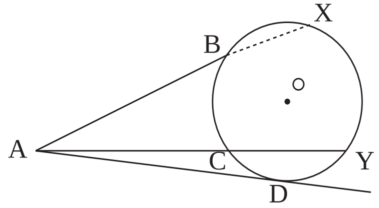
22 Tangents

After studying this chapter you will be able to acquire knowledge and the applications of the following :

- properties of angles related to tangents.
- properties of tangents drawn to a circle from an external point.
- The angle between the tangent and a chord at the point of contact and the angle in the alternate segment.

22.1 Tangent and the point of contact



AB meets the circle and when produced cuts the cricle again at X.
ACY cuts the circle at C and Y.
AD meets the cricle at D and D is the only point common to the AD and the circle.

A line such as AD is known as a tangent to the circle.

When a straight line cuts a circle, if the two intersecting points coincide, then the straight line is a tangent to the circle. This common point is called the points of contact. According to the above diagram, the straight line AD, touches the circle with centre O, at D.

Given below are some important results connecting circles and their tangents.

(A) The shortest distance from the centre of a circle to a tangent drawn to the circle, is the radius at the point of contact.

(B) The line drawn perpendicular to a radius at a point on the circle is a tangent to the circle.

(C) The converse of the theorem (B) is as follows. The tangent drawn to a circle at any point is perpendicular to the radius drawn at that point.

(D) All the points on a tangent, other than the point of contact, lie outside the circle.

22.1 Activity

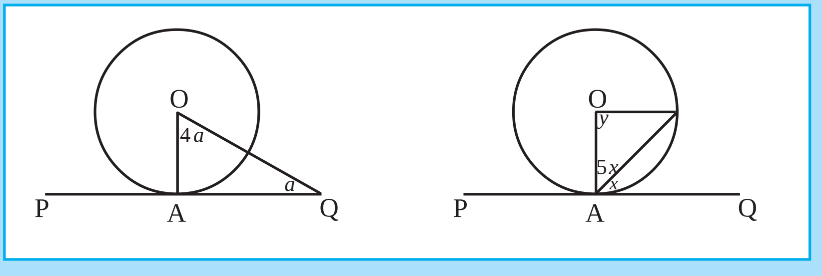
- Draw a circle with centre O and radius 3 cm.
- Mark point A on the circle
- Join OA
- Construct a perpendicular to radius OA at A.
  Name it as PA, and produce PA to Q
- Mark a point X on the line PAQ
- Join OX
- Measure and find the length of OX.
  Can it be written as $OX > OA$?
- Compare the distance from the centre to any point on the line PAQ, other than A, with the radius.

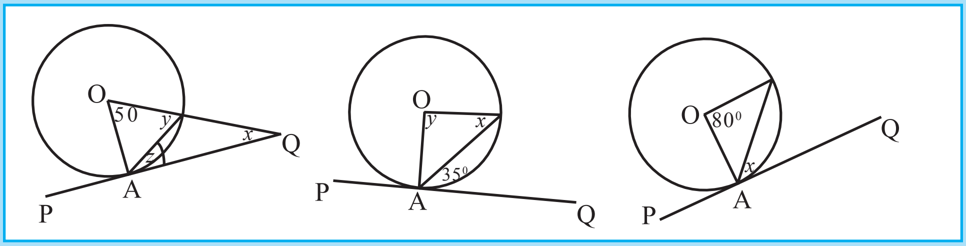
22.2 Activity

- Draw a circle with radius 5 cm on a bristol board and mark its centre as O.
- Mark any point A outside the circle and fix a pin at that point. Tie a piece of thread to the pin and fix another pin at the other end of the thread. Keep the thread taut and move it, when the thread just touches the circle, mark that point as P.
- Join OP. Measure and find the angle $O\hat{P}A$

