09 Percentages

By studying this lesson you will acquire knowledge on the following :
- Calculating the compound interest for an allocated period of time
- Buying goods under hire purchase
- Finding the interest rate when a loan is obtained with the interest being based on the reducing balance

9.1 Compound interest

A report released by a financial institute which pays a 10% annual rate of interest to a depositor is given below.

Date	Details	Deposit Rs	Withdrawal Rs	Balance Rs
2004-01-01	Cash Deposit	5000.00	-	5000.00
2004-12-31	Interest year 2004	500.00	-	5500.00
2005-01-01	Balance as at date	-	-	5500.00
2005-12-31	Interest year 2005	550.00	-	6050.00
2006-01-01	Balance as at date	-	-	6050.00
2006-12-31	Interest year 2006	605.00	-	6655.00

When examining this report, we can see the way interest is paid. The interest paid for the year 2004 at the annual rate of 10% is Rs 500. This interest and the deposit are added and this amount is considered as the deposit for the next year. That is Rs 5500. In the second year, this Rs 5500 will get Rs 550 as the interest. The interest for the next year is calculated for the sum of Rs 5500 and the interest of Rs 550.

When calculating the interest in this manner, the interest is calculated on the interest too. This is known as **compound interest.**

When calculating simple interest for several years,the interest for the money deposited is calculated for one year and multiplied by the number of years.

For free distribution

Thus the interest for 1 year for Rs 5000 $= \text{Rs } 5000 \times \dfrac{10}{100}$

$= \text{Rs } 500$

The interest for Rs 5000 for 3 years $= \text{Rs } 500 \times 3$

$= \text{Rs } 1500$

∴ The amount at the end of the 3rd year is $= \text{Rs } 5000 + 1500$

$= \underline{\text{Rs } 6500}$

This is less than the amount gained when calculating the compound interest.

Example 1

A person borrows Rs 12000 at a compound interest rate of 25% per annum from a commercial bank. How much should he pay to settle the loan at the end of two years?

Solution

Loan $= \text{Rs. } 12000$

Rate of interest $= 25\%$

Period $= 2$ years

The interest that should be paid at the end of the first year $= \text{Rs } 12000 \times \dfrac{25}{100}$

$= \text{Rs } 3000$

The amount of the loan at the begining of the second year $= \text{Rs } 12000 + \text{Rs } 3000$

$= \text{Rs } 15000$

The interest for the second year $= \text{Rs } 15000 \times \dfrac{25}{100}$

$= \text{Rs } 3750$

The amount to be paid at the end of the second year $= \text{Rs } 15000 + \text{Rs } 3750$

$= \underline{\text{Rs } 18750}$

For free distribution

Example 2

A person deposited Rs. 100 000 on compound interest at 12% per annum as a fixed deposit. Find the total amount at the end of 3 years ?

Solution

The principal	$= Rs \ 100 \ 000$
Interest for the first year	$= Rs \ 100 \ 000 \times \dfrac{12}{100}$
	$= Rs \ 12 \ 000$
Amount for the second year	$= Rs \ 100 \ 000 + Rs \ 12 \ 000$
	$= Rs \ 112 \ 000$
Interest for the second year	$= Rs \ 112 \ 000 \times \dfrac{12}{100}$
	$= Rs \ 13 \ 440$
Amount for the third year	$= Rs \ 112 \ 000 + Rs \ 13 \ 440$
	$= Rs \ 125 \ 440$
Interest for the third year	$= Rs \ 125 \ 440 \times \dfrac{12}{100}$
	$= Rs \ 15 \ 052.80$
The amount at the end of the third year	$= Rs \ 125 \ 440 + Rs \ 15 \ 052.80$
	$= Rs \ 140 \ 492.80$

> The procedure of calculating the interest for time unit by time unit and adding it to the principal and calculating interest on that amount for the next period is called the method of **compound interest**

9.2 Hire Purchase

Pay Rs.5000 initally to buy a television set and pay the balance in 12 instalments of Rs. 1200 each."

You may have often seen such commercial advertisements on television or in newspapers.

When we need to buy certain commodities, if we cannot afford to pay for an outright purchase, we are able to pay a part of the total value and pay the balance in instalments with an interest. This method of purchasing is called **hire - purchase.**

Since, after the initial payment, the balance is paid in installements, interest is calculated on the diminishing balance. Diminishing balance is where the loan amount is reduced when the monthly instalment is paid.Then the interest is calculated only on the outstanding loan amount.

