19 Matrices

After studying this chapter you will be able to acquire knowledge and the application of :

- the concept of matrices
- the addition of matrices
- the subtraction of matrices
- Multiplying a matrix by an integer

19.1 Introduction of Matrices

Numbers are used to give information on magnitude of a quantity. Matrices can be used to give information on magnitude of two or more quantities. Such an occasion is given below.

To three students Kamal, Akhila and Niroshan in a grade eleven class, the mathematics teacher gave each of them a parcel as prize for their abilities shown in mathematics in the first term test.

There were,

6 exercise books and 2 pens in Kamal's parcel
4 exercise books and 3 pens in Akhila's parcel
5 exercise books in Niroshan's parcel.
There were no pens in Niroshan's parcel

Let us show this information in a table.

	Exercise books	Pens
Kamal	6	2
Akhila	4	3
Niroshan	5	0

There are three rows and two columns in this table. The order of rows are Kamal, Akhila and Niroshan and the order of columns are exercise books and pens. Hence the information in the table above can be forwarded as below

$$\begin{pmatrix} 6 & 2 \\ 4 & 3 \\ 5 & 0 \end{pmatrix}$$

Thus in a formation of numbers arranged in a rectangular way is called a matrix. The numbers in a matrix are in rows and columns and are enclosed with a bracket. The numbers included here are called elements or components.
Some examples for matrices are given below:

Example 1

(i) $\begin{pmatrix} 2 & 1 & 3 \end{pmatrix}$ (ii) $\begin{pmatrix} -1 & 5 & 7 \\ 2 & -1 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} 2 & -1 \\ -1 & -3 \end{pmatrix}$ (iv) $\begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$

19.2 Order of a matrix

There are 3 rows and 2 columns in the information matrix that was mentioned earlier. Therefore it is written as 3 x 2 matrix. It is also called as a three by two matrix. 3 x 2 is expressed as the order of the matrix. Hence the order of a matrix shows the number of rows and the number of columns in that matrix.

Now let us examine the order of some matrices:

(i) $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 4 & 3 \end{pmatrix}$ order of the matrix is 2 x 3

(ii) $\begin{pmatrix} 2 & 4 & 1 \end{pmatrix}$ order of the matrix is 1 x 3, (a matrix of one by three) there is only one row. So it is called a **row matrix.**

(iii) $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ order of this matrix is 2 x 1
(a matrix of two by one) it has only one column. So that it is called a **Column matrix**.

(iv) $\begin{pmatrix} 3 & 1 & 2 \\ 0 & 4 & 6 \\ 2 & -1 & 5 \end{pmatrix}$ order of the matrix is 3 x 3
(a matrix of three by three) It has equal number of rows and columns.Therefore is called a **Square matrix**.

(v) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ This is a square matrix of the order 2 x 2. All elements of the main diagonal are 1 and the other elements are zero. It is called a **Unit matrix**.

(vi) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ This is also a unit matrix of order 3 x 3.

> If the number of rows is *m* and the number of columns is *n* of a matrix, A, then the order of A is *m* x *n*

19.3 Naming matrices

Generally a matrix is denoted in English capital letters

Example 2

(i) $A = \begin{pmatrix} 3 & -1 \\ 1 & 0 \\ 2 & 4 \end{pmatrix}$

(ii) $B = \begin{pmatrix} -3 & 1 & 0 \end{pmatrix}$

(iii) $C = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$

(iv) $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Unit matrix is always named as I

19.4 Equal matrices

When the corresponding pairs of elements are equal in two equal order matrices, they are called **Equal matrices**.

Example 3

(i) When $p = a$, $q = b$, $r = c$, and s = d

$$X = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \quad \text{If and} \quad Y = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

then X and Y are equal matrices. It can be written as $X = Y$

(ii) $$P = \begin{pmatrix} 3 & 4 \\ -1 & 5 \\ 0 & 2 \end{pmatrix} \quad \text{and} \quad Q = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$$

Find a, b, c, d, e, f when P = Q

The order of P and Q are the same .And P = Q only when the corresponding elements are equal.

$\therefore$ $a = 3$, $b = 4$, $c = -1$, $d = 5$, $e = 0$, $f = 2$

In equal matrices the order is equal while the corresponding elements are equal

Exercise 19.1

(1) Determine whether the following expressions are true or false.

(i) The number of elements in a matrix including 3 rows and 2 columns is 6.

