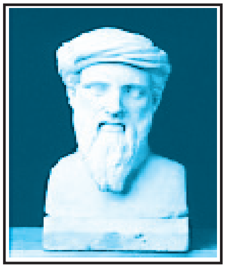
17 Pythagoras' Theorem

After studying this chapter you will be able to acquire the knowledge and application about the following :

- to solve problems using Pythagoras' relationship.
- confirm the validity of the Pythagoras' relationship.



Pythagoras was a Greek philosopher who lived in the period 580 - 500 BC. He was a mathematician and also an astronomist. He believed the concept that everything in this world can be explained through mathematics. He even experimented on mathematical nature in music through numbers.

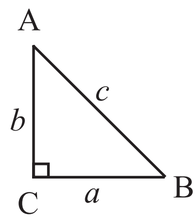
The relationship that Pythagoras discovered was first proved as a theorem only after about 300 years by a mathematician named Euclid. This relationship has been pictorially shown in more than 400 different ways by mathematicians all over the world.

Pythagoras' theorem is an important relationship in mathematics. Not only in mathematics, this is used in other fields as well.

Pythagoras' theorem

In a right angled triangle, the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on the other two sides which contain the right angle.

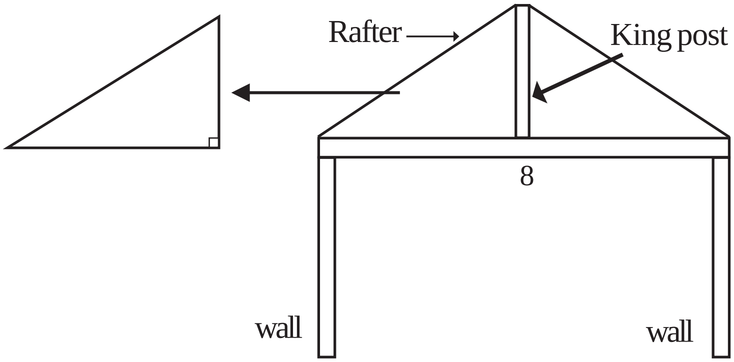
$$a^2 + b^2 = c^2$$



This relationship is used by architects, masons, technical officers, surveyors etc.

Try to solve the problem given below using the knowledge you gained in previous grades.

A building is 8 m wide. A king-post of height 1m is fixed in the middle of a beam, which is supported by the side walls. Let us find the length of the slant plank needed to fix the king-post to the middle of the beam.



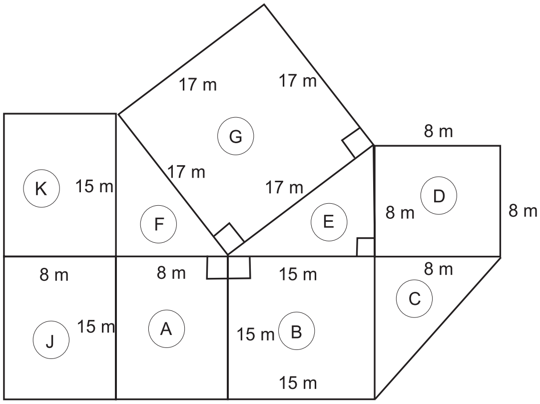
Consider a right angled triangle here. The distance from one end to the middle of the beam is 4 m. The length of the king post fixed at the middle, perpendicular to it is 1 m. The rafter makes the hypotenuse of the right angled triangle thus formed.

Let the length of the hypotenuse be x metres.
According to Pythagoras' relationship,

$$x^2 = 4^2 + 1^2$$
$$x^2 = 16 + 1$$
$$x^2 = 17$$
$$x = \sqrt{17}$$

$\therefore$ The lenght of the slant plank = $\sqrt{17}$ m

Given below is a combination of squares, rectangles and right angled triangles.