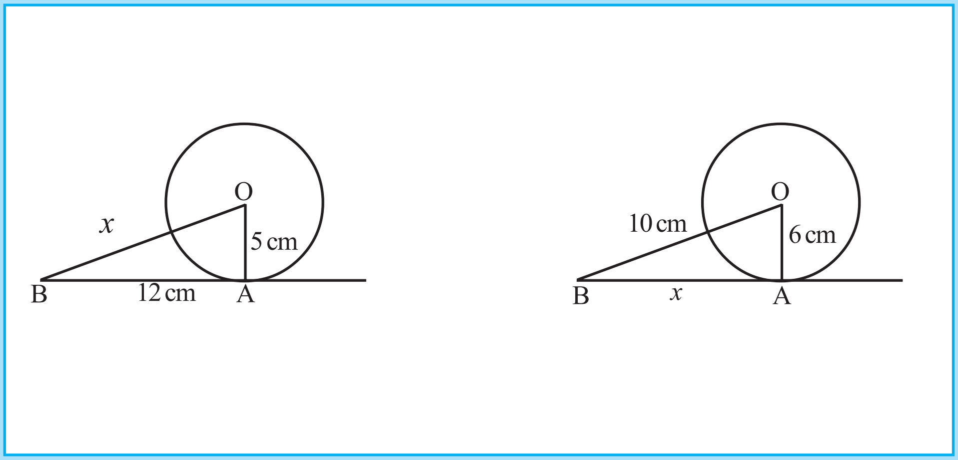
Exercise 22.1

(1) In each of the diagram given below, PAQ is a tangent to the circle. Using the data given in the diagrams find the magnitude of angles denoted by simple letters.

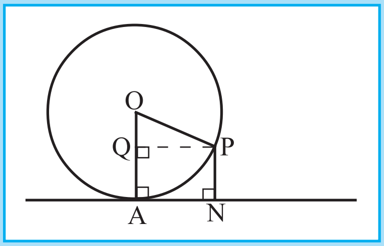




(2) In each of the circle given below, the centre is O and a tangent is drawn at the point A. Find the length marked by x.



(3) AN is the tangent drawn at A to the circle with centre O. $A\hat{N}P = 90^0$
If AN = 15 cm and PN = 9 cm.
(i) Find the length of PQ
(ii) Find the length of AQ
(iii) Find the length of OQ if the radius of the circle is r
(iv) Find the radius of the circle



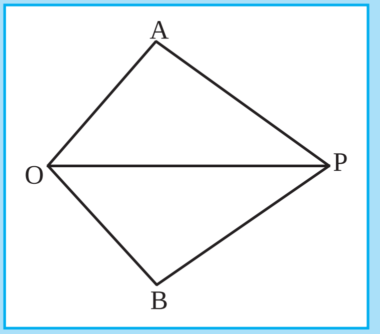
(4) The diagram shows two right angled triangles OAP and BOP. Here OA = OB.

(i) Show that Δ OPA and Δ BOP are congruent

(ii) Name an angle equal to $O\hat{P}A$

(iii) Name an angle equal to $A\hat{O}P$

(iv) Name a side equal in length to AP



(5) O and C are centres of two intersecting circles. OC produced cuts the circle with centre C at T. The two circles intersect at A and B. $A\hat{O}B = 60^0$

(i) Join OA, OB, TA, TB

(ii) Show that Δ OAT and Δ OBT are congruent

(iii) Name an angle equal to angle $O\hat{T}A$

(iv) Name an angle equal to $T\hat{O}A$

22.2 Tangents drawn to a circle from an external point

22.3 Activity

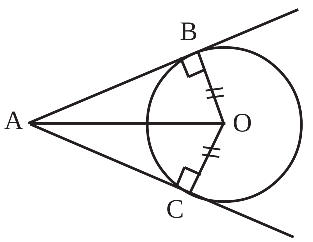
- Draw a circle with radius 3 cm. Mark its centre as O
- Mark two points B and C on the circle, as not to lie on the same diameter. Draw two tangents to the circle at these points.
- Mark the intersection point of the two tangents as A
- Join OA

- Cut the triangles AOB or AOC thus formed
- See whether the two triangles coincide
- State the conclusion that can be made from this

Two tangents can be drawn to a circle from an external point. The theorem given below related to these tangents is very important.

Theorem
When two tangents are drawn to a circle from an external point, then
(i) two tangents are equal in length
(ii) the angles subtended by the tangents at the centre of the circle are equal
(iii) the line joining the centre to the external point bisects the angle between the tangents.

Data : AB and AC are tangents to the circle with centre O. B and C are the points of contact.



