

15 Representation and Interpretation of Data

After studying this chapter you will be able to acquire knowledge and the application of :

- drawing a histogram for data with equal class intervals.
- drawing a histogram for data with unequal class intervals
- drawing a frequency Polygon using the histogram
- drawing a Cumulative frequency curve
- finding quartiles and the inter quartile range
- representing data by graphs

15.1 Histogram of a frequency distribution with equal class intervals

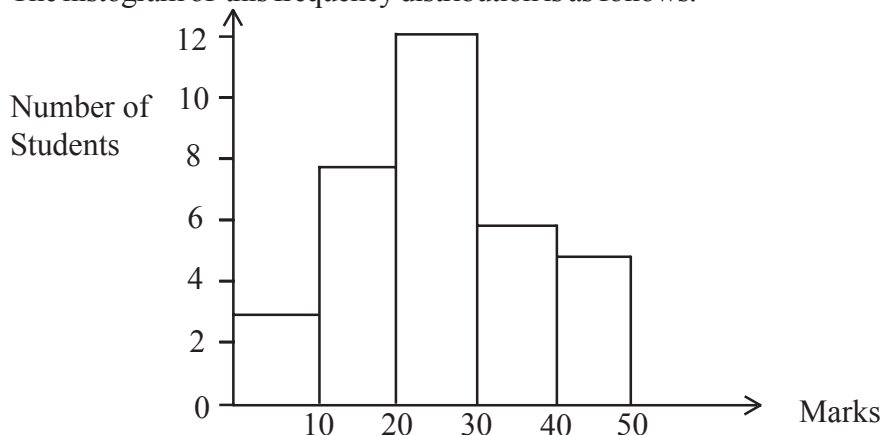
You have learnt earlier how to draw a histogram for a frequency distribution with equal class intervals. Consider the following example.

Example 1

Marks obtained by the students in a class for a test, out of 50 marks is shown by the frequency distribution given below. Draw a histogram to represent this data.

Marks	0-10	10-20	20-30	30-40	40-50
No:of students (frequency)	3	8	12	6	5

The histogram of this frequency distribution is as follows.



- The width of the columns are equal in this histogram.
- The frequency of each class interval is represented by the height of the column.
- The area of the rectangle relevant to each class interval is proportional to the frequency.

Exercise 15.1

- (1) The time taken by each athlete to finish a race is given below. Draw a histogram to represent this information. Here 55 - 65 means 55 or over and less than 65.

Time (to the nearest minute)	55-65	65-75	75-85	85-95	95-105
Frequency (No. of athletes)	10	18	12	6	4

- (2) The data obtained by measuring the mass of students in a class on a certain day are given in the following distribution. Draw a histogram to represent this data.

Weight (W) Kg	30-35	35-40	40-45	45-50	50-55
Frequency (No. of Students)	5	7	10	5	3

- (3) The information about the mass of group of players is shown in the table given below. Represent this data in a histogram.

Mass (Kg)	40 - 45	45-50	50-55	55-60	60-65	65-70
No. of Players (Frequency)	5	7	10	16	8	4

(4) The number of packets of a new brand of ice cream, sold during the first 200 days, after introducing it to the market, is shown in the frequency table given below.

Represent this data in a histogram.

No. of packets sold	0-50	50-100	100-150	150-200	200-250	250-300
Frequency (No. of days)	20	35	60	40	25	20

(5) The data obtained by measuring the circumference of trees loaded to a lorry are shown in the table given below.

Circumference (cm)	0-25	25-50	50-75	75-100	100-125
Frequency (No of trees)	8	10	12	20	18

- (i) Represent this data in a histogram.
- (ii) What is the modal class of this distribution?

15.2 The histogram of a set of data with unequal class intervals

Example 2

A Company manufactures metal rods in different lengths. The table given below shows information of a day's production of the company.

length (cm)	10-20	20-30	30-40	40-50	50-70	70-100	100-140
No. of metal rods (Frequency)	6	7	8	10	10	9	8

The size of the first four intervals is equal but the sizes of the 5th, 6th and the 7th are unequal.

