20 Inequalities

After studying this chapter you will be able to acquire knowledge and the applications of :

- represent the solution of an inequality on a number line.
- solve inequalities of the form $ax + b > cx + d$
- solve problems involving inequalities.

20.1 Introduction

You will have to refresh your knowledge about solving equations before you start studying inequalities.

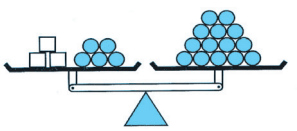
Solving the equation $3x + 5 = 14$ is shown below.

The unkonwn term in the above equation is x. The aim of solving the equation is to find the value of x.

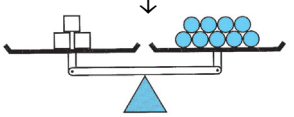
The balance scale is a good model to solve equations. See how the equation $3x + 5 = 14$ is represented in the balance.

Here represents the unknown term x.

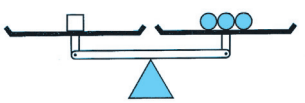
Visual representation



Remove 5 beads from each side



Separate the beads into portions of three's and the boxes into portions of one's



Algebraic representation

$3x+5=14$

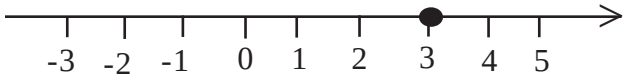
step 1 $3x+5-5=14-5$ — Subtract 5 from each side

$3x=9$

step 2 $\frac{3x}{3}=\frac{9}{3}$ — Divide both sides by 3

$x=3$

The value of x which satisfies the equation $3x + 5 = 14$ is 3. In a linear equation there is only one value to satisfy the unknown term. ie. the solution set of a linear equation will have only one element. Now see how that value is shown on a number line. On the number line, the point showing 3 is shaded.



When solving equations, to isolate the unknown term x, the other numbers are removed. Here to remove + and - first the additive inverse is added to each side. Then to remove divisions and multiplications we apply the inverse mathematical operations.

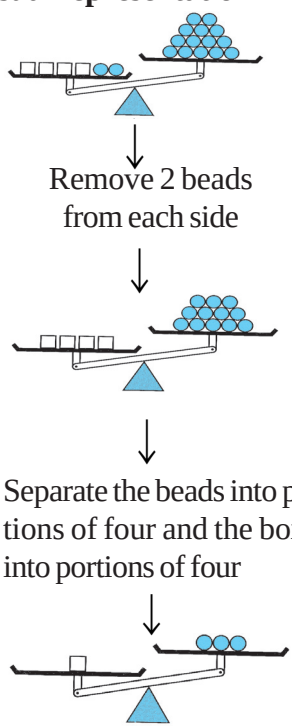
20.2 Inequalities of the form $ax + b > c$

Now look at the inequality given below.

$$4x + 2 > 14$$

Here we have to find the values that x can have. So that the expression on the left hand side $4x+2$ will be greater than 14. We can isolate x, by applying inverse mathematical operations adding, subtracting, multiplying, dividing the same way we did in solving equations.

Visual representation



Algebraic representation

$4x+2>14$

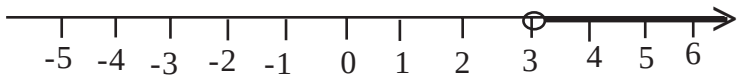
step 1 $4x+2-2>14-2$ — Subtract 2 from each side

$4x>12$

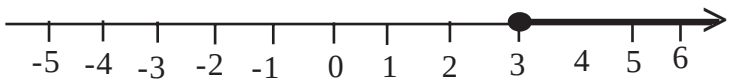
step 2 $\frac{4x}{4} > \frac{12}{4}$ — Divide both sides by 4

$x>3$

This means that for any value of $x > 3$, $4x + 2 > 14$. Hence we can see that for an inequality, there are many solutions. ie. the solution set of x will have an infinite number of elements. It is represented on a number line as shown below. Here x cannot take the value 3. Therefore the point $x = 3$ is circled but not shaded.



Now when we solve the inequality $4x + 2 \geq 14$ in the same way the solution is $x \geq 3$. Therefore $x = 3$ is also included in the solution set and is represented on a number line as shown below. The point $x = 3$ is circled and shaded.



But every inequality cannot be solved the way the equations are solved. Look at the problem given below.

$-2x = 10$ To solve this we multiply both sides by multiplicative inverse of (-2)

$$-2x \times \left(\frac{-1}{2}\right) = 10 \times \left(\frac{-1}{2}\right)$$

$x = -5$

Now look at the inequality $-2x > 10$

When we multiply both sides by the multiplicative inverse of (-2) the answer is $x > -5$

Then you will think that the value of x should be greater than -5.