To prove that : (i) AB = AC

(ii) $A\hat{O}B = A\hat{O}C$

(iii) $O\hat{A}B = O\hat{A}C$

Proof : $O\hat{B}A = O\hat{C}A = 90^0$ (Tangent is perpendicular to the radius)
In right angled triangles OBA and OCA
OB = OC (radius)
OA = OA (common side)
$\therefore \Delta\ OAB \equiv \Delta\ OCA$ (hypotenuse-side)

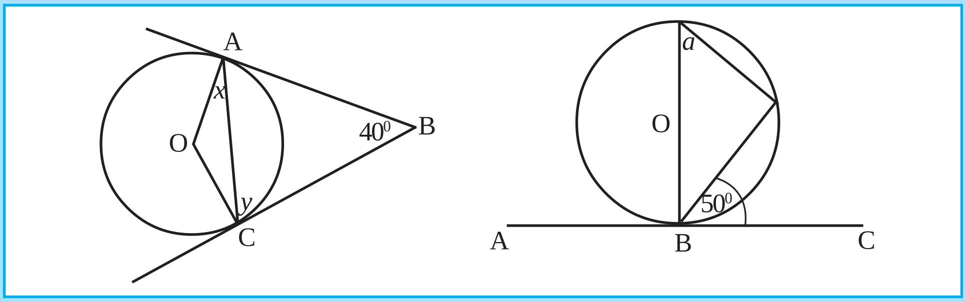
$\therefore$ AB = AC

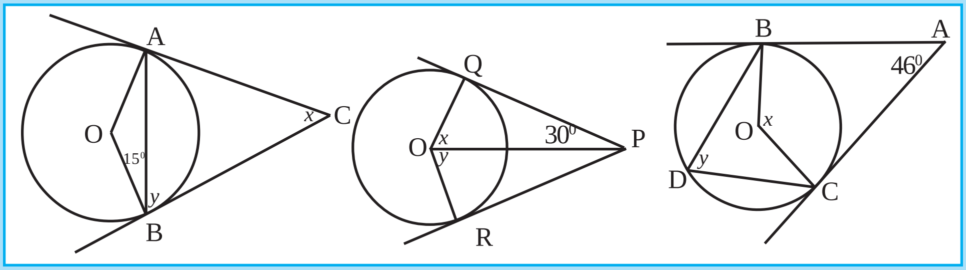
$A\hat{O}B = A\hat{O}C$

and $O\hat{A}B = O\hat{A}C$

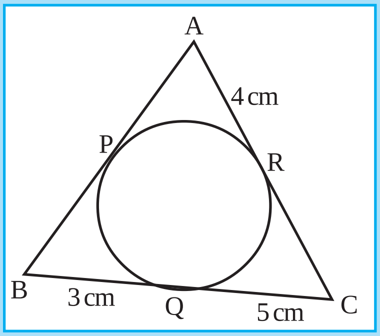
Exercise 22.2

(1) In each diagrams, the tangents drawn to a circles from an external point are shown. Find the angles marked by symbols.



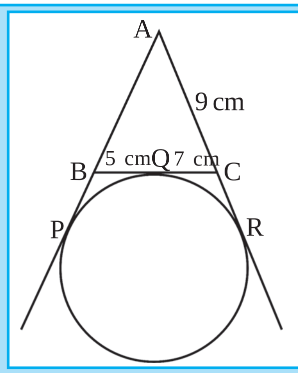


(2) AB, BC and CA are tangents to a circle.



- (i) Find the length of AP
- (ii) Find the length of BP
- (iii) Find the length of CR
- (iv) Find the perimeter of the triangle ABC

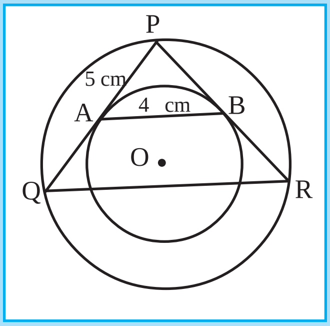
(3) AP, AR and BC are tangents to a circle



- (i) Find the length of BP
- (ii) Find the length of CR
- (iii) Find the length of AR
- (iv) Find the length of AP
- (v) Find the length of AB

