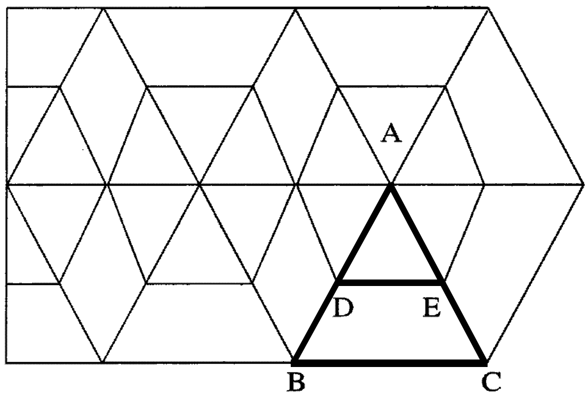
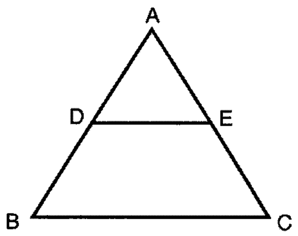
12 Equi - Angular Triangles

By studying this lesson you will acquire knowledge on the following :

- Using the theorem and it s converse that'a straight line drawn parallel to a side of a triangle divides the other two sides proportionally.'
- Corresponding sides of equi-angular triangles are proportional.



A portion of a gate is illustrated in the figure given above. There are various plane figures to be seen. A boy who wants to find the length of wire used in the gate observed it carefully. Let's draw the triangle given in dark lines separately.



ABC is an equilateral triangle and D and E are the midpoints of AB and AC respectively. What can you say about DE and BC? According to the mid point theorem you have learnt that DE// BC and

$DE = \frac{1}{2} BC$. (Mid point theorem)

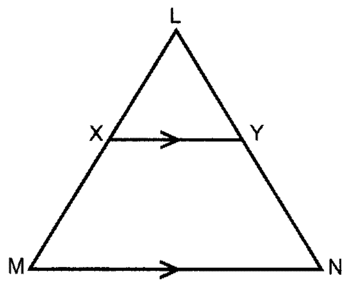
Thus, $\frac{AD}{DB} = 1$ and $\frac{AE}{EC} = 1$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

Theorem : **A straight line drawn parallel to a side of a triangle divides the other two sides proportionally.**

Thus, in triangle LMN, where XY // MN,

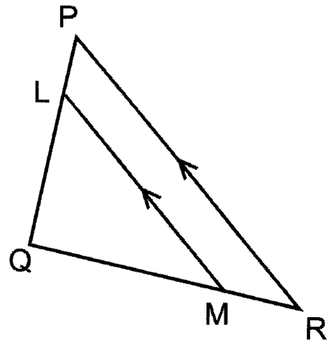
$$\frac{LX}{XM} = \frac{LY}{YN}$$



Activity 12.1

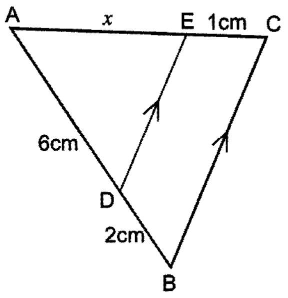
To verify the above theorem,first go through the activity given below.

Draw the triangle PQR slightly bigger in your exercise book. Mark a point L on PQ and draw a line parallel to PR though L. By measuring the angles $Q\hat{L}M$ and $L\hat{P}R$ or by any other method,verify that PR and LM are parallel. Now measure the lengths PL, QL, QM, MR and obtain the values of $\frac{QL}{PL}$ and $\frac{QM}{RM}$. Hence see whether the theorem is true. Repeat the same procedure with obtuse angled triangles, right angled triangles and acute angled triangles.



Now study the following example carefully.

Example 1



In the diagram DE// BC. Find x.

