11 Mid Point Theorem

By studying this lesson you will acquire knowledge on the following :

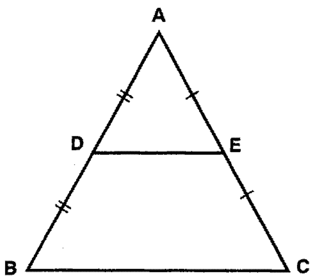
- The mid point theorem
- Proof of the mid point theorem
- Converse of the mid point theorem
- Proof of the converse of the mid point theorem
- Solving problems using the above two theorems

11.1 Introduction

There is a direct relationship between the line joining the mid points of two sides of a triangle and its third side.

11.2 Mid point theorem

The line joining the mid points of two sides of a triangle is parallel to the third side and is half the size of the third side.



D and E are the mid points of the sides AB and AC respectively of the triangle ABC. Then

$$DE \parallel BC \text{ and } DE = \frac{1}{2}BC.$$

Activity 11.1

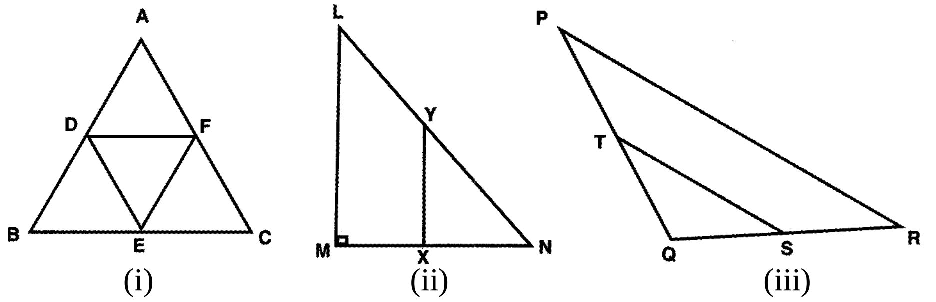
(1) Draw a triangle ABC. AB = 6 cm, BC = 5 cm, AC = 4 cm. Mark the mid points D and E of any two sides and join them. By measuring DE or by any other method, compare it with the other side.

(2) In a quadrilateral ABCD, AB//DC and AD//BC. What can you say about the quadrilateral ABCD?

(3) In a quadrilateral PQRS, PS = QR and PS//QR.What can you say about the quadrilateral PQRS ?

Exercise 11.1

(1)

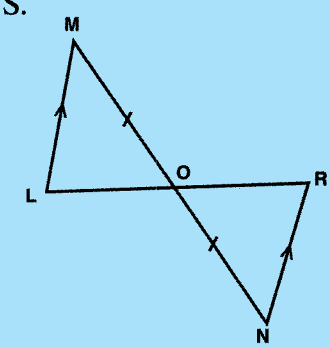


(i) In triangle ABC, AB = 8 cm, BC = 7 cm, AC = 6 cm. D, E and F are the mid points of AB, BC, and CA respectively. Find the lengths of the sides of the triangle DEF.

(ii) In triangle LMN, LM = 8 cm, MN = 6 cm and $L\hat{M}N = 90^0$. X and Y are the mid points of MN and LN respectively. Find $Y\hat{X}N$ and YN

(iii) In triangle PQR, S and T are the mid points of QR and PQ respectively. PR = 24 cm, QR = 13 cm and the perimeter of $\triangle$PQR is 52 cm. Find the lengths of QT and TS.

(2)

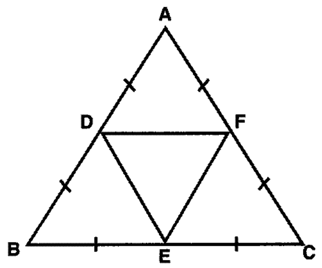


In the figure LM//NR, LR and MN intersect at O. MO = ON. Prove that the triangles LMO and NRO are congruent.

Hence write down the corresponding equal sides of the two triangles.

Example 1

Show that the triangle obtained by joining the mid points of the sides of an equilateral triangle, too is an equilateral triangle.



Data : ABC is an equilateral triangle
AD = DB, BE = EC and AF = FC

To prove that : DEF is an equilateral triangle

Proof : In the triangle ABC,
AD = DB and AF = FC. (Data)

$\therefore$ DF = $\frac{1}{2}$BC ——— ① (Mid point theorem)

In the triangle ABC,
AD = DB and BE = EC. (Data)

$\therefore$ DE = $\frac{1}{2}$AC ——— ② (Mid point theorem)

In the triangle ABC,
BE = EC and AF = FC.

