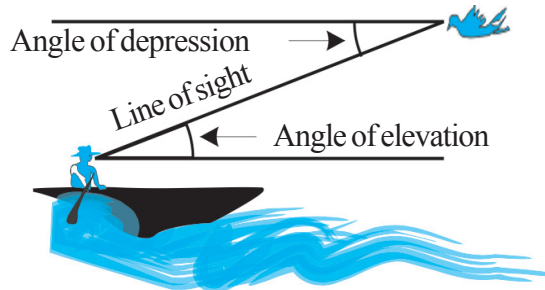


(19) Inclination of a straight road running along the slope of a mountain is $\frac{4}{17}$. Find the angle, the road makes with the horizontal.

- (20) A straight road running along the slope of a mountain is inclined at an angle of 12° to the horizontal. What is the vertical distance from the horizontal to a point 250 m long along this road?

18.8 Directions in a vertical plane

Eye level (Horizontal)

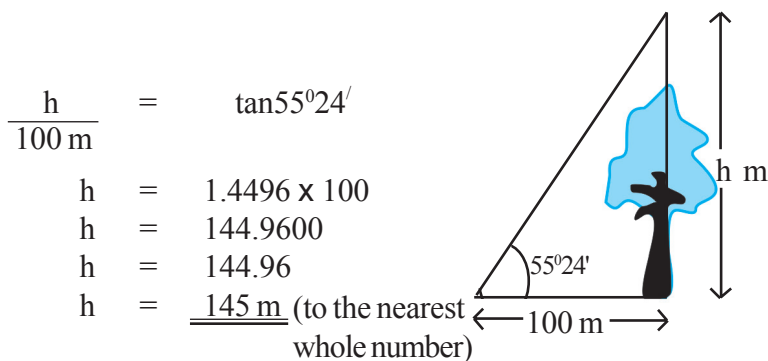


Angle of elevation or the angle of depression can be found using a clinometer.

- An angle similar to the angle at which the man sees the bird is referred to as an angle of elevation.
- An angle similar to the angle at which the bird sees the man is referred to as an angle of depression.

Example 12

A person standing 100 m away from a tree, observes a weather balloon, right above the tree, at an angle of elevation $55^\circ 24'$ from him. Find the height of the balloon above the ground at this moment



Example 13

A man watching from a light house, 60 m high, observes the angle of depression of a boat to be $42^{\circ}40'$. At the same time he notices an aircraft vertically above the boat at an angle of elevation $20^{\circ}27'$.

Calculate:

- (i) The distance (x) from the boat to the foot of the light house
- (ii) The height at which the aircraft was flying at that moment.

If the distance from the boat to the foot of the light house is x ,

From $\triangle ABT$,

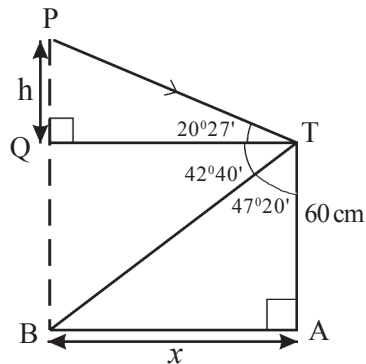
$$\begin{aligned}\frac{x}{60 \text{ m}} &= \tan 47^{\circ}20' \\ x &= 1.0850 \times 60 \\ &= 65.1000\end{aligned}$$

\therefore The distance from the boat to the foot of the light house = 65 m

Also from $\triangle PQT$,

$$\begin{aligned}\frac{h}{65.1} &= \tan 20^{\circ}27' \\ h &= 0.3729 \times 65.1 \text{ m} \\ h &= 24.28 \text{ m} \\ BP &= (h + 60) \text{ m} = (24.28 + 60) \text{ m} \\ BP &= 84.28 \text{ m} \\ \underline{BP} &= \underline{84 \text{ m}}\end{aligned}$$

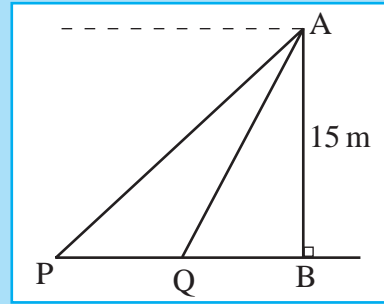
\therefore The air craft was about 84 m above the boat, in the sky.



Exercise 18.7

- (1) A flag post is fixed on a horizontal ground. From a distance of 20 m from the foot of the flag post, the angle of elevation to the top of the flag post is found to be 50° . Find the height of the flag post.