(ii) The order of a matrix is denoted by combining the number of rows and number of columns with the sign of multiplication.

(iii) (3 1) is a row matrix. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is a column matrix.

(iv) Order of a matrix with m rows and n columns is $m \times n$.

(v) The order of a certain matrix is $n \times 1$. This is a column matrix.

(vi) $\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$

(vii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is a square matrix

(viii) A and B are two matrices. If A = B, the order of A = order of B

(2) In a mathematical test Sanjeewa got 45, Rajan got 58 and Sarojni got 51 marks. Express this information in a column matrix.

(3) A_3, A_4 and A_5 are three sizes of papers internationally recognized. The length of an A_3 paper is 42 cm and the width is 29.7 cm this information can be expressed in a row matrix as [(42 29.7)].

(i) The length of an A_4 paper is 29.7 cm and the width is 21 cm. Express this in a row matrix.

(ii) The length of an A_5 paper is 21 cm and the width is 14.8 cm. Express this information in a row matrix.

(iii) Express the information of the three types of papers A_3 A_4, and A_5 in a 3×2 matrix.

(iv) Express the lengths of these three types of papers in a column matrix and widths in a column matrix.

(4) In a housing complex there are three types of houses named H, M and L. In a H type house there are 4 bedrooms, 2 bathrooms and a garage. In a M type house there are 3 bedrooms, a bathroom and a garage. There are 2 bedrooms, 1 bathrooms and no garage in a L type house.

(i) Express this information in a matrix and name it as A

(ii) Write the order of A

(iii) What kind of a matrix is A?

(5) If $\begin{pmatrix} 1 & -1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} x & p+q \\ x+y & p-q \end{pmatrix}$, find the values of *x*, *y*, *p* and *q*.

(6) $\begin{pmatrix} 3 & 2 & -1 \\ 3 & 0 & -2 \end{pmatrix}$ Write the order of the matrix.

Write the matrix obtained by interchanging the rows as columns. Are the two matrices equal? Give reasons to your answer

19.5 Addition of matrices

The table of information and the corresponding matrix about the exercise books and pens in the prize parcels obtained by Kamal, Akhila and Niroshan in grade eleven for their abilities shown in mathematics in the first term test is given below and is named A.

	Exercise books	Pens
Kamal	6	2
Akhila	4	3
Niroshan	5	0

$$\Longrightarrow A = \begin{bmatrix} 6 & 2 \\ 4 & 3 \\ 5 & 0 \end{bmatrix}$$

Again in the second term, each of them got a prize parcel for their abilities shown in mathematics. The information table and the corresponding matrix about the number of exercise books and pens in each parcel is given below and named as B.

	Exercise books	Pens
Kamal	4	3
Akhila	5	3
Niroshan	3	2

$$\Longrightarrow B = \begin{pmatrix} 4 & 3 \\ 5 & 3 \\ 3 & 2 \end{pmatrix}$$

Let us build up the information matrix about the number of exercise books and pens gifted to each student on both occasions. For this purpose let us add the two matrices above. It is clear that to add the two matrices, the corresponding elements of the matrices should be added. Then the result is also a matrix of the same order.

$$A+B = \begin{pmatrix} 6 & 2 \\ 4 & 3 \\ 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ 5 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 6+4 & 2+3 \\ 4+5 & 3+3 \\ 5+3 & 0+2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 10 & 5 \\ 9 & 6 \\ 8 & 2 \end{pmatrix}}}$$

Study the following examples about the addition of matrices.

Example 4

If $B = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $C = \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$ find B + C

$$B+C=\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1-3 \\ 2+1 \\ -2-1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix}}}$$

Example 5

$$(2\ 1\ 3)+(-1\ \ -5\ \ 1)$$
$$(2\ 1\ 3)+(-1\ \ -5\ \ 1)=(2-1\quad 1-5\quad 3+1)=\underline{\underline{(1\ -4\quad 4)}}$$

Example 6

$$\begin{pmatrix} 2 & -1 \\ -3 & 5 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -1 & -2 \end{pmatrix}$$
$$\begin{pmatrix} 2 & -1 \\ -3 & 5 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 2+0 & -1+2 \\ -3-1 & 5-2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 & 1 \\ -4 & 3 \end{pmatrix}}}$$

To add of two matrices, the order of the two matrices should be equal. A matrix of the same order is obtained as the sum of the two matrices by adding the corresponding elements.