In triangle ABC, BC// DE. According to the theorem that, a straight line drawn parallel to a side of a triangle divides the other two sides propotionally,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{6}{2} = \frac{x}{1}$$

$$x = \underline{\underline{3}}$$

Example 2

In the diagram AB // QR. Find x using the given information

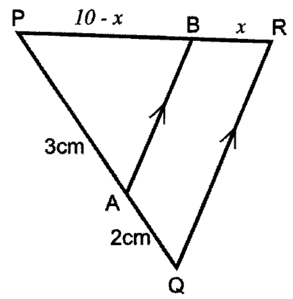
$$\frac{3}{2} = \frac{10 - x}{x} \quad \text{given}$$

$$3x = 2(10 - x)$$

$$3x = 20 - 2x$$

$$5x = 20$$

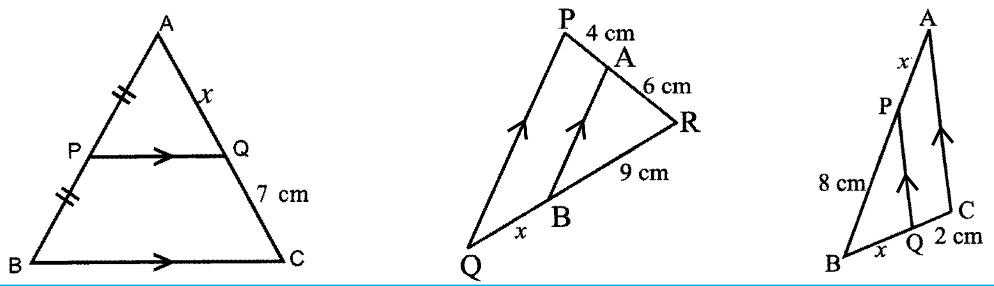
$$x = \underline{\underline{4}}$$

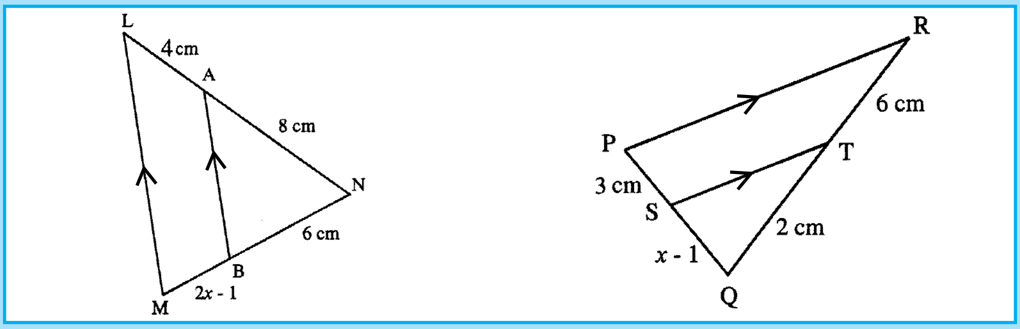


Do the following exercises using this theorem.

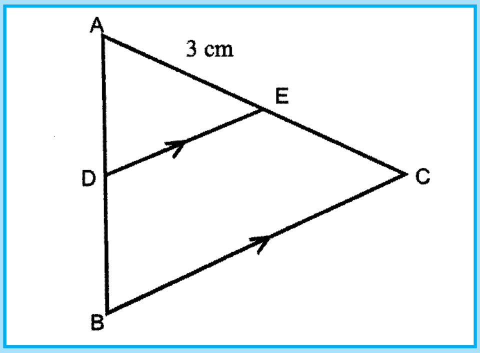
Exercise 12.1

(1) Find the lengths of the line segments marked by x in the following triangles.

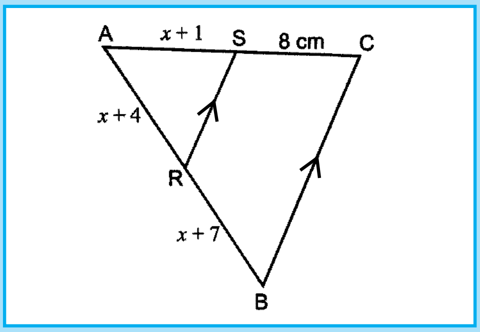




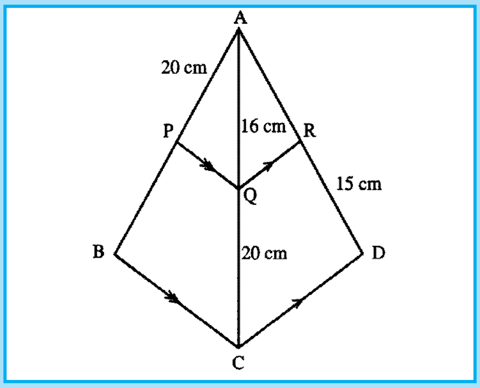
(2) In triangle ABC, AB = 12, AC = 8. Find AD.