As the size of the first four intervals is 10, the area of the first four rectangles are 60,70,80,100 respectively.

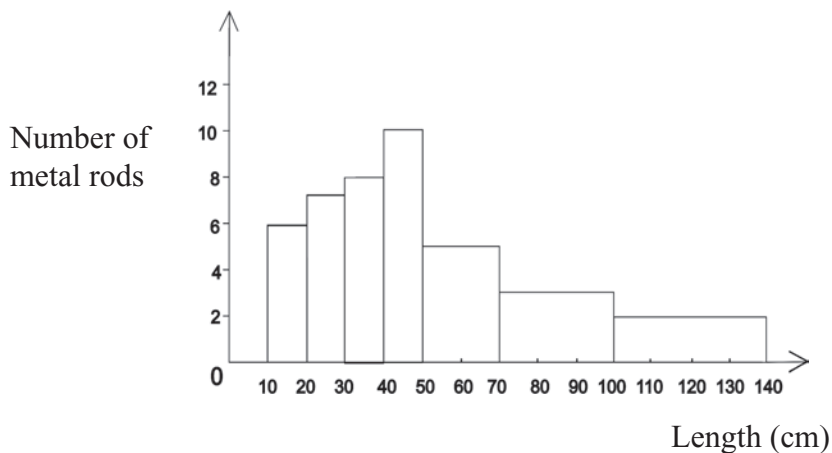
If you consider the size 10 of the class interval as one unit, the height of the rectangle corresponding to the class 50 - 70 can be obtained by dividing

the frequency by 2 ie $\frac{10}{2} = 5$

Therefore the heights of the rectangles of the given distribution should be adjusted as shown below.

Length cm	No. of metal rods (f)	Height of the rectangles
10-20	6	6
20-30	7	7
30-40	8	8
40-50	10	10
50-70	10	$10 \div 2 = 5$
70-100	9	$9 \div 3 = 3$
100-140	8	$8 \div 4 = 2$

The histogram drawn according to the above table is shown below.

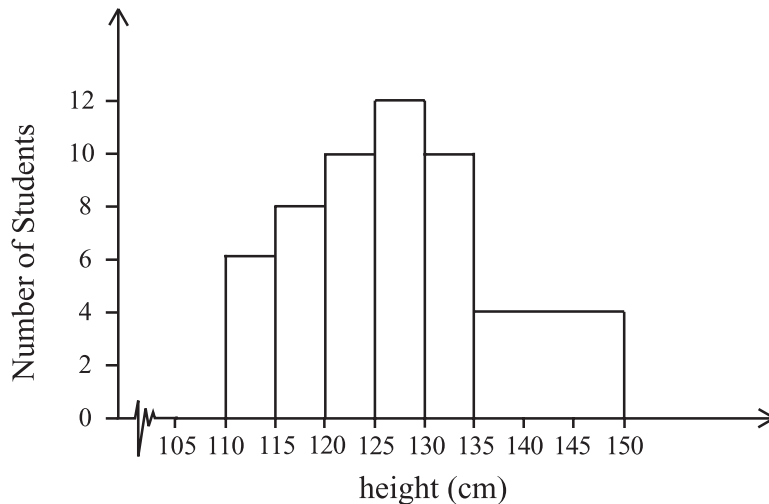


Example 3

Draw a histogram to represent the data given below about the heights of a group of students.

Height (cm)	Frequency (No. of students)	Height of Rectangles
110-115	6	6
115-120	8	8
120-125	10	10
125-130	12	12
130-135	10	10
135-150	12	$12 \div 3 = 4$

See how the heights of the rectangles are calculated taking the width of a class 5 cm as one unit. Accordingly the width of the class 135-150 is 3 units. For the area to be proportional to the frequency, the frequency 12, when divided by 3, gives the height of the rectangle, 4



So far we have considered drawing histograms for continuous data.