In this procedure, there are a lot of advantages and disadvantages. One advantage is that the commodity can be used before paying the total price. The disadvantages of seasonal price fluctuations will also be less. A disadvantage is that both money and the item will be reclaimed when the due instalment is not paid on time. Another disadvantage is that when paying in instalments, the total amount paid is more than when making an outright purchase.

Example 3

A washing machine which can ebe purchased outright at Rs 15500 can also be bought on hire purchase by making an initial payment of Rs 3500 and paying the balance by in 6 equal monthly instalments at an interet rate of 24% per annum. Find the value of a monthly instalment.

Interest rate per annum	$= 24\%$
Interest rate per month	$= 24\% \times \dfrac{1}{12}$
	$= 2\%$
Cash Price of the machine	$= Rs\ 15\ 500$
Down Payment	$= Rs\ 3\ 500$
The balance to be paid	$= Rs\ (15\ 500 - 3\ 500\ ) = Rs\ 12\ 000$
Time period to pay the balance	$= 6\ months$
One instalment	$= \dfrac{Rs\ 12000}{6}$
	$= Rs\ 2\ 000$
The interest for an instalment per month	$= Rs\ 2000 \times \dfrac{2}{100}$
	$= Rs\ 40$

The loan amount for the 1st month	$= Rs\ 12\ 000 = 6 \times 2000$
The loan amount for the 2nd month	$= Rs\ 10\ 000 = 5 \times 2000$
The loan amount for the 3rd month	$= Rs\ 8\ 000 = 4 \times 2000$
The loan amount for the 4th month	$= Rs\ 6\ 000 = 3 \times 2000$
The loan amount for the 5th month	$= Rs\ 4\ 000 = 2 \times 2000$
The loan amount for the 6th month	$= Rs\ 2\ 000 = 1 \times 2000$

The interest for the 1st month $= Rs\ 6 \times 2\ 000 \times \dfrac{2}{100} = 40 \times 6$

The interest for the 2nd month $= Rs\ 5 \times 2\ 000 \times \dfrac{2}{100} = 40 \times 5$

The interest for the 3rd month $= Rs\ 4 \times 2\ 000 \times \dfrac{2}{100} = 40 \times 4$

The interest for the 4th month $= Rs\ 3 \times 2\ 000 \times \dfrac{2}{100} = 40 \times 3$

The interest for the 5th month $= Rs\ 2 \times 2\ 000 \times \dfrac{2}{100} = 40 \times 2$

The interest for the 6th month $= Rs\ 1 \times 2\ 000 \times \dfrac{2}{100} = 40 \times 1$

Total interest

$$= Rs\ (6 \times 40) + (5 \times 40) + (4 \times 40) + (3 \times 40) + (2 \times 40) + (1 \times 40)$$
$$= Rs\ 40 \times (1 + 2 + 3 + 4 + 5 + 6)$$
$$= Rs\ 40 \times \dfrac{6}{2}(1 + 6)$$
$$= Rs\ 40 \times 3 \times 7 = Rs\ 840$$

Here, 1 + 2 + 3 + ... + 6 is an arithmatic progression.

∴ the sum can be obtained by applying the formula, sum $= \dfrac{n}{2}(n + 1)$

this is called the **total number of month units**

Total interest = (Interest for an instalment) × Total number of month units
∴ Amount to be paid in instalments $\quad$ = Amount to pay + total interest
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ = Rs 12000 + 840
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ = Rs 12840

Because it has to be paid in 6 equal instalments,

the money that should be paid per instalment $= Rs \dfrac{12\,840}{6}$ (As it is paid in equal instalments)

$$= Rs.2140$$

Example 4

A Rs 40000 down payment and 16 monthly instalments can be arranged to purchase a furniture set under hire purchase which can be purchased outright at Rs 120000. The rate of interest per annum is 18%

(i) What is the loan amount that should be paid monthly?
(ii) What is the number of month units?
(iii) What is the total interest that should be paid?
(iv) What is the total amount that should be paid?
(v) What amount is to be paid as an instalment?