(3) In triangle ABC, RS // BC. Prove that x can take only one value and find it.



(4) According to the information given in the diagram find PB and AR.

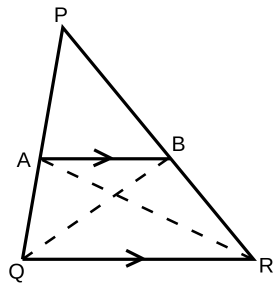


Theorem :

A straight line drawn parallel to a side of a triangle divides the other two sides proportionally.

Proof of the Theorem

A line drawn parallel to one side of a triangle divides the other two sides proportionally.



Data : In triangle PQR
AB // QR

To prove that : $\frac{PA}{AQ} = \frac{PB}{BR}$

Construction : Join AR and BQ

Proof : As the triangles PAB and ABQ are of the same altitude,

$$\frac{PAB\,\Delta}{ABQ\,\Delta} = \frac{PA}{AQ}$$

Similarly, the triangles PAB and BRA have the same altitude,

$$\frac{PAB\,\Delta}{BRA\,\Delta} = \frac{PB}{BR}$$

But,

$ABQ\,\Delta = BRA\,\Delta$ (between the same pair of parallels AB, and QR, and on the same base AB)

$$\therefore \frac{PAB\,\Delta}{ABQ\,\Delta} = \frac{PAB\,\Delta}{BRA\,\Delta}$$

$$\therefore \frac{PA}{AQ} = \frac{PB}{BR}$$

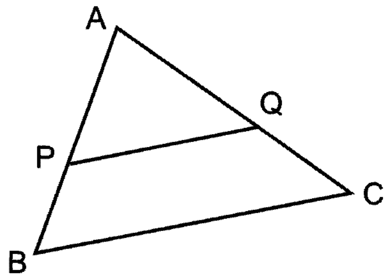
Converse of the Theorem :

If a straight line divides two sides of a triangle proportionally then it is parallel to the third side

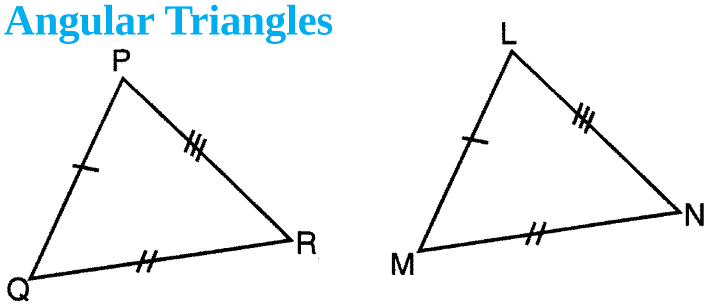
That is, if

$$\frac{AP}{PB} = \frac{AQ}{QC},$$

then PQ // BC.



12.2 Equi - Angular Triangles



Look at the two triangles PQR and LMN. You know that these triangles are congruent.

$\therefore$ in the two triangles PQR and LMN,

$$\hat{P} = \hat{L}$$

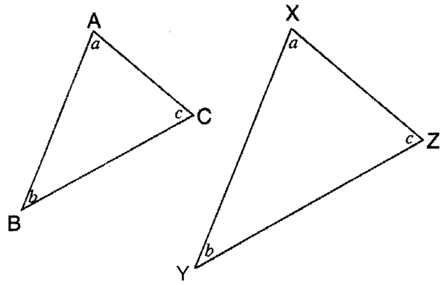
$$\hat{Q} = \hat{M}$$

$$\hat{R} = \hat{N}$$

If three angles of a triangle are equal to the three angles of another triangle, then, they are **equi-angular triangles**.

While the triangles PQR and LMN are congruent they are equi-angular too.