$\therefore$ EF = $\frac{1}{2}$AB ——— ③ (Mid point theorem)

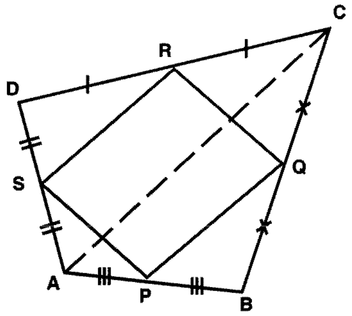
But AB = BC = AC (ABC is an equilateral triangle)

From (1), (2) and (3): DF = DE = EF
Hence Δ DEF is an equilateral triangle.

Example 2

ABCD is a quadrilateral. P,Q,R and S are the mid points of AB, BC, CD and DA respectively. Show that PQRS is a parallelogram.

Data : In the quadrilateral ABCD
AP = PB, BQ = QC,
CR = RD and AS = DS



To prove that : PQRS is a parallelogram

Construction : Join A and C

Proof : In triangle ADC

DS = AS (S is the mid point of AD)

DR = RC (R is the mid point of DC)

$\therefore$ $SR = \frac{1}{2} AC$ - - - ——— ① (Mid point theorem)

And SR//AC - - - - ——— ② (Mid point theorem)

In the triangle ABC

AP = BP (P is the mid point of AB.)

BQ = CQ (Q is the mid point of BC.)

$\therefore$ $PQ = \frac{1}{2} AC$ - - - ——— ③ (Mid point theorem)

And PQ//AC ——— ④ (Mid point theorem)

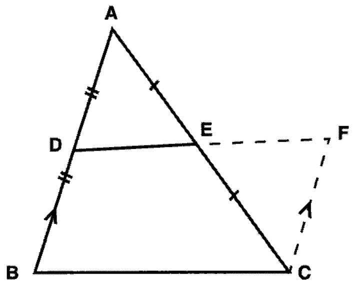
From ① and ③: PQ=SR

From ② and ④: PQ//SR

Hence PQRS is a parallelogram.

Proof of the mid point theorem.

Data : ABC is a triangle, D and E are the mid points of AB and AC respectively.



To prove that : DE//BC and

$DE = \frac{1}{2}BC$

Construction : Draw a line through C parallel to BA to meet DE produced at F

Proof : In triangles ADE and CEF,

	AE = EC	(data)
	$A\hat{D}E = E\hat{F}C$	(alternate angles, BA // CF)
	$A\hat{E}D = C\hat{E}F$	(vertically opposite angles)
/	$\Delta ADE \equiv \Delta CFE$	(A A S)

$\therefore$ AD = CF and DE = EF

But AD = BD; $\therefore$ BD = CF.

In the quadrilateral BDFC, BD = CF and BD//CF

$\therefore$ BDFC is a parallelogram.

/ DF = BC and DF//BC

But DE = EF (corresponding sides of congruent triangles)

$\therefore DE = \frac{1}{2}DF$

$\therefore DE = \frac{1}{2}BC$ (Since DF = BC)

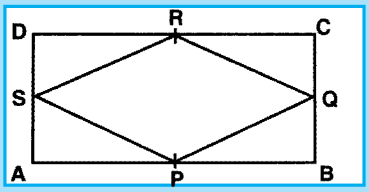
Hence DE // BC

and $DE = \frac{1}{2}BC$

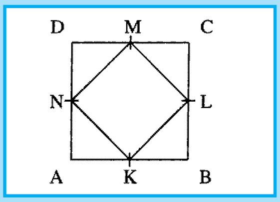
Exercise 11.2

(1) ABC is an isosceles triangle. P, Q and R are the mid points of AB, BC and CA respectively. Show that PQR is an isosceles triangle.

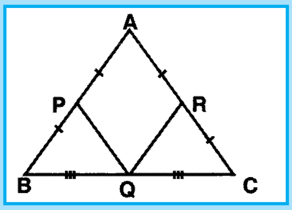
(2) ABCD is a rectangle. P, Q, R and S are the mid points of AB, BC, CD and DA respectively. Show that PQRS is a rhombus.



(3) ABCD is a square. K, L, M and N are the mid points of AB, BC, CD and DA respectively. Show that KLMN is a square.