Now Consider the frequency distribution given below.

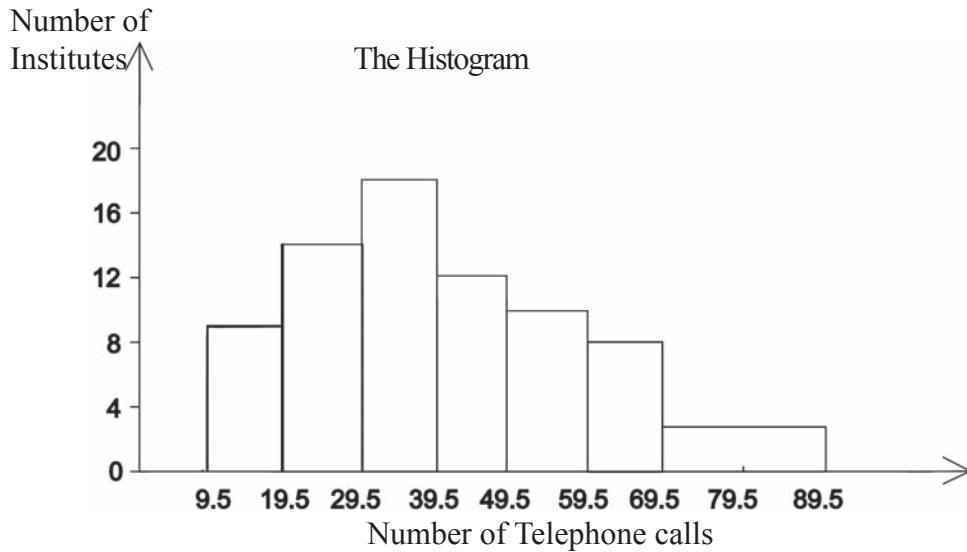
Example 4

The table given below shows the number of telephone calls taken by 77 institutes in a certain day.

Number of Telephone calls	Number of Institutes
10 - 19	9
20 - 29	14
30 - 39	18
40 - 49	12
50 - 59	10
60 - 69	8
70 - 89	6

These are called **discrete data**. Therefore we cannot draw a histogram for this data as discussed earlier. So the histogram should be drawn by rewriting this data as continuous data using class boundaries and calculating the heights of rectangles.

No. of Telephone Calls	Class boundaries	No. of Institutes	Height of the rectangles
10 - 19	9.5 - 19.5	9	9
20 - 29	19.5 - 29.5	14	14
30 - 39	29.5 - 39.5	18	18
40 - 49	39.5 - 49.5	12	12
50 - 59	49.5 - 59.5	10	10
60 - 69	59.5 - 69.5	8	8
70 - 89	69.5 - 89.5	6	6 2 3



Exercise 15.2

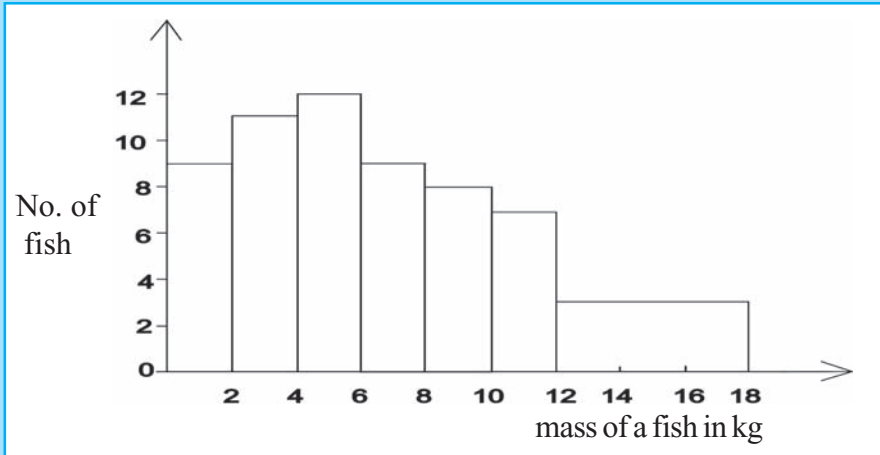
- (1) The table shows the results obtained from an investigation done regarding the life span of a certain brand of electric bulbs. Draw a histogram to represent this data.