19.6 Subtraction of matrices

Subtracting a matrix means, subtracting the elements of the second matrix from the corresponding elements of the first. Accordingly it is clear that in subtraction also the order of the matrices should be equal.

Example 7

simplify

$$\begin{pmatrix} 5 & 1 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 1 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 5-2 & 1-(-3) \\ 3-1 & 2-4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 4 \\ 2 & -2 \end{pmatrix}}}$$

Example 8

If $A = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ find A - B.

$$A - B = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3-1 \\ -2-(-3) \\ 0-2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}}}$$

To subtract two matrices, the order of the two matrices should be equal. A matrix of the same order is obtained as the difference of the two matrices by subtracting the corresponding elements.

Exercise 19.2

(1) Simplify

(i) $\begin{pmatrix} 1 & 3 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 4 & -2 \end{pmatrix}$

(ii) $\begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

(iii) $\begin{pmatrix} 4 & -1 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} 6 & 4 \\ 2 & 1 \end{pmatrix}$

(iv) $\begin{pmatrix} 1 & -2 \\ -3 & 2 \\ 0 & 4 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ 2 & -4 \\ 1 & -1 \end{pmatrix}$

(v) $\begin{pmatrix} 1 & -2 & 3 \\ -3 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 2 \\ 4 & 9 & -8 \end{pmatrix}$

(vi) $\begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & -4 \\ 0 & -4 & 5 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$

(2) Simplify

(i) $\begin{pmatrix} 10 \\ 8 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

(ii) $\begin{pmatrix} 2 & 3 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -1 & -5 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 4 & -3 \end{pmatrix} - \begin{pmatrix} 5 & 2 & -4 \\ -2 & 1 & -1 \end{pmatrix}$

(iv) $\begin{pmatrix} 3 & 2 \\ -5 & 7 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ -6 & 0 \end{pmatrix}$

(v) $\begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 3 & -1 & -4 \end{pmatrix} - \begin{pmatrix} -5 & -1 & 0 \\ 2 & 6 & -7 \\ 3 & -4 & 8 \end{pmatrix}$

(vi) $\begin{pmatrix} 2 & 1 \\ -1 & 3 \\ 0 & -2 \end{pmatrix} - \begin{pmatrix} 0 & -2 \\ 3 & 0 \\ 4 & 1 \end{pmatrix}$

(3) If $A = \begin{pmatrix} 6 & 1 & 5 \\ 2 & 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 2 & 3 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 0 & 2 \\ 4 & -2 & 1 \end{pmatrix}$

find the matrices given below.

(i) A+B (ii) A+C (iii) B+C (iv) A+B+C (v) A+A
(vi) B+B+B (vii) A-B (viii) A-C (ix) B-C (x) C-B

(4) (i) If $\begin{pmatrix} 2 & -3 \\ 1 & 5 \\ 7 & 3 \end{pmatrix} + \begin{pmatrix} x & 2 \\ y & a \\ -4 & -3 \end{pmatrix} = \begin{pmatrix} 5 & b \\ -3 & 2 \\ z & c \end{pmatrix}$ find a, b, c, x, y and z

(ii) If $\begin{pmatrix} 1 & 3-1 \\ 2-2 & 0 \\ 0 & 4 & 2 \end{pmatrix} - \begin{pmatrix} \text{a} & 1 & \text{e} \\ 0 & b & 1 \\ d-2 & c \end{pmatrix} = \begin{pmatrix} 2 & p & 2 \\ r & 3 & s \\ -1 & q & 4 \end{pmatrix}$ find the values of a, b, c, d, e, p, q, r, and s that satisfy this equation.

(iii) If $\begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix} - A = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ Find the matrix A.

(5) If $\begin{pmatrix} x & -2y \\ 3p & -q \end{pmatrix} + \begin{pmatrix} y & x \\ -q & 2p \end{pmatrix} = \begin{pmatrix} 3 & -3 \\ 3 & 4 \end{pmatrix}$ find x, y, p and q.

(6) The vertices of a triangle ABC drawn on a Cartesian plane are $\text{A} = (2,0), \text{B} = (-2,-4), \text{C} = (-1,1)$

(i) build up the matrix with A,B,C as rows and the coordinates x,y as columns.

(ii) Build up new matrix if the point A is translated by(2,0),and B by(-1,-2) and C by (0,-2).