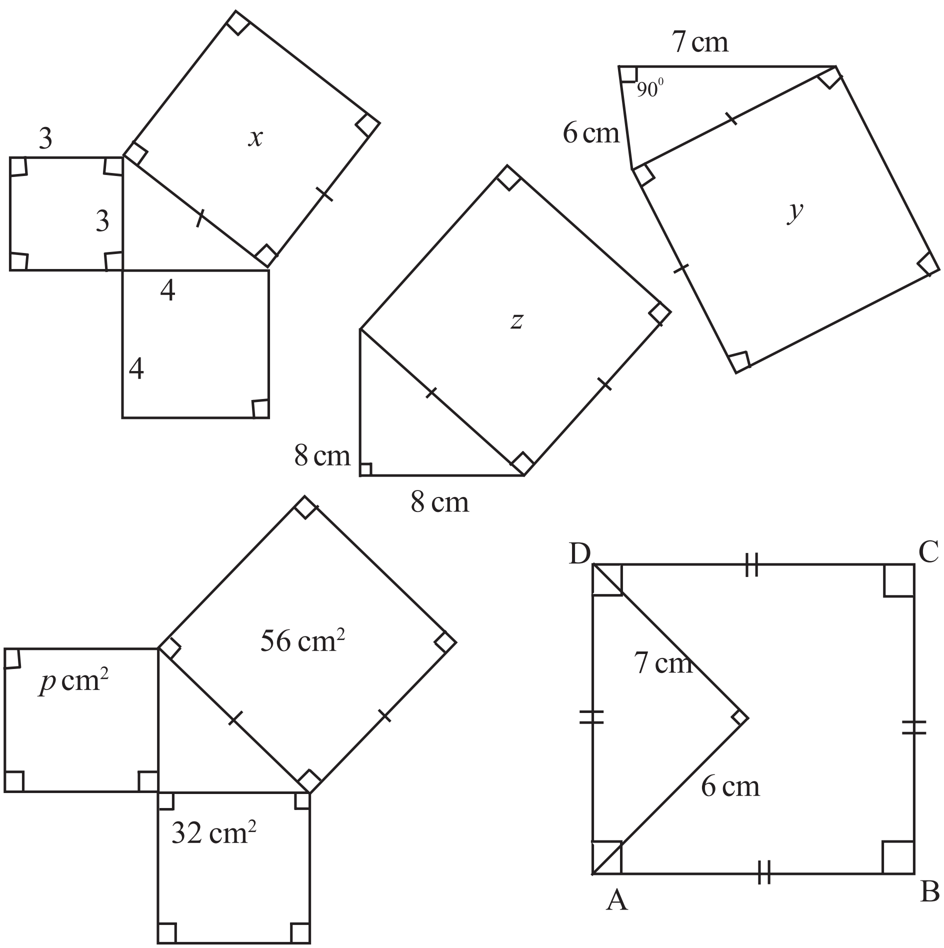
Find the area of the figures A,B,C,D,E,F,G.

Write the sum of the areas of figers B and D. Write down the relationship between the sum of areas of B and D and the area of G.

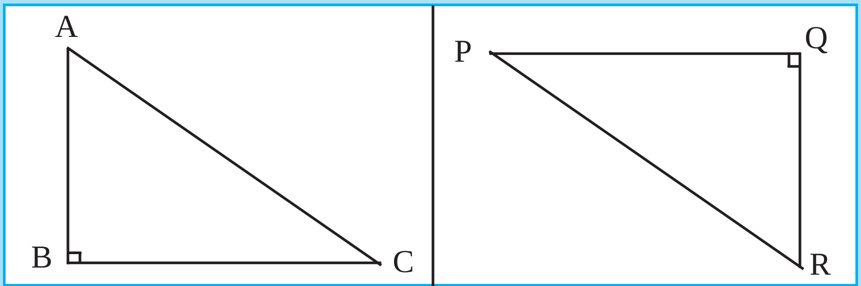
Is the sum of the areas of A and K equal to the area of G ? Why is it so? What fraction of the total area is the sum of areas of B and D?