Accordingly congruent triangles are always equi-angular



Look at the two triangles ABC and XYZ. In these triangles,

$$\hat{A} = \hat{X}$$

$$\hat{B} = \hat{Y}$$

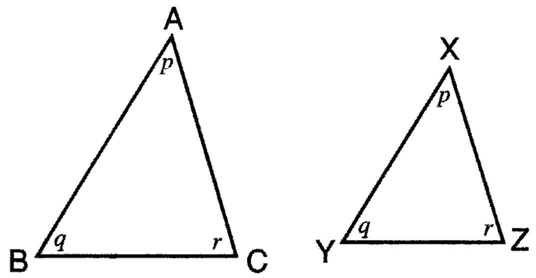
$$\hat{C} = \hat{Z}$$

The three angles of triangle ABC are equal to the three angles of the triangle XYZ.

Are the two triangles congruent?
You can see that they are not congruent but they are equi-angular. They are equal in shape, but not congruent.

Equi-angular triangles are not always congruent

Theorem :
The corresponding sides of two equi-angular triangles are proportional

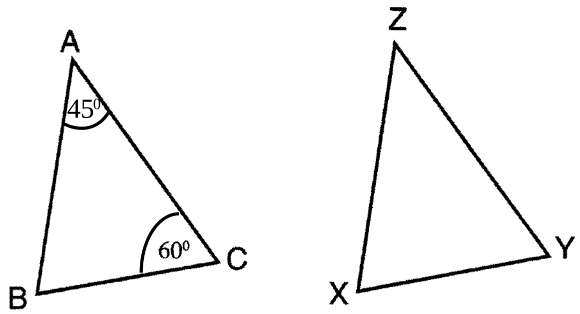


ABC and XYZ are equi-angular triangles.

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

Activity 12.2

To verify that the corresponding sides of two equi-angular triangles are proportional, go through the following activity.



Construct triangle ABC having the angles equal to the angles in the given triangle and AC = 4 cm.

Constuct the triangle XYZ such that YZ = 8 cm, $\hat{Y} = \hat{C} = 60^0$, and $\hat{Z} = \hat{A} = 45^0$. By measuring the remaining angles in both triangles verify that the triangles ABC and XYZ are equi-angular.

Now, measure the sides AB, BC, XY and XZ.

According to the construction, we know that, $\frac{AC}{YZ} = \frac{4}{8}$.

Now, finding the values of $\frac{AB}{XZ}$ and $\frac{BC}{XY}$, test whether the above theorem is true. Try to verify this theorem further by drawing various types of equi-angular triangles.

> **Converse of the theorem.**
> If the corresponding sides of two triangles are proportional, then the triangles are equi-angular.

Study carefully, the problems solved by using this theorem.

Example 3

In the diagram, AB//CD. Find the length of CD.

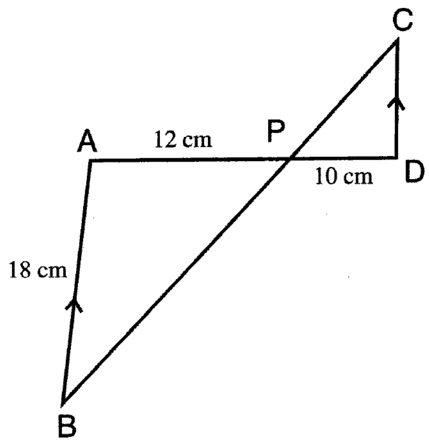
In triangles APB and PDC,

$A\hat{P}B = C\hat{P}D$ (vertically opposite angles)

$A\hat{B}P = P\hat{C}D$ (Alternate angles, AB // CD)

$\therefore B\hat{A}P = P\hat{D}C$ (Remaining angles of the triangles)

$\therefore$ Triangles APB and PDC are equi-angular.



$$\frac{AP}{PD} = \frac{AB}{CD} = \frac{BP}{PC}$$

AP = 12 cm, PD = 10 cm, AB = 18 cm

$$\frac{12}{10} = \frac{18}{CD}$$

$$CD = \frac{18 \times 10}{12}$$

$$= \underline{\underline{15 \text{ cm}}}$$

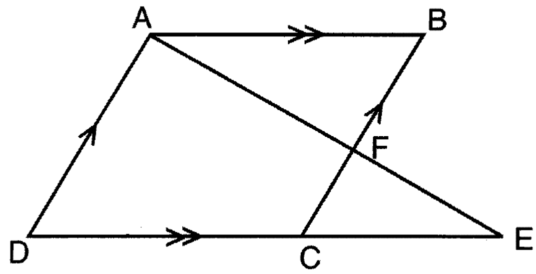
Example 4

F is a point on side BC of the parallelogram ABCD. AF and DC produced meet at E.