Thus as $x = -4$ is a solution, substitute $x = -4$ in

$-2x > 10$

Left hand side = $-2x$ Right hand side = 10

$= -2 \times (-4)$

$= 8$

ie $8 > 10$ which is not true.

Therefore when an inequality is divided or multiplied by a negative number, the inequality sign should be changed to the opposite direction.

See the examples given below.

Example

-15 > -40 is true

Divide both sides by -5 and change the inequality to the opposite direction.

$$\frac{-15}{-5} < \frac{-40}{-5}$$

$3 < 8$ This is true.

Example

-6 < -2 This is true.

Multiply both sides by (-4) and change the inequality sign.

$(-6) \times (-4) > (-2) \times (-4)$

$24 > 8$ which is true.

If you did not change the inequality sign, then you will get $24 < 8$ which is not true.

If $a > b$

(i) $a + c > b + c$

(ii) $a - c > b - c$

If $c > o$, $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

If $c < o$, $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

Do the following exercise to revise what you have learnt earlier.

Exercise 20.1

(1) Substitute and see whether the given value of x satisfies each expression.

(i) $x + 2 > 5$, $x = 3$ (ii) $2x + 1 < 3$, $x = 2$

(iii) $2x + 1 < 3$, $x = 0$ (iv) $3x - 2 > -8$, $x = 3$

(v) $3x - 2 > -8$, $x = -3$ (vi) $3x - 4 < 5$, $x = 4$

(2) By dividing and multiplying each inequality by the given number, see how the inequality changes

(i) $-10 < 60 \quad (-2)$ (v) $-18 < -12 \quad (-3)$

(ii) $-20 < 8 \quad (-4)$ (vi) $20 < 30 \quad (-5)$

(iii) $-12 < 8 \quad (-2)$ (vii) $20 < 30 \quad (5)$

(iv) $-12 < 8 \quad (2)$ (viii) $24 > -6 \quad (-2)$

(3) Show the solution set of each inequality on a number line.

(i) $3x + 2 > 5$ (ii) $2x - 5 > 3$ (iii) $2x + 1 > -5$

(iv) $4x - 3 > -7$ (v) $5x + 5 > -7$

(4) Show the solution set of each inequality on a number line.

(i) $2x - 7 < 3$ (ii) $3x + 5 < 2$ (iii) $4x + 3 < -5$

(iv) $5x - 3 < -13$ (v) $4x - 1 < 0$

(5) Show the solution set of each inequality on a number line.

(i) $4x - 3 \geq 5$ (ii) $2x + 1 \leq 5$ (iii) $6x + 5 \leq -7$

(iv) $3x - 7 \leq 7$ (v) $2x - 3 \geq 7$

(6) Nimal has Rs 155. He bought 5 mangoes at the rate of Rs x each and 4 apples at the rate of Rs 30 each. Build up an inequality on this information and find the maximum price of a mango.

(7) In a bag there are x number of 500 g sugar packets and 8 of 100 g tea packets. If the total weight of the bag is less than 2 kg, What is the maximum number of sugar packets it can have ?

(8) The minimum mass of a loaf of bread should be 450 g. If 5 slices weight 40 g and 5 slices weight x g are cut from a loaf, what is the maximum value of x.

Example 1

Show the solution set of $7-3x \geq 1$ on a number line.

$$7-3x \geq 1$$

Additive inverse of 7 i.e. -7 is added to both sides

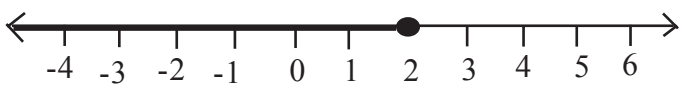
$$7-3x-7 \geq 1-7$$

$$-3x \geq -6$$

Now diving both sides by (-3)

$$\frac{-3x}{-3} \leq \frac{-6}{-3}$$ (Inequality sign is changed)

$$x \leq 2$$



i.e. 2 or any value less than 2 satisfies the inequality $7-3x \geq 1$

Verify $7-3x \geq 1$

Left hand side $= 7 - 3x$ Right hand side $=1$

$= 7-3\times(-5)$

$= 7+15$

$= 22$

Thus $7-3x \geq 1$

$\therefore$ Left hand side $\geq$ Right hand side

Example 2

(1) Show the solution set of the following inequalities on a number line.