- (2) A person standing on the top of a tower of 15 m high observes two children at points P and Q, of which the angles of depressions are 30° and 60° . Calculate the distance between the two children. (P, Q, B are one straight line.)

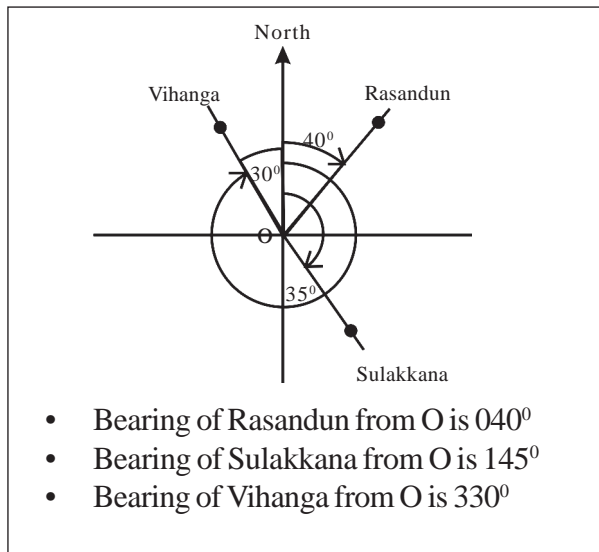


- (3) A ladder is kept against a wall with its upper end just touching the top of the wall. The horizontal distance between the foot of the ladder and the wall is 1.5 m. If the ladder makes an angle of 60° with the ground, find the length of the ladder and the height of the wall.
- (4) A man watching from the top of a tower observes the angles of depression to the top and the bottom of a building of height 7 m to be 45° and 60° respectively. Find the height of the tower.
- (5) If it is found that when the altitude of the sun (angle of elevation of the sun rays) is 45° , a tower, on a horizontal ground, casts a shadow 10 m longer than when the altitude of the sun is 60° . Find the height of the tower.
- (6) A rocket initially travels 10 km vertically upwards. Then it travels 20 km in a direction inclined at 15° to the vertical. Thereafter it travels 60 km in a direction making an angle of 26° to the vertical. Find the vertical height from the starting point to the rocket at the end of the third movement.

-
- (7) A flag post fixed on roof of a building is being observed from a point A on horizontal ground. It was noted that the angles of elevation to the top and the bottom of the flag post from the point A are 47° and 39° respectively. If the flag post is of height 15 m, find the height of the building.
- (8) A helicopter is flying vertically upwards at a uniform speed. A man standing 150 m away on the place where the helicopter was stationed, observes that at a certain moment the angle of the elevation of the helicopter is $48^\circ 36'$ and after 10 seconds the angle of elevation of the helicopter is $72^\circ 24'$. Find the speed at which the helicopter is flying.
- (9) A telephone tower is fixed to the roof of a building 42.5 m away from an observation point. The height of the building is 41 m. The difference between the angle of elevation of the top of the tower and the angle of elevation of the foot of the tower is 15° . Find the height of the telephone tower.
- (10) A person in a boat observes that the angle of elevation of the top of a cliff on the coast, 245 m away from him is 8° . Then he rows his boat a distance of 65 m straight towards the cliff. Find the angle of elevation of the top of the cliff from where he is now.
- (11) When an aircraft is 300 m vertically above a buoy P, the pilot observes another buoy Q at an angle of depression 65° . Find the distance between P and Q. What will be the angle of depression of another buoy R which is placed 150 m from Q on the same direction PQ, as seen by the pilot.

18.9 Directions on a horizontal plane

Bearing

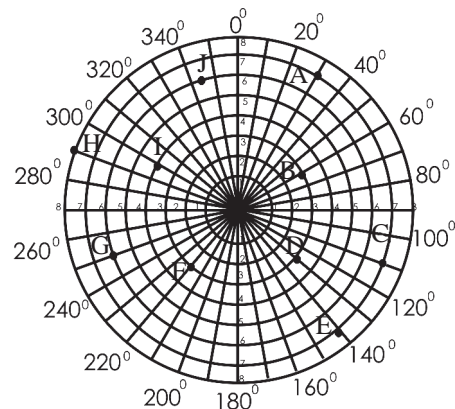


- Bearing can be measured by a clinometer or a compass
- Bearing is always measured starting from the north direction in a clockwise direction and is stated using three digits.
- Trigonometrical ratios can be used in solving problems involving bearing too.

Activity 18.5

Tabulate the bearing of all the points from A to J shown on the radar screen given below. This is prepared by drawing lines making angles of 10° at one point.