(i) Show that triangles ECF and ABF are equi-angular.

(ii) Show that EC : AB = FC : BF

Let us first draw a diagram to show this data.



In triangles, ABF and ECF,

$A\hat{F}B = C\hat{F}E$ (Vertically opposite angles)

$A\hat{B}F = F\hat{C}E$ (Alternate angles, AB // CE)

$B\hat{A}F = F\hat{E}C$ (Alternate angles, AB // CE)

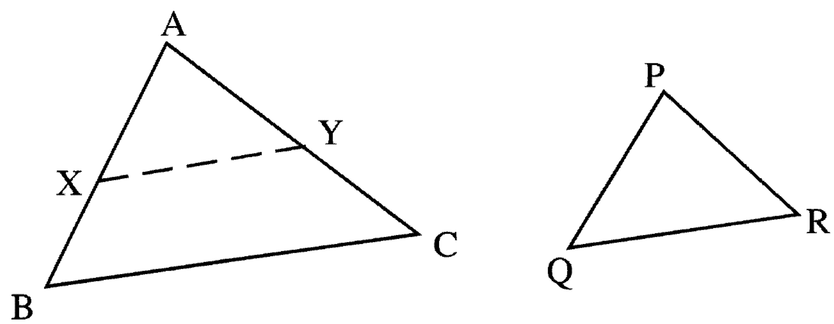
$\therefore$ Triangles ABF and ECF are equi-angular.

According to the above theorem,

$$\frac{AB}{EC} = \frac{BF}{FC} = \frac{AF}{FE}$$

$\therefore$ EC : AB = FC : BF

Let us prove the theorem that, the corresponding sides of two equi-angular triangles are proportional.



Data : In triangles ABC and PQR,
$\hat{A} = \hat{P}$; $\hat{B} = \hat{Q}$; $\hat{C} = \hat{R}$

To prove that : AB : PQ = BC : QR = CA : RP

Construction : Mark the points X and Y on AB and AC respectively such that AX = PQ and AY = PR

Proof : In triangles AXY and PQR

$\hat{A} = \hat{P}$ (Data)

AX = PQ (Construction)
AY = PR (Construction)

$\therefore \Delta AXY \equiv \Delta PQR$ (S, A, S)

$\therefore A\hat{X}Y = P\hat{Q}R$

But $P\hat{Q}R = A\hat{B}C$ (Data)

$\therefore A\hat{X}Y = A\hat{B}C$

As they are corresponding angles

XY // BC.

$$\therefore \frac{BX}{XA} = \frac{CY}{YA}$$

$$\frac{BX}{XA} + 1 = \frac{CY}{YA} + 1 \quad \text{(adding 1 to both sides)}$$

$$\frac{BX+XA}{XA} = \frac{CY+YA}{YA}$$

$$\frac{AB}{AX} = \frac{AC}{YA}$$

But AX = PQ (Construction)
AY = PR (Construction)