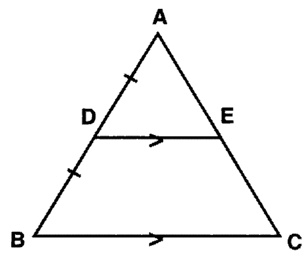
(4) In a triangle ABC, AB = AC. P, Q and R are the mid points of AB, BC and CA respectively. Show that APQR is a rhombus.



Converse of the mid point theorem:

Theorem:
In a triangle a line drawn through the mid point of one side, parallel to another side bisects the third side.

In triangle ABC, if AD=BD and DE//BC then AE=EC

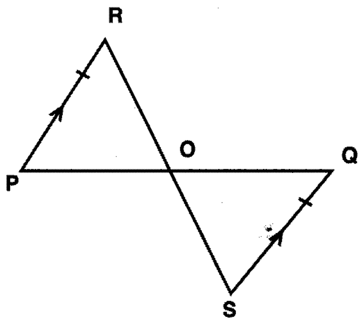


Activity 11.2

(1) Draw triangle ABC, with AB=6 cm, BC=5 cm, AC=4 cm. Mark the mid point D of AB. Draw a line through D parallel to BC. Let the parallel line cut AC at E. Measure the length AE and find a relationship between AE and EC.

(2) State the properties of a parallelogram

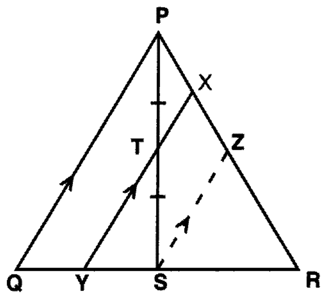
(3) In the figure PR//SQ and PR=SQ. PQ and SR intersect at O. Prove that the triangles POR and SOQ are congruent. Hence write the corresponding sides of the two triangles.



Example 3

In a triangle PQR, S is the mid point of QR and T is the mid point of PS. The line drawn through T parallel to PQ intersects PR and QR at X and Y respectively.

Show that $XY = \frac{3}{4}PQ$.



Data : In the triangle PQR, QS = SR, PT = TS
QP // YX.

To prove that : $XY = \frac{3}{4}PQ$.

Construction : Draw a line SZ, parallel to QP to meet PR at Z.

Proof : In triangle PSZ

PT = TS

and TX // SZ

$\therefore$ PX = XZ

and $TX = \frac{1}{2}SZ$

If TX = a, then SZ = $2a$ ——— ①

In triangle PQR,

QS = SR

and QP // SZ

$\therefore$ PZ = ZR; $SZ = \frac{1}{2}PQ$

Since SZ = $2a$, PQ = $4a$ ——— ②

In triangle PQS,

PT = TS and YT//PQ

$\therefore$ QY = YS and TY = $\frac{1}{2}$PQ

$\because$ PQ = 4a, TY = 2a ——— (3)

From (1), (2) and (3)

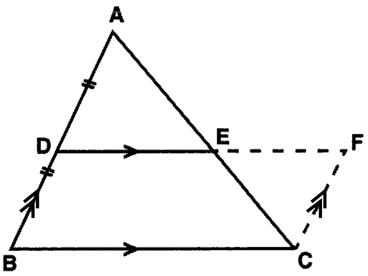
XY = TX + TY = a + 2a = 3a,

PQ = 4a,

$\therefore$ XY = $\frac{3}{4}$PQ

$XY = 3a$
$XY = 3\,\frac{PQ}{4}$
$XY = \frac{3}{4}PQ$

Proof of the converse of the mid point theorem



Data : In triangle ABC, AD = BD and DE // BC

To prove that : AE = EC

Construction : Draw a line through C parallel to DB to meet DE produced at F.

Proof : In the quadrilateral BCFD

BC // DF

BD // CF

$\therefore$ BCFD is a parallelogram.

$\therefore$ BD = CF

But, BD = AD

$\therefore$ AD = CF.