Duration Period (Hrs)	300-400	400-500	500-600	600-700	700-800	800-1000
Frequency (Number of electric Bulbs)	10	15	12	10	8	4

- (2) The table shows, the marks obtained by a group of students at an examination. Using the class boundaries draw a histogram to represent this data.

Class Interval (Marks)	1-10	11-20	21-30	31-40	41-50	51-80
Frequency (Number of students)	6	5	8	7	5	9

(3) A histogram representing information about the mass of fish is shown below.



- (i) How many fish are there weighing from 6 to 8 kg ?
- (ii) How many fish are there weighing from 6 to 10 kg ?
- (iii) What is the total number of fish weighed to obtain this data ?

15.3 The Frequency Polygon

A Frequency Polygon can be used to represent data as well as the histogram. There are two ways of constructing a Frequency Polygon.

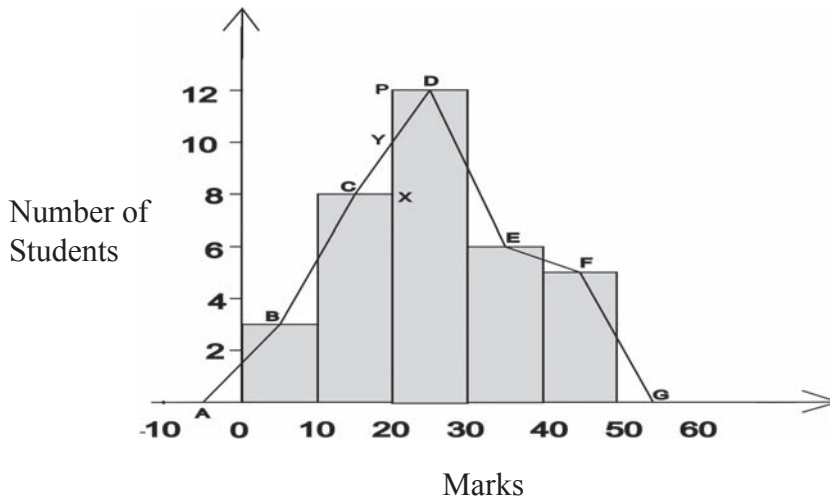
1. Using a histogram
2. Using the mid value of each class interval and the relevant frequency

When the class intervals are equal, the frequency polygon can be obtained by drawing straight line segments, joining a point half the breadth of a column away from the first column and the mid point of the top of each rectangle then a point half the breadth of a column after the last column.

Let us consider the information about the marks obtained at the examination in example 1.

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students (Frequency)	3	8	12	6	5

Let us draw a frequency polygon to represent this data. First we have to draw the histogram.