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$$

Similarly $\frac{AB}{PQ} = \frac{BC}{QR}$

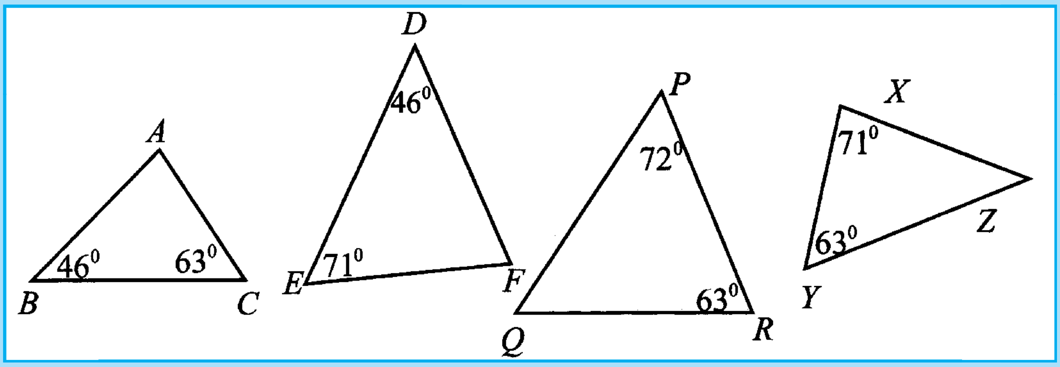
$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

$\therefore$ AB : PQ = BC : QR = CA : RP

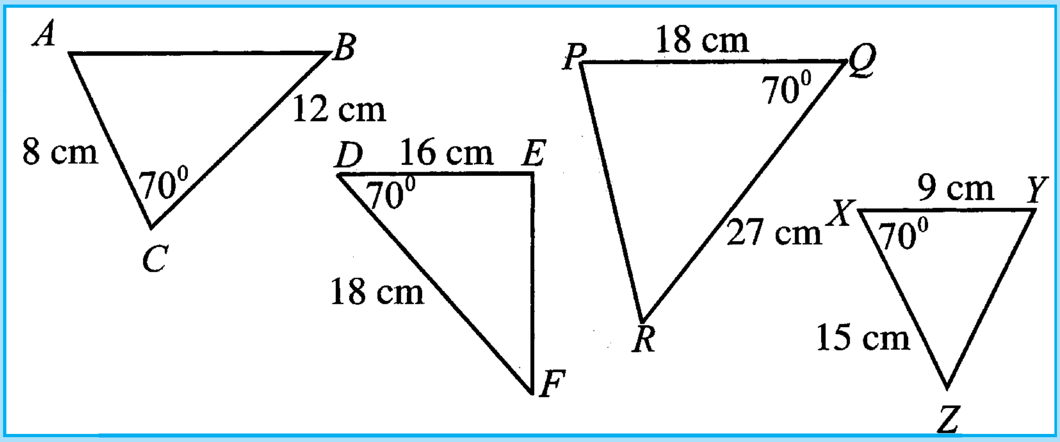
Exercise 12.2

1. Choose and name the equiangular triangles from the triangles given below. Give reasons why they are equianguler.

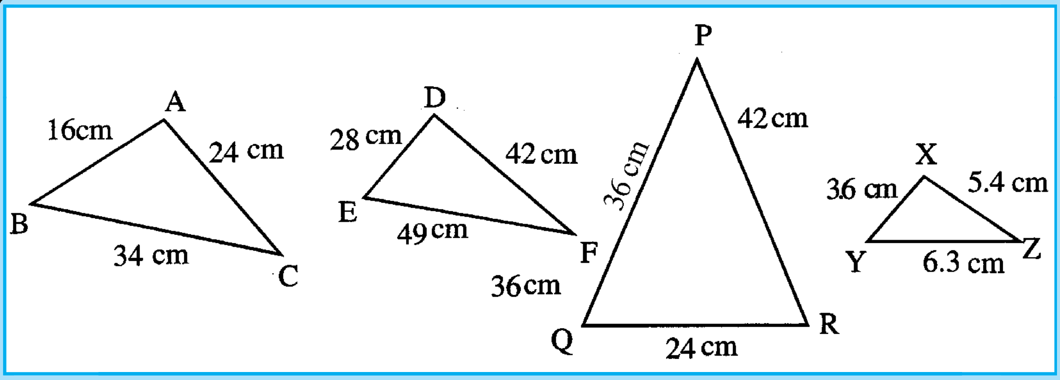
(i)



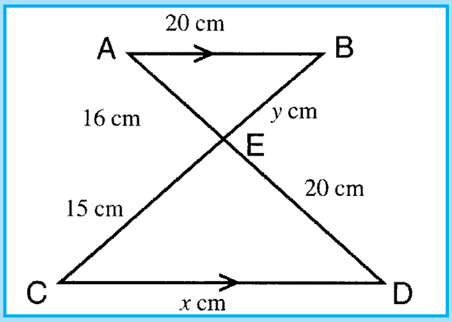
(ii)