In the triangles ADE and CFE

AD = CF

$A\hat{D}E = E\hat{F}C$ (alternate angles, DA // CF)

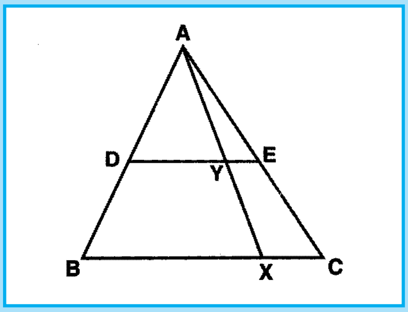
$A\hat{E}D = C\hat{E}F$ (vertically opposite angles)

$\therefore \Delta ADE \equiv \Delta CFE$ (AAS)

$\therefore AE = EC$

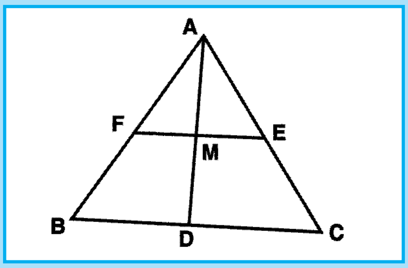
Exercise 11.3

(1) ABC is a triangle. D and E are the midpoints of AB and AC respectively. X is any point on BC. AX and DE intersect at Y. Show that AY=YX.



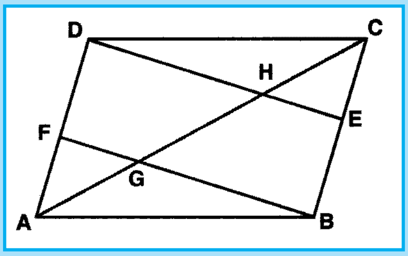
(2) ABC is a triangle. D, E, and F are the mid points of BC, CA and AB respectively. AD and FE intersect at M. Prove that

i. AM = MD and
ii. FM = ME.

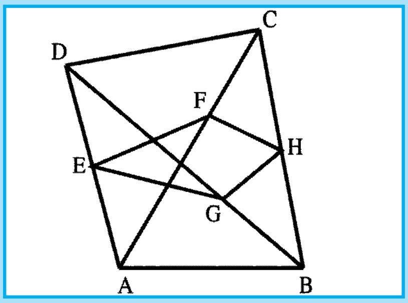


(3) ABCD is a parallelogram. E and F are the mid points of BC and AD respectively. The diagonal AC meets BF and DE at G and H respectively. Prove that

i. BF // ED
ii. AG = GH = HC.

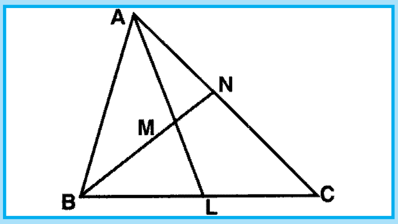


(4) In a quadrilateral ABCD, E, F, G and H are the mid points of AD, AC, BD and BC respectively. If AB = DC, prove that EFHG is a rhombus and hence deduce that EH is perpendicular to FG.

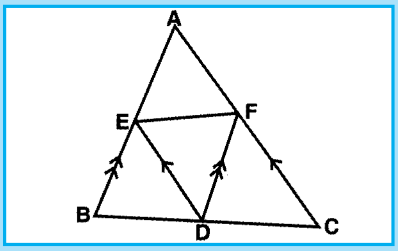


(5) In triangle ABC, L is the mid point of BC. M is the mid point of AL. BM produced a meets AC at N. Prove that

$$AN = \frac{1}{3} AC$$

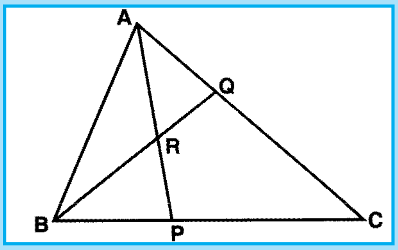


(6) ABC is a triangle and D is the midpoint of BC. A line drawn through D, parallel to CA meets AB at E and a line drawn through D, parallel to BA meets CA at F. Prove that $EF = \frac{1}{2}$ BC and $DE = \frac{1}{2} AC$.

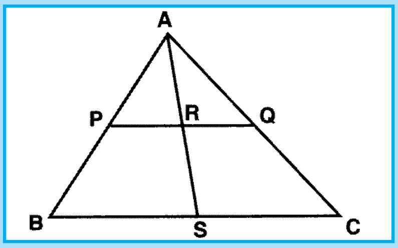


(7) In triangle ABC, P is the midpoint of BC. Q is a point on AC such that AQ:QC = 1:2. AP and BQ intersect at R. Prove that AR=RP and

$$RQ = \frac{1}{4} BQ$$



(8) In triangle ABC, P and Q are the mid points of AB and AC respectively. R is the mid point of PQ. AR produced meets BC at S. Prove that BS = SC.



(9) ABCD is a parallelogram. The side CB is produced to E so that CB=BE. DE meets AB at G. Prove that G is the midpoint of AB.