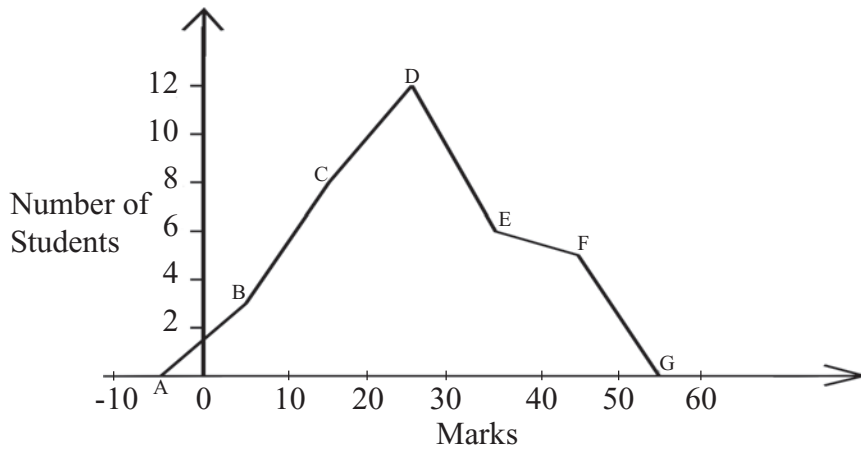


The point A which is the mid point of a class interval to the left of O is joined to the mid point B of the first column. In the same way join B, C, D, E, F and finally join the point G which is the mid point of a class interval to the right of the last class interval and complete the polygon.

This type of polygon is called a frequency polygon. A frequency Polygon has following properties.

- The area of the Frequency Polygon is equal to the area of the Histogram. (Considering the triangles CYX and PDY it is clear that the area removed from the histogram is equal to the area added.)
- One side of the polygon is the AG - axis

Without drawing a histogram, the frequency polygon can be drawn by taking the mid value of each class interval and the corresponding frequency as an ordered pair and by plotting and joining the points in order.

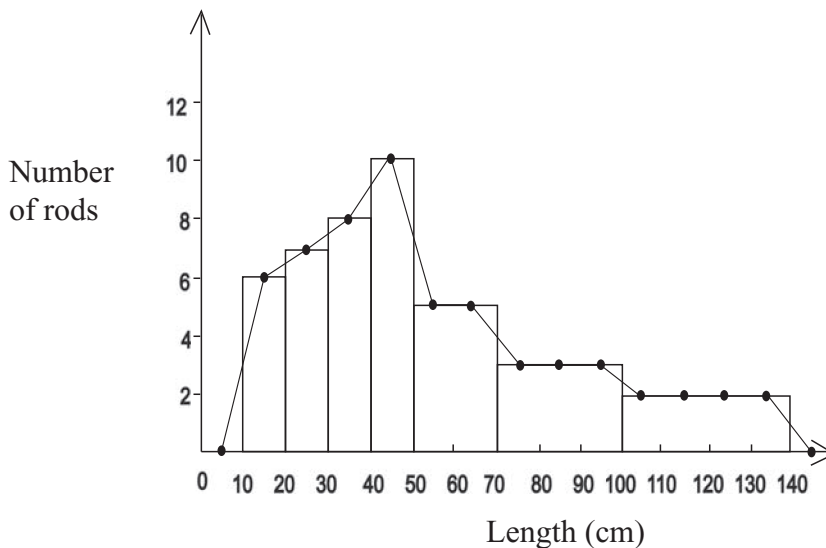


The frequency polygon can be obtained by joining the points A, (5,3), (15,8), (25,12), (35,6), (45,5) and G respectively.

15.4 The frequency Polygon for a frequency distribution with unequal class intervals

Example 5

Let us draw a frequency polygon using the histogram for the frequency distribution in example 2



The frequency Polygon can be drawn in the same way by separating the classes of unequal sizes to equal sizes. It is shown in the diagram above.

Exercise 15.3

- (1) The table below gives the information about the ages of a group of children in a play ground.