(i) Outstanding loan after the initial payment $= Rs\ 120\,000 - Rs\ 40\,000$
$= Rs\ 80000$

   Time period $= 16$ months

∴ The part of the loan to be paid monthly $= Rs\ \dfrac{80\,000}{16}$

$= Rs\ 5000$

(ii) The number of month units $= \dfrac{16}{2}(16+1)$

$= 8 \times 17$

$= 136$

(iii) The total interest to be paid $= Rs\ 5\,000 \times \dfrac{18}{100} \times \dfrac{1}{12} \times 136$

$= Rs\ 10\,200$

(iv) The total amount that should be paid $= Rs\ 80\,000 + Rs\ 10\,200$
$= Rs\ 90\,200$

(v) $\therefore$ Instalment $\qquad = Rs \dfrac{90\,200}{16}$

$\qquad\qquad\qquad\qquad = \underline{Rs\ \underline{5637.50}}$

Exercise 9.2

1. The monthly salary of a person is Rs.18 000. He was granted ten month salary by his department as a distress loan, at an interest rate of 5% per annum. This loan is deducted from the salary in 60 monthly instalments.
    (i)   What part of the loan is to be paid monthly?
    (ii) What is the interest for the part of the loan to be paid monthly?
    (iii) What is the number of month units for which the interest should be calculated?
    (iv) What is the total interest?
    (v)   How much is to be paid as an instalment?

2. A computer is priced at Rs 59 000. To buy this under hire purchase, Rs.5000 should be paid as a down payment and the balance in 18 equal monthly instalments. The rate of interest per annum is 15%

    (i) Find the number of month units for which interest should be calculated.
    (ii) What is the total interest?
    (iii) How much is to be paid as an instalment?

3. A person was granted Rs 396000 by a bank as a housing loan and he agreed to pay it back in 3 years in equal monthly instalments at an interest rate of 24% per annum on the diminishing balance. Find how much is to be paid as an instalment?

4.   Ramesh was granted a loan of Rs 57 600 by a bank, at an interest rate of 15 per annum. This loan and the interets on the diminishing balance should be paid by equal monthly instalments in 36 months. Find how much should be paid as an instalment.

Example 5

A down payment of Rs 5000 and 14 monthly instalments of Rs 1650 each should be paid to purchase a colour television set under hire purchase which can be bought outhrigt at Rs 26000

1. What is the outstanding balance?
2. What portion should be paid monthly?
3. How much is the total amount that should be paid in instalments?
4. Find the total interest that should be paid.
5. Find the number of month units
6. What is the interest per month unit?
7. Find the rate of interest per annum.

Solution

1. Outstanding balance
$$= \text{Rs } 26000 - 5000$$
$$= \text{Rs } 21000$$

2. The part of the loan to be paid monthly
$$= \text{Rs } \frac{21000}{14}$$
$$= \text{Rs } 1500$$

3. The total amount that should be paid in instalments
$$= \text{Rs } 1650 \times 14$$
$$= \text{Rs } 23100$$

4. The total interest that should be paid
$$= \text{Rs } 23100 - \text{Rs } 21000$$
$$= \text{Rs } 2100$$

5. The number of month units
$$= \frac{14}{2}(14+1)$$
$$= 7 \times 15$$
$$= 105$$

6. The interest per month unit
$$= \text{Rs } \frac{2100}{105} = \underline{\text{Rs } 20}$$

7. The rate of interest per annum
$$= \frac{20}{1500} \times 12 \times 100 \%$$
$$= 16\%$$

Exercise 9.3

1. A motor car worth Rs 1 400 000 can be bought by making an intial payment of Rs 200 000 and by paying the balance in 24 equal monthly instalments of Rs 59 175.

   (i) Find the total amount paid in 24 instalments?
   (ii) Find the total interest paid
   (iii) What is the monthly instalment of the loan without the interest ?
   (iv) Find the number of month units.
   (v) Find the interest per month unit.
   (vi) Find the rate of interest.

2. The price of a refrigerator is Rs 20 720.
   The refrigerator can be obtained by paying Rs 2000 as down payment and the rest by monthly instalments of Rs 942.50 each. If the number of month units for which the interest should be paid is 12 x 25,

   (i) what is the number of months allowed for paying the outstanding balance?
   (ii) what part of the loan should be paid monthly?
   (iii) how much interest is to be paid?
   (iv) find the rate of interest per annum.

3. The price of a sewing machine is Rs 16 750. This can be bought under hire purchase by paying Rs 1750 as down payment and the rest by 10 monthly instalments of Rs 1582.50 each.

   (i) What is the amount paid in 10 months?
   (ii) What is the interest that should be paid?
   (iii) What is the number of month units?
   (iv) What is the interest per month unit?
   (v) What part of the loan should be paid monthly?
   (vi) What is the rate of interest per annum?