Point	Bearing



Example 14

Kavya starts from point A walks on a bearing of 25° for 70 m and comes to point B

How far is Kavya from

- (i) North of A
- (ii) East of A

(i) From $\triangle AXB$,

$$\frac{AX}{70} = \cos 25^\circ$$

$$AX = 0.9063 \times 70$$

$$\underline{AX = 63.4410 \text{ m}}$$

\therefore Kavya is 63.441 m North of A.

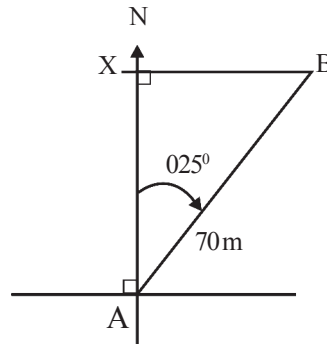
(ii) Similarly from $\triangle AXB$,

$$\frac{BX}{70} = \sin 25^\circ$$

$$BX = 0.4226 \times 70$$

$$\underline{BX = 29.5820 \text{ m}}$$

\therefore Kavya is 29.582 m East of A.



Example 15

A ship sets out from a port and sails due north for 12 km and then another 12 km on a bearing of 080° . What is the distance to the ship from the port now? What is its bearing?

In $\triangle OBA$, $OB = BA$

$$\therefore \hat{BAO} = \frac{180^\circ - \hat{OBA}}{2} = \frac{180^\circ - 100^\circ}{2} = 40^\circ$$

In the right angled triangle AXB,

$$\frac{AX}{12} = \cos 40^\circ$$

$$AX = 0.7660 \times 12$$

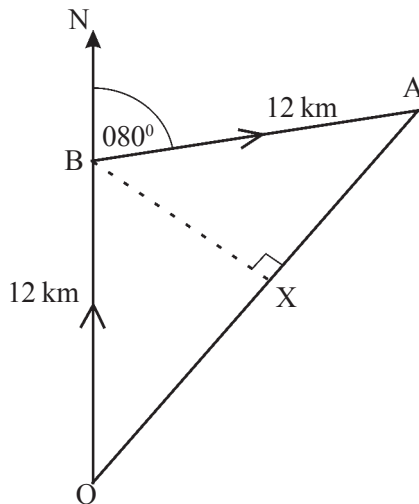
$$AX = 9.1920$$

$$OA = 9.1920 \times 2$$

$$= 18.3840 \text{ km}$$

$$= \underline{\underline{18.4 \text{ km}}}$$

Now the ship is 18.4 km from the port on a bearing of 040° .



Example 16

A ship leaves harbour A on a bearing of 042° and sails for 22 km in this direction. Then it sails on a bearing of 090° for another 30 km and reaches the harbour B. Find the bearing of B from A and also the distance between the two harbours.

From $\triangle ADE$,

$$\frac{DE}{22} = \sin 42^\circ$$

$$\therefore DE = 0.6691 \times 22 = 14.72$$

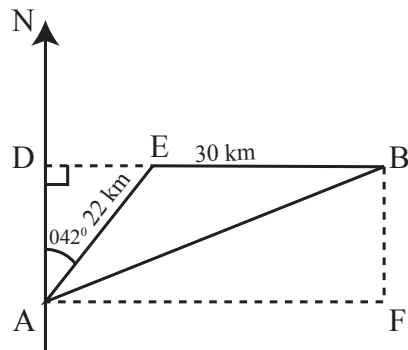
Distance from D to E = 14.72 km

Similarly from $\triangle ADE$,

$$\frac{AD}{22} = \cos 42^\circ$$

$$\therefore AD = 0.7431 \times 22 \text{ km} = 16.3482$$

$$= \underline{\underline{16.35 \text{ km}}}$$



By applying Pythagoras' theorem to right angled triangle ABF,

$$AB^2 = AF^2 + BF^2$$

Also $AF = DE + BE = 14.72 + 30 = 44.72 \text{ km}$

and

$$BF = AD = 16.35 \text{ km} \quad (\text{to the nearest second decimal})$$

$$\therefore AB^2 = (44.72)^2 + (16.35)^2 = 2267.20$$

$$AB = \sqrt{2267.20}$$

$$\underline{AB = 47.6 \text{ km}} \quad (\text{nearest to the first decimal place})$$