Age (Years)	2-4	4-6	6-8	8-10	10-12	12-14
Frequency (No:of Children)	2	7	8	10	5	3

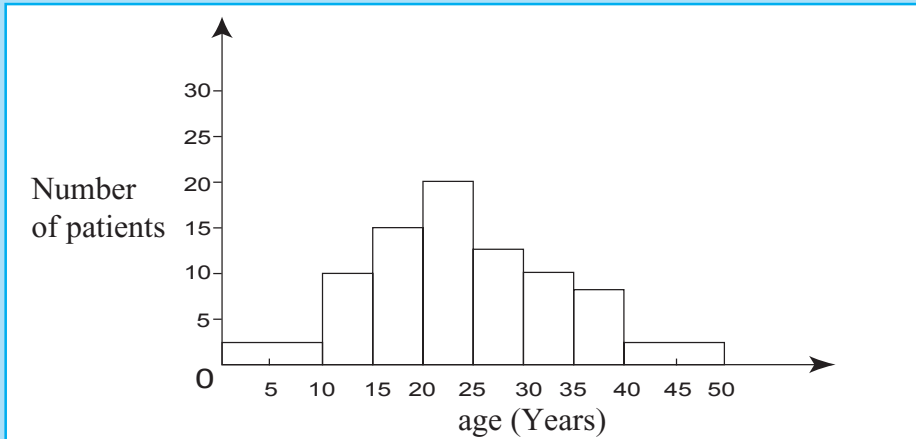
- (i) Draw a histogram to represent this data.
- (ii) Draw the Frequency Polygon with the use of the histogram.
- (iii) Comment on the area of the histogram and the area of the frequency Polygon?

- (2) The information about the heights of 80 students in a school is given below.

Class intervals (height) cm	110-115	115-120	120-125	125-130	130-135	135-140	140-145	145-150	150-155	155-165
Frequency (No of students)	5	10	14	16	18	6	5	4	3	4

- (i) Draw a histogram to represent the above data and using it draw the frequency polygon.

(3) A histogram drawn to represent the information about ages of patients who had come to a clinic to get treatment is shown below.



- (i) If there were 15 patients between the ages 15-20. find the number of patients relevant to each age group according to the histogram.
(ii) Draw the frequency polygon using the histogram.

(4) The information obtained from a survey about ages of farmers in a village are given below.

Age in years	15-19	20-24	25-29	30-34	35-39	40-44	45-49
Frequency (No of farmers)	05	8	10	12	7	4	2

- (i) Considering the class boundaries, draw the histogram to represent this data.
(ii) Draw the frequency polygon using the histogram.

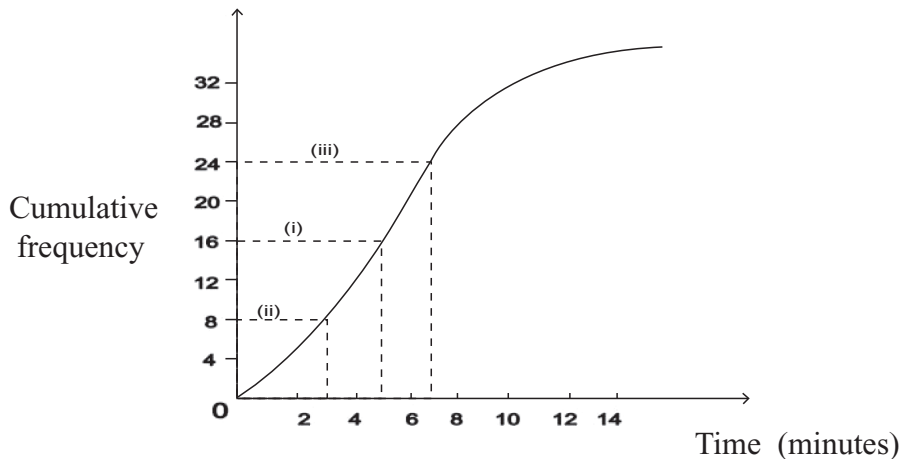
15.5 Cumulative Frequency Curve and Quartiles

Look at the table given below. It gives information about the time taken for a telephone call by each caller at a post office during a day.

(1) Class Interval time (minutes)	(2) Frequency (number of customers)	(3) Cumulative Frequency
0-2	4	4
2-4	6	10
4-6	10	20
6-8	8	28
8-10	3	31
10-12	1	32

Each value in the cumulative frequency column is obtained by adding the frequency in that row and all the frequencies in the rows above it. The frequency thus obtained is called the **cumulative frequency**. The table with cumulative frequencies is called a **cumulative frequency table**.