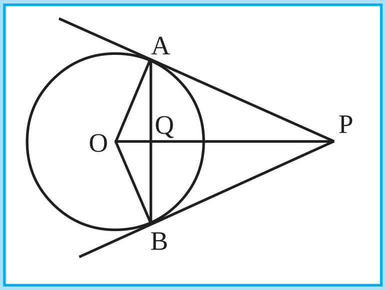
(4) O is the centre of two concentric circles. PQ and PR are the two tangents to the inner circle. P, Q and R are point on the outer circle. PQ = 10 cm PR = 10 cm

(i) Find the magnitude of $O\hat{A}P$
(ii) Find the length of BR
(iii) Find the length of PB
(iv) Find the length of QR
(v) Find the perimeter of Δ PQR



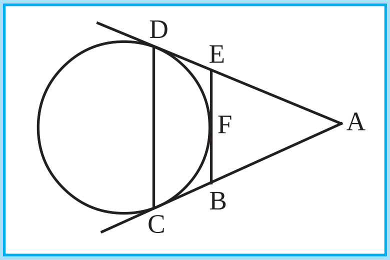
(5) PA and PB are the tangents to the circle with centre O.

(i) Show that $P\hat{A}Q = P\hat{B}Q$
(ii) Show that AQ = BQ
(iii) Show that $P\hat{Q}A = P\hat{Q}B$
(iv) Find the magnitude of the angle $P\hat{Q}A$
(v) Is OP, the perpendicular bisector of AB ?

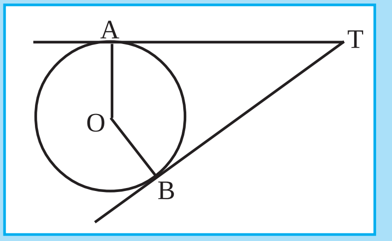


(6) As shown in the diagram, CBA, DEA and BFE are tangents to the circle at the points C, D, and F
E is on AD and B is on AC.

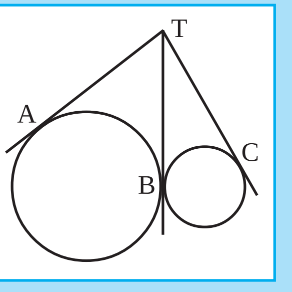
(i) Write AD as the sum of two line segments.
(ii) Write AC as a sum of two line segments
(iii) Name a line segments equal in length to ED
(iv) Name three line segments equal in length to BC
(v) Show that AD + AC = AB + BE + AE



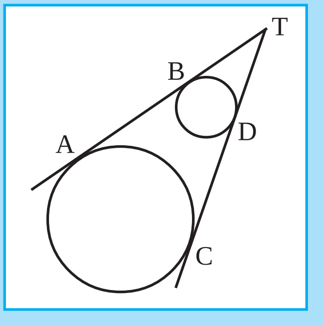
(7) TA and TB are two tangents drawn to the circle with centre O from an external point T. Show that AOBT is a cyclic quadrilateral.



(8) As shown in the diagram, TB is the common tangent to the two circles. TA is a tangent to the big circle and TC is a tangent to the small circle.
Show that TA = TC

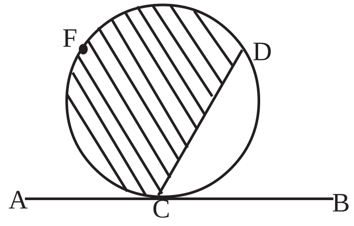


(9) TBA and TDC are two common tangents to the two circles as shown in the diagram. showthat
(i) AB = CD
(ii) AC // BD

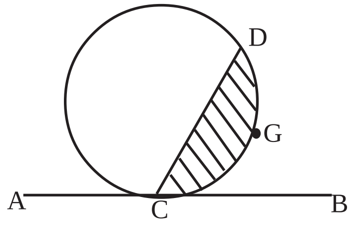


22.3 Angles in the alternate segment

(i) The shaded part of the circle, DFC which is opposite to the angle $B\hat{C}D$ is the alternate segment with reference to $B\hat{C}D$.