Exercise 17.1

(1) Find the areas of the squares named as x, y, z, and p in the following diagrams, and the ABCD



(2) Using each right angled triangle given below, fill in the blanks



Area of square on AB = AB^2
Area of square on BC = . . .
Area of square on AC = . . .

AC^2 = . . . + . . .
(According to Pythagoras' theorem)

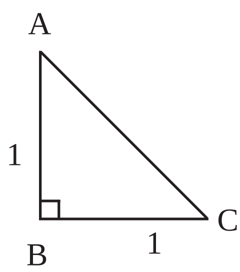
Area of square on PQ = . . .
Area of square on QR = . . .
Area of square on PR = . . .

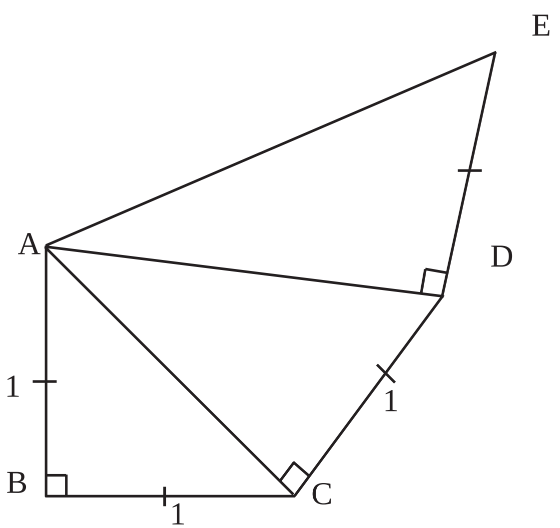
$PQ^2 + QR^2$ = . . .

Activity 1

Draw an isosceles right angled triangle on a page in your exercise book, (Draw it towards the lower middle port of the page.) equal in length.
If one of these sides is equal to 1 unit of length,

$$AC = \sqrt{1^2 + 1^2} = \sqrt{2}$$





As BC = AB

$$AC = \sqrt{1^2 + 1^2}$$
$$AC = \sqrt{2}$$

Now, as shown in the figure, draw a perpendicular to AC at C and mark D on it 1 unit away from C.
Now calculate AD.

$$AD^2 = AC^2 + DC^2$$
$$AD^2 = (\sqrt{2})^2 + 1^2$$
$$AD^2 = 2 + 1$$
$$AD^2 = 3$$
$$AD = \sqrt{3}$$

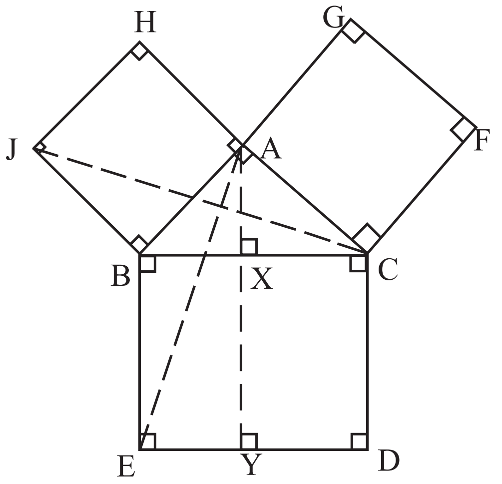
Similarly by drawing a perpendicular at D to AD mark point E on it as before. Repeat this activity and go on drawing right angled triangles. Use different colours to draw the hypotenuse of each right angled triangle. Draw as many right angled triangles as possible this way and examine the locus of point C. This is called Archimedes spiral. In this the length of each hypotenuse gives the square root of each counting number starting from 1.See how Pythagoras relationship is applied here.

17.1 Proof of Pythagoras Theorem

It is already mentioined that the proof of Pythagoras' theorem was first introduced by a mathematician named Euclid, 300 years after Pythagoras.