Considering the first and the third columns of the table, the number of customers who had taken less than 2 minutes is 4 and those who have taken less than 4 minutes is 10. When they are written as ordered pairs (2,4), (4,10), (6,20), (8,28), (10,31), (12,32) are obtained. By plotting these points on a co-ordinate plane and joining them, a smooth curve as shown below is obtained.



This is known as the **Cumulative Frequency Curve**

Observe the graph. Horizontal line (i) is drawn through 16, which is exactly half the total number of callers. The x co-ordinate of the point where the line (i) meets the curve is 5. What is this value ?

You already know that the value corresponding to the middle score is the median. Hence according to the given information, the median is 5 minutes. Now look at the horizontal lines (ii) and (iii). Line (ii) is drawn through the mid value of the

first half of the cumulative frequency. .ie through 8 which is the $\frac{1}{4}$ of 32. The x co-ordinate of the point where this line meets the curve is 3. This is called the first quartile and is denoted by Q_1

$$\therefore Q_1 = 3 \text{ Minutes}$$

The horizontal line (iii) is drawn through the mid value of the second half of the cumulative frequency.

i.e Through 24 which is $\frac{3}{4}$ of 32

The x co-ordinate of the point where the line (iii) meets the curve is 7. This is called the third quartile and is denoted by Q_3

$$Q_3 = 7 \text{ minute}$$

The value $Q_3 - Q_1$ is known as the **Interquartile Range**

$$\begin{aligned} \text{Here } Q_3 - Q_1 &= 7 - 3 \\ &= 4 \end{aligned}$$

$$\therefore \text{Inter quartile range} = 4$$

Example 6

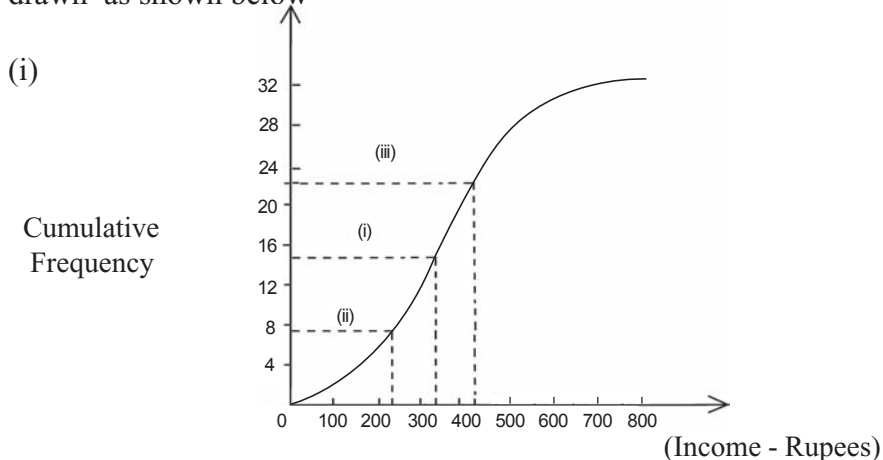
A direct marketing salesman had recorded his daily income for 30 days in his note book. The table given below was constructed using his notes.

Income (Rupees)	0-100	100-200	200-300	300-400	400-500	500-600	600-700
No. of days	2	4	5	8	6	3	2

- Draw a cumulative frequency curve to represent the above data .
- Find the median income per day.
- Find the inter quartile range.