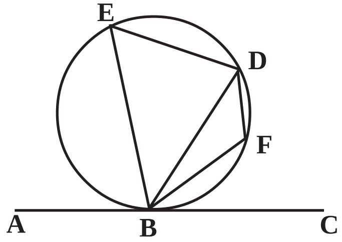
(ii) The shaded part of the circle DGC which is opposite to $A\hat{C}D$ is the alternate segment with reference to angle $A\hat{C}D$.



Now we will see what is meant by an alternate segment and an angle in the alternate segment in a circle.

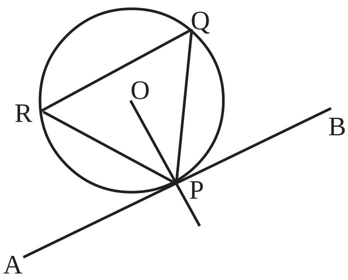
(iii) In the figure below, ABC is a tangent to the circle with centre O. The point of contact is B. BD is a chord. BD divides the circle into two segments.

- $D\hat{E}B$ is the angle in the alternate segment with reference to angle $D\hat{B}C$
- $D\hat{F}B$ is the angle in the alternate segment with reference to angle $D\hat{B}A$



22.4 Activity

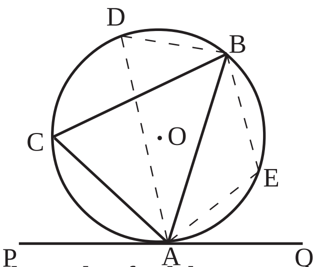
- Draw a circle with centre O and radius 5 cm on a bristol board
- Mark a point P on the circle. Draw a tangent at P. Name it as APB
- Draw a chord PQ
- In the alternate segment of $Q\hat{P}B$ mark a point R on the circumference
- Complete the triangle PQR
- Cut out the angle $Q\hat{P}B$ and keep it on Δ PQR , it will coincide with $P\hat{R}Q$
- What is the conclusion that can be arrived from the result ?



22.5 Activity

The centre of a circle is O. The tangent PAQ touches the circle at A.

$Q\hat{A}B = 60^0$



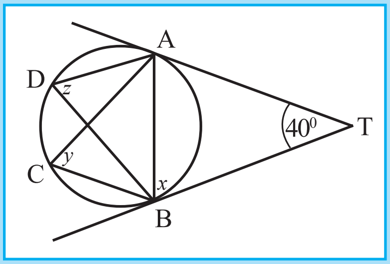
(i) What is the value of $A\hat{D}B$.
State reasons
(ii) Find the value of $B\hat{C}A$
(iii) Find the value of $B\hat{A}Q$
(iv) What is the value of $B\hat{C}A$. Write the result used to find this
(v) Are the angles in the alternate segment of angle $B\hat{A}Q$ equal ?
(vi) What is the value of $B\hat{E}A$. Write the result used to find this
(vii) Find the value of $B\hat{A}P$
(viii) Are the angles in the alternate segment of $B\hat{A}P$ equal ?

Theorem

When a straight line touches a circle and a chord is drawn from point of contact the angles made by the chord and the tangent are equal to angles in the alternate segments.

Exercise 22.3

(1) Tangents drawn to a circle at A and B intersect at T.



(i) Name a line segment equal to BT

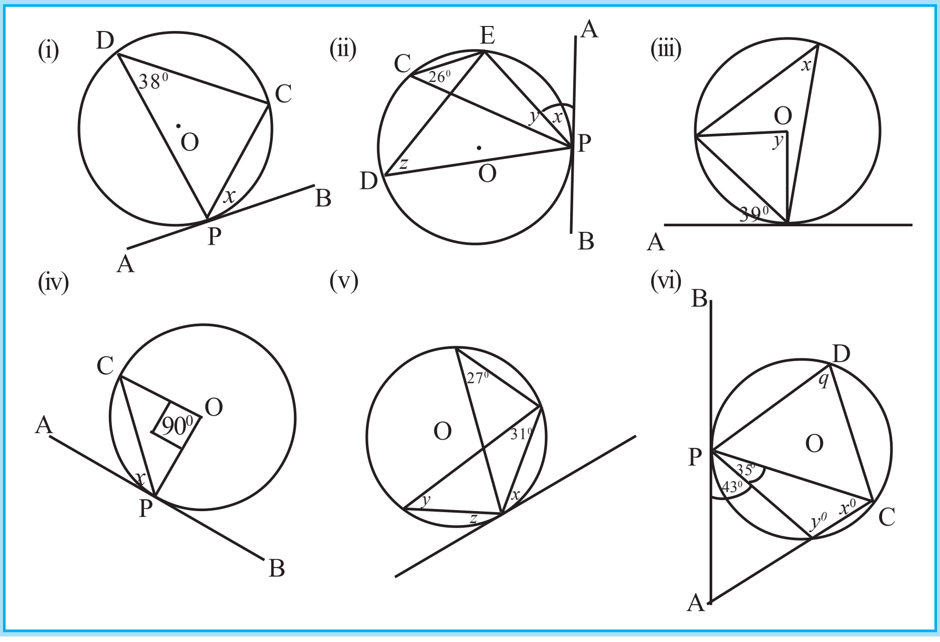
(ii) What type of a triangle is Δ ATB ?