(iii) if the corresponding points after translating the three points A,B,C are P,Q and R. build up the matrices including the coordinates of the vertices P,Q and R.

19.7 Multiplication of a matrix by a whole number

Let us take $\text{A} = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}$

If we denote matrix A when multiplied by 2 as 2A, letus find a method to find 2A.

$$2A = A + A = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} + \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} = \begin{pmatrix} 2a & 2d \\ 2b & 2e \\ 2c & 2f \end{pmatrix}$$

Accordingly $$2A = 2\begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} = \begin{pmatrix} 2a & 2d \\ 2b & 2e \\ 2c & 2f \end{pmatrix}$$

Thus when multiplying the matrix A by 2, it is clear that all the elements of it should be multiplied by 2.

Similarly, $$3A = 3\begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} = \begin{pmatrix} 3a & 3d \\ 3b & 3e \\ 3c & 3f \end{pmatrix}$$

> When a matrix is multiplied by an integer, all the elements of it should be multiplied by the particular integer.

Let us consider the following examples:

Example 9

$$4\begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 4\times 2 & 4\times 1 \\ 4\times(-1) & 4\times 0 \\ 4\times 3 & 4\times(-2) \end{pmatrix} = \underline{\underline{\begin{pmatrix} 8 & 4 \\ -4 & 0 \\ 12 & -8 \end{pmatrix}}}$$

Example 10

$$-2\begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} -2\times 1 & -2\times 2 \\ -2\times(-3) & -2\times 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 & -4 \\ 6 & -8 \end{pmatrix}}}$$

Example 11

$$-1\begin{pmatrix}1 & -3 & 2\end{pmatrix}=\begin{pmatrix}-1\times1 & -1\times(-3) & -1\times2\end{pmatrix}=\underline{\underline{\begin{pmatrix}-1 & 3 & -2\end{pmatrix}}}$$

Example 12

If $4A-\begin{pmatrix}3 & 7\\10 & 6\end{pmatrix}=\begin{pmatrix}-7 & 5\\-2 & 14\end{pmatrix}$ find A

$$4A=\begin{pmatrix}-7 & 5\\-2 & 14\end{pmatrix}+\begin{pmatrix}3 & 7\\10 & 6\end{pmatrix}=\begin{pmatrix}-4 & 12\\8 & 20\end{pmatrix}$$

$$4A=4\begin{pmatrix}-1 & 3\\2 & 5\end{pmatrix}$$

$$\therefore A=\underline{\underline{\begin{pmatrix}-1 & 3\\2 & 5\end{pmatrix}}}$$

Exercise 19.3

(1) Multiply the each matrix h given below by the integers outside the matrix.

(i) $3\begin{pmatrix}2\\7\\5\end{pmatrix}$ (ii) $2\begin{pmatrix}3 & -1 & -2\end{pmatrix}$ (iii) $5\begin{pmatrix}4 & -1\\-3 & 0\end{pmatrix}$

(iv) $-2\begin{pmatrix}3\\2\\1\end{pmatrix}$ (v) $-3\begin{pmatrix}1 & 2 & -1\\3 & -1 & -3\end{pmatrix}$ (vi) $-4\begin{pmatrix}1 & -1 & -2\\2 & -3 & 0\\0 & -1 & 3\end{pmatrix}$

(2) If $A=\begin{pmatrix}2 & 3\\1 & 5\end{pmatrix}$, $B=\begin{pmatrix}-1 & 2\\-2 & 0\end{pmatrix}$ and $C=\begin{pmatrix}3 & -1\\-2 & 4\end{pmatrix}$ find

(i) 2A (ii) 3B (iii) -2C (iv) A+ 4B (v) A+2B+3C (vi) 3B-4C

(3) Find $2\begin{pmatrix}3 & -1 & 0\end{pmatrix} - 3\begin{pmatrix}1 & 3 & -1\end{pmatrix}$.

(4) If $2A + 3\begin{pmatrix}-2 \\ -1\end{pmatrix} = \begin{pmatrix}4 \\ -1\end{pmatrix}$ find A.

(5) If $-1\begin{pmatrix}1 & 3 & 2 \\ -2 & 0 & -1 \\ 2 & -3 & -1\end{pmatrix} + 4B = \begin{pmatrix}3 & 5 & -14 \\ 2 & -4 & 1 \\ 2 & -5 & -11\end{pmatrix}$ find the matrix B.