We will consider different methods of proving this theorem.

Method I



Data :- In triangle ABC, angle A is a right angle. The square on BC is BCDE, the square on AC is ACFG and the square on AB is ABJH.

To prove that :- The area of BCDE = The area of ACFG + The area of ABJH

Construction :- Draw AXY perpendicular to BC to meet BC at X and ED at Y.
Join AE and JC

Proof :- $J\hat{B}A = C\hat{B}E = 90^{\circ}$ (Data)

When $A\hat{B}C$ is added to both sides, of the equatian above

$$J\hat{B}C = A\hat{B}E$$

Consider

Δ JBC and Δ ABE,

BJ = AB (sides of ABJH square)

BC = BE (sides of BCDE square)

$J\hat{B}C = A\hat{B}E$ (Proved)

$\therefore$ Δ JBC $\equiv$ Δ ABE (S.A.S)

$\therefore$ Area of Δ JCB = Area of Δ ABE

HAC is a straight line (sum of two adjecent angles)

Δ JBC and square JBAH are on the same base JB and are between the parall lines JB and HAC.

$$\therefore \text{ Area of } \Delta \text{ JBC } = \frac{\text{Area of squae JBAH}}{2}$$

Similarly, Δ ABE and the rectangle BEYX are on the same base BE and between the parallel lines BE and AXY.

$$\therefore \text{ Area of } \Delta \text{ ABE} = \frac{\text{Area of rectangle BEYX}}{2}$$

Areas of Δ JBC and Δ ABE are equal. Area of square JBAH = Area of rectangle BEYX

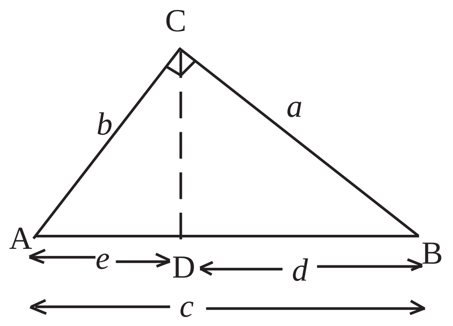
Similary by joining AD and BF it can be shown that

Area of the square CFGA = Area of the rectangle CDYX

$\therefore$ the area CDYX + BEYX = JBAH + CFGA

ie. Area BEDC = JBAH + CFGA

Method II



Data :- In Δ ABC, angle C is a right angle. CD is drawn prependicular to AB.

Construction :- AD:DB = e : d

let length of AB be c, AC be b and BC be *a*

To prove that :- $a^2 + b^2 = c^2$

Proof :- As Δ ABC and Δ ACD are similar, the corresponding sides are proportional.

ie $\frac{c}{b} = \frac{b}{e}$

$\therefore\ b^2 = ce$ ——— (1)

Similarly as Δ ABC and Δ BCD are similar, the corresponding sides are proportional.

i.e. $\dfrac{c}{a} = \dfrac{a}{d}$

$a^2 = cd$ ——— (2)

Add (1) and (2),

$$a^2 + b^2 = ce + cd$$
$$= c\,(e+d)$$

but $e + d = c$

$$\therefore\ a^2 + b^2 = c \times c$$

$$\underline{\underline{a^2 + b^2 = c^2}}$$

Exercise 17.2

1. A ladder is kept against a vertical wall in such a way that the foot of the ladder is 3 m away from the base of the wall on a horizontal ground and the top of the ladder is at a height of 4 m from the base of the wall. Find the length of the ladder.

2. An electric post is of height 10 m. A supporting wire fixed to a point 2 m below the top of the past is fixed at the other end to a point on the horizontal ground 6 m away from the foot of the post. Find the length of the wire. (ignore the knots)

3. The diagonals of a rhombus are of lengths 24 cm and 10 cm. Find the length of a side of the rhombus.

4. In Δ ABC, AB = AC = 17 cm. The length of the perpendicular drawn from A to BC is 15 cm. Find the area of the triangle.