(iii) What is the value of x ?

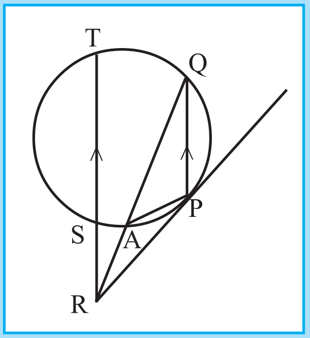
(iv) Name the angles in the alternate segment to ABT

(v) Find the value of y and z

(2) In each diagram given below the tangent drawn at the point P on the circle with centre O is APB. Using data given in each diagram, find the value of angles denoted by symbols.



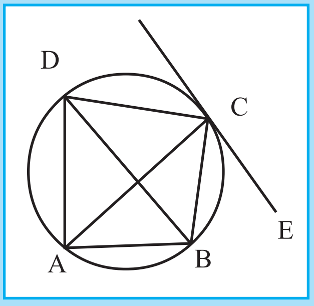
(3) A, P, Q, T, S are points on a circle. RST and RAQ are secants. PR touches the circle at P. PQ // RT



(i) Name an angle equal to angle $R\hat{P}A$.

(ii) Name an angle equal to angle $P\hat{Q}A$

(4) ABDC is a cyclic quadrilateral. AC is the bisector of angle $\hat{A}$. CE is the tangent drawn at C.



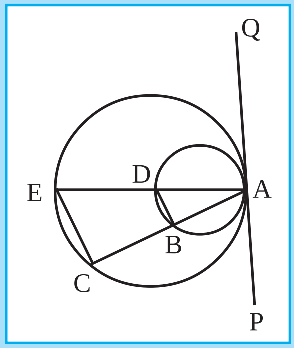
(i) Name two angles in the alternate segment with reference to angle $B\hat{C}E$

(ii) If $B\hat{C}E = x^0$, mark two angles equal to x in the alternate segment

(iii) Show that $D\hat{A}C = D\hat{B}C$

(vi) show that BD // CE ?

(5) Two circles touch internally at A. The common tangent at A is PAQ. E and C are two points on the larger circle. AE and AC cuts the smaller circle at D and B.



(i) What is the angle smaller circle equal to $Q\hat{A}D$?

(ii) What is the angle in the bigger circle equal to $Q\hat{A}D$?

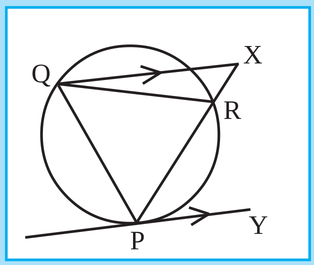
(iii) Show that BD // CE ?

(iv) If AB = BC, show that AD = DE

(v) If it is given that $D\hat{A}Q = 90^0$, show that the centre of the bigger circle is D

(7) In the diagram the tangent PY touches the circle at P. Q and R are two points on the circle. Line drawn through Q parallel to PY meets PR produced at X.

Show that $P\hat{Q}R = P\hat{X}Q$



(8) K, L and M are point on a circle. The tangent drawn at L meets KM produced at P.

Show that $K\hat{L}P = L\hat{M}P$