Class Interval (Income - Rupees)	Frequency (days)	Cumulative Frequency
0 - 100	2	2
100 - 200	4	6
200 - 300	5	11
300 - 400	8	19
400 - 500	6	25
500 - 600	3	28
600 - 700	2	30

According to the table above the cumulative frequency curve can be drawn as shown below



(ii) According to the graph the median income - Rs 350

(iii) 1st quartile (Q_1) = 220

3rd quartile (Q_3) = 450

$$\begin{aligned}\text{The inter quartile range} &= Q_3 - Q_1 = (450 - 220) \\ &= \underline{\underline{230}}\end{aligned}$$

Exercise 15.4

(1) The number of shirts completed within a week by a group of girls in a garment factory is given below.

No of shirts	20	23	25	26	27	28	29	30	32	35	38	40
Frequency (No of girls)	2	4	3	3	4	6	5	4	2	4	3	4

(i) Find the first, second and the third quartiles of the number of shirts completed within a week.

(ii) Find the inter quartile range.

(2) The number of days in which a group of student absent from school during one year are given below.

12, 9, 10, 11, 12, 0, 11, 2, 9, 8, 9, 7, 0, 13, 11

Find the

(i) first quartile

(ii) second quartile

(iii) third quartile

(iv) inter quartile range

(3) In a school, where there are parallel classes, the marks obtained by 240 students in grade 11 classes in a test are shown below.

Marks	0 - 8	9 - 17	18 - 26	27 - 35	36 - 44	45 - 53	54 - 62	63 - 71	72 - 82
Frquency	15	18	38	40	48	30	26	14	11

- (i) If 25% of the students who obtained lowest marks are to be chosen what is the mark that should be taken as the limit ?
- (ii) If 25% of the students who obtained highest marks are to be chosen what is the mark that should be taken as the limit ?

(4) Information collected from a research done regarding a certain disease is given in the table. A sample of 2 to 34 years was considered .

Age of patients (years)	2- 6	6 - 10	10 - 14	14 - 18	18 - 22	22 - 26	26 - 30	30 - 34
No: of Patients (Frequency)	8	12	20	40	55	38	20	7

Find the,

- (i) Quartiles
- (ii) Inter quartile range of the distribution.

(5) Given below is the information about the lengths of pieces of waste ribbon in a factory

(Class interval) length of a piece in cm	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
(Frequency) No. of pieces	8	11	15	22	27	25	20	16	10

- (i) Prepare a cumulative frequency table.
- (ii) Draw a cumulative frequency curve and hence find the median and the interquartile range.

- (6) The time duration of telephone calls taken from a post office by outside callers is shown in the table given below.

Class interval (Time in seconds)	0 - 60	60 - 120	120 - 180	180 - 240	240 - 300	300 - 360	360 - 420	420 - 480	480 - 540	540 - 600
Frequency (No. of calls)	6	16	19	23	29	33	22	20	19	13

- (i) Prepare the cumulative frequency table.
(ii) Draw the cumulative frequency curve and hence find the
- (a) median
 - (b) 1st and the 3rd quartiles
 - (c) inter quartile range
- (7) In a certain school, the grade 6 admission test was taken by 150 students. Information about the marks they obtained is given below.

Class Interval (Marks)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Frequency (No. of students)	12	15	17	18	21	19	16	13	11	8

- (i) Prepare the cumulative frequency table on this information.
(ii) Draw the cumulative frequency curve and hence find the median of the distribution, first and the third quartiles and the inter quartile range.

- (8) Given below is the information about the monthly salaries of employees in a certain institution .By drawing a cumulative frequency curve on this information, find the median and the inter quartile range.

Class Interval (salary-Rupees)	5000 - 10000	10000 - 15000	15000 - 20000	20000 - 25000	25000 - 30000	30000 - 35000	35000 - 40000
Frequency (No of employees)	8	16	25	38	25	14	5

- (9) Information collected by a survey done in 90 days about the price of a kilogram of beans is given below.

Class interval (Price in Rupees)	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50
Frequency (No. of days)	7	13	28	17	14	8	3

Draw the cumulative frequency curve to illustrate the above data.
Hence find the median price of a kilogram of beans and the inter quartile range.