5. In the quadrilateral ABCD, $A\hat{B}C = 90^0$, AB = 12 cm, BC = 9 cm, CD = 8 cm, and AD = 17 cm.

   Show that $A\hat{C}D = 90^0$

6. In the triangle AOB, AO = OB and $A\hat{O}B = 90^0$. Show that the area of the square on the hypotenuse is 4 times the area of the triangle.

7. In the triangle ABC, AB>AC. The perpendicular AX is drawn from A to BC. Show that $AB^2 - AC^2 = BX^2 - CX^2$

8. Show that the area of the square drawn on a diagonal of a square is twice the area of the original square.

9. In the equilateral triangle XYZ, the mid point of YZ is O. Show that $3YZ^2 = 4\ OX^2$

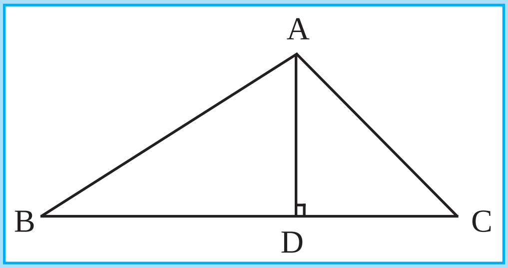
10. X is any point inside the rectangle ABCD show that $XA^2 + XC^2 = XB^2 + XD^2$

11. The diagonals of the quadrilateral PQRS, intersect each other at right angles. Show that $PQ^2 + RS^2 = QR^2 + PS^2$

12. In the triangle PQR, angle $P\hat{Q}R = 90^0$. The mid point of QR is X and the mid point of PQ is Y.
Show that $4\,(PX^2 + RY^2) = 5PR^2$

13. In the equilateral triangle ABC, P is a point on BC. Such that $BP = \frac{1}{3}BC$
Show that $9AP^2 = 7AB^2$

14.



ABC is a triangle. AD is a prepandicular to BC Prove that
$AC^2 = AB^2 + BC^2 - 2\ BC.BD$

Pythagorain Triods

In any right angled triangle, if the sides containing the right angle are *a* and *b* and the hypotenuse is c, then the values of *a*, b and c which satisfy the relationship $a^2 + b^2 = c^2$ are called Pythagoras' triples.

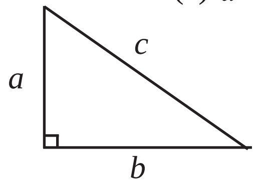
The simplest Pythagoras triple, which is a whole number triple is (3, 4, 5) The multiples of this such as (6, 8, 10) , (9, 12, 15) , (30, 40, 50) are also Pythagoras triples.

(8, 15, 17), (7, 24, 25), (9, 40, 41) are some more Pythagoras tripples. Pythagoras triple too can be written as their multiples.

In Pythagoras triples other than their multiples, you will find that one value is always an even number and the other two are odd numbers.

Euclid has introduced three equations which can be used to find Pythagoras triples. These three equatious are

(1) $a = x^2 - y^2$, (2) $b = 2xy$, (3) $c = x^2 + y^2$



As mentioned before, if the lengths of the sides of a right angled triangle are given by *a*, *b* and *c*, then

Fill in the blanks in the table given here for given values of *x* and *y*.

x	y	$x^2 - y^2$	$2xy$	$x^2 + y^2$
2	1	3	4	5
3	2	. . .	. . .	. . .
3	1	. . .	. . .	. . .
4	1	. . .	. . .	. . .
4	3	. . .	. . .	. . .
5	4	. . .	. . .	. . .
5	2	. . .	. . .	. . .
6	1	. . .	. . .	. . .
6	5	. . .	. . .	. . .
7	2	. . .	. . .	. . .

Find more values for x and y to form Pythagoras triples.