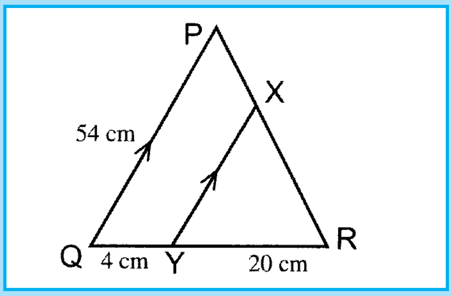
(iii)



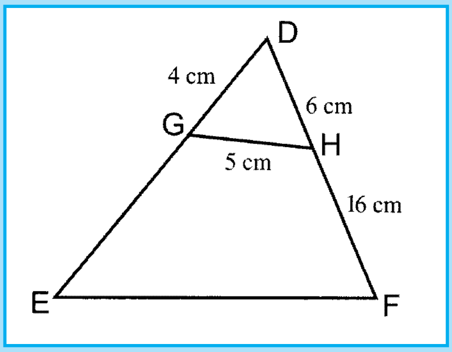
2. In the diagram AB//CD. Show that the triangles AEB and EDC are equiangular and find the lengths of CD and BE.



3. In triangle PQR, PR=36cm and PQ//XY. Find the lengths of XY and RX.



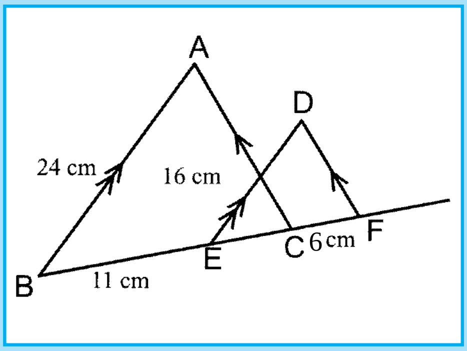
4. In the diagram $D\hat{G}H = E\hat{F}D$. Show that the triangles DGH and DFE are equiangular.
Find the lengths of the sides GE and EF



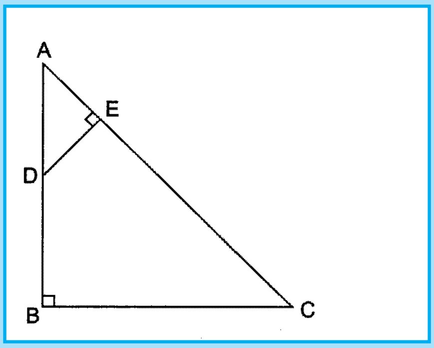
5. In the diagram AB//DE and AC//DF. Show that the triangles ABC and DEF are equiangular and find the length of EC.

AB = 24 cm BE = 11 cm

DE = 16 cm CF= 6 cm



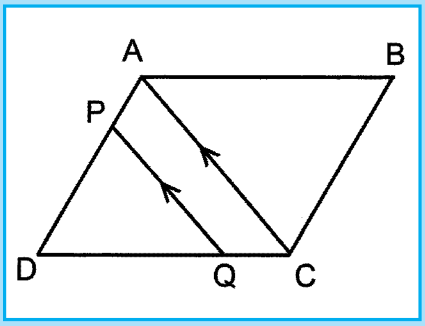
6. In the diagram $A\hat{B}C = A\hat{E}D = 90^0$. Prove that the triangles AED and ABC are aqui-angular.
Show that AE : AB = DE : BC



7. AE is the perpendicular drawn from A to the diagonal BD of rectangle ABCD. Prove that AD:BD = DE : BC

8. A is a right angle in triangle ABC. The perpendicular drawn from A to BC is AD. Prove that $AB^2 = BD.BC$.

9. ABCD is a parallelogram. AC // PQ . Prove that AB.PQ = AC.DQ



(10) In triangle ABC, AB = BC. P and Q are points on AC. The line drawn parallel to AB through P meets BC at S. The line drawn parallel to CB through Q meets PS at R. Prove that AB:AC = PR:PQ.