$$\hat{D}AB = \hat{A}BF \quad (\text{Alternate angles})$$

$$\tan \hat{A}BF = \frac{AF}{BF} = \frac{44.72}{16.35} = 2.7352$$

$$\therefore \underline{\hat{A}BF = 69^{\circ}55'}$$

\therefore B is 47.6 km from A on a bearing of $069^{\circ}55'$

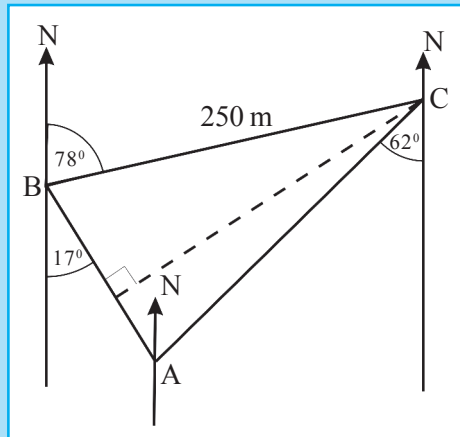
Exercise 18.8

- (1) (a) Find the bearing of each given in the diagram using the information of the following

- (i) C from B
- (ii) A from B
- (iii) B from A
- (iv) B from C

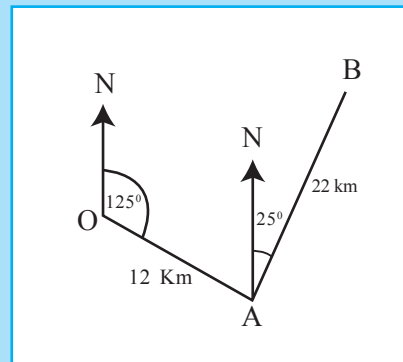
- (b) Find the magnitude of the angles of the triangle ABC

- (c) Find the length of the sides AB and AC of the triangle ABC



- (2) A motor car travels on a bearing of 125° for 12 km and then another 22 km on a bearing of 025° . Find the current location of the motor car with respect to the initial position

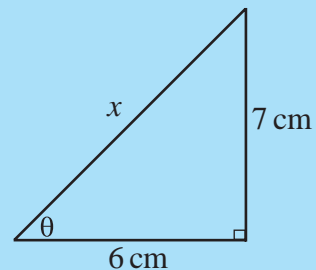
- (i) to the east is the motor car?
- (ii) to the north is the motor car?
(from the initial position)



- (3) On a horizontal ground the tree A is situated 225 m North of the tree C. Tree B is situated 375 m East of the tree C
Find the bearing of (a) B from A (b) A from B
- (4) At a certain instant, three ships A, B and C are located as follows. A is 25 km West of C. B is located to the South of C, B is located $25^{\circ}10'$ to the South of A from East.
- Find the distance between A and B
 - Find the distance between B and C
 - Find the bearing of A from B
- (5) A ship leaves harbour P on a bearing of 306° and sails for 7 km and then sails on a bearing of 070° for 11 km. Find the distance from the ship to the harbour P at this moment.

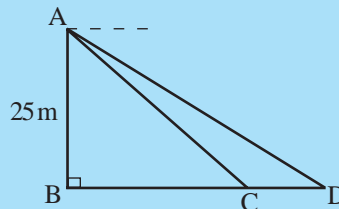
Mixed Exercise 18.9

- (1) According to the information given in the figure,
- Find the value of θ
 - Find the value of x



- (2) In the figure the angle of depression of C from A is 55° and the angle of depression of D from A is 36° . $AB=25$ m
Find,

- BC
- DC
- AD



- (3) In a rectangle, the length of a diagonal is 27 cm. Length of one side is 24 cm. Calculate the angles between a diagonal and the sides.
- (4) Shakya leaves her home at a bearing of 015° and travels for 50 m in this direction and from there she travels 70 m at a bearing of 040° . Then she travels 100 m at a bearing of 330° completing her journey. Find how far North and how far West is she from her home.
- (5) A person rowing a boat towards a lighthouse observes that the angle of elevation to the top of the light house is $24^{\circ}30'$, 320 m away from the light house. Find the angle of elevation to the top of the light house after traveling for 120 m towards the light house. Find the height of the light house?
- (6) On a certain day, at 3.30 pm a ship which was detected 1km due south of a light house at the point A, sailing at a constant speed on a bearing of 060° and at 3.40 pm it was seen at a point B due East of the light house. The shortest distance from the light house to the path of the ship is LC.
- State the above information in a diagram
 - Find the length LC
 - Find the length AC
 - At what time did the ship reach the point C?
- (7) From two points A and B on a straight road running from West to East a ship S is detected on bearings of 060° and 330° .
- Give reasons for $\hat{A}SB$ to be a right angle.
If $AB = 5$ km
 - Calculate the distance AS
 - Calculate the distance BS