(i) $2-3x>-5$ (ii) $5-x<6$

(iii) $6-5x\geq 3$ (iv) $8-3x\leq -4$

(v) $11-2x<-1$ (vi) $1-5x>11$

(2) Vimal bought 5 mangoes at Rs x each and 12 plantains at Rs 2 each. Kamala bought 3 mangoes at Rs x each and 10 plantains at Rs 3 each. If the amount spent by Vimal is more than the amount Kamala spent, what is the minimum price of a mango. If x is a whole number.

(3) Parcel A contains 6 cups and sauces each weighing 125 g and 3 tumblers each weighing x g. Parcel B contains 2 plates each weighing 300 g and 6 tumblers each weighing x g. If the mass of parcel A is less than that of B what is minmum mass of a tumbler?

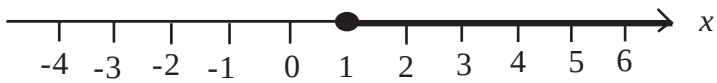
(4) Ravindra fills the tank A with 7 pots full of water and 10 buckets full of water. Leelasena uses the same pot and the same bucket and fills the tank B with 10 pots of water and 2 buckets of water. Capacity of the pot is 8 litres and the capacity of the bucket is x litres. If the tank A has more water than tank B what is the minimum capacity that the bucket can have?

(5) Motor car A travels at a speed of x kmh^{-1} for 2 hours and at a speed of 40 kmh^{-1} for 3 hours. Motor car B travels at a speed of x kmh^{-1} for 4 hours and at a speed of 25 kmh^{-1} for 4 hours. If B has travelled a distance greater than A What is the minimum value of x. ?

(6) Sunila has 15 blue tiles of area 16 cm^2 of each and 10 white tiles of area x cm^2 of each. Ranjani has 20 blue tiles and 5 white tiles with same area. If the total area of the tiles Sunila has is more than the total area of tiles Ranjani has, What is the minimum area of a white tile. ?

Express the solution set of $3x + 4 \geqslant x + 6$ on a number line.

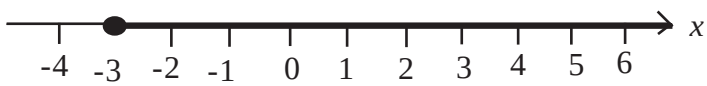
$$\begin{aligned} 3x + 4 &\geqslant x + 6 \\ 3x + 4 - 4 &\geqslant x + 6 - 4 \\ 3x &\geqslant x + 2 \\ 3x - x &\geqslant x + 2 - x \\ 2x &\geqslant 2 \\ x &\geqslant 1 \end{aligned}$$



Example 3

Express the solution set of $2x - 8 \leqslant 5x + 1$ on a number line

$$\begin{aligned} 2x - 8 &\leqslant 5x + 1 \\ 2x - 8 + 8 &\leqslant 5x + 1 + 8 \\ 2x &\leqslant 5x + 9 \\ 2x - 5x &\leqslant 5x + 9 - 5x \\ -3x &\leqslant 9 \\ \frac{-3x}{-3} &\geqslant \frac{9}{-3} \qquad \text{(Inequality sign is changed)} \\ x &\geqslant -3 \end{aligned}$$



Exercise 20.2

Exercise 20.3

(1) Represent the solution sets of the following inequalities on a number line.

(i) $3x+5>x-1$ (ii) $4x-3<2x+1$

(iii) $5x-2\le 2x-8$ (iv) $3x+2\ge x+6$

(v) $7x+4>3x$

(2) Represent the solution sets of the following inequalities on a number line.

(i) $3x-1>5x+1$ (ii) $x+2\le 4x-1$

(iii) $2x+3\ge 6x-5$ (iv) $3x-4>5x$

(v) $x-7<2x+1$

(3) Represent the solution sets of the following inequalities on a number line.

(i) $3-2x>3x-7$ (ii) $4-x\ge x-4$

(iii) $5+3x\le x+1$ (iv) $7+2x>3+x$

(v) $3x+7\ge -3-2x$

(4) Represent the solution sets of the inequalities below on a number line and write 3 whole number solutions for each inequality.

(i) $2x+3<x-1$ (ii) $3x-1\ge 7-x$

(iii) $5x-2>2x+4$ (iv) $4x+5\ge x+8$

(v) $3x+7\le -3-2x$

(5) Find the minimum whole number value of x that satisfies the inequality $3x-2\ge 8-2x$.

(6) Find the minimum whole number value of x that satisfies the inequality $6x-7>2x+1$.

(7) Does the set of values of x which satisfy the inequality $5-3x\le 6-4x$ have a maximum value or a minimum value? Find it.