6 Binomial Expressions

By studying this lesson you will acquire knowledge on the following :

- Expansion of $(a+b)^3$
- Expansion of $(a-b)^3$
- Use of cubic expressions.

6.1 Expansion of $(a + b)^3$

The length, breadth and height of a container, the shape of which is a cube of side *a* cm are increased by *b* cm each. Let us find the new volume.

Initial volume $= a \times a \times a \text{ cm}^3$

$= a^3 \text{ cm}^3$

Volume after increase in size $=(a + b) \times (a + b) \times (a + b) \text{ cm}^3$

$= (a + b)^3 \text{ cm}^3$

To simplify the above expression, we should expand $(a+b)^3$.

$$(a+b)^3 = (a+b)(a+b)(a+b)$$

$$(a^2+2ab+b^2)(a+b)$$

$$= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$$

$$= a^3+3a^2b+3ab^2+b^3$$

$$\boxed{(a+b)^3 = a^3+3a^2b+3ab^2+b^3}$$

Accordingly, $(x+a)^3 = x^3+3x^2a+3xa^2+a^3$

6.2 Expansion of $(a - b)^3$

As above, let's get the expansion of $(a-b)^3$

$$
\begin{aligned}
(a-b)^3 &= (a-b)(a-b)(a-b) \\
&= (a^2-2ab+b^2)(a-b) \\
&= a^3-2a^2b+ab^2-a^2b+2ab^2-b^3 \\
&= a^3-3a^2b+3ab^2-b^3
\end{aligned}
$$

$$\boxed{(a-b)^3 = a^3-3a^2b+3ab^2-b^3}$$

Accordingly $(y-n)^3 = y^3-3y^2n+3yn^2-n^3$

Exercise 6.1

1. Copy the table given below and complete it

	Expansion of $(a+b)^3$	Expansion of $(a-b)^3$
number of terms	....	....
highest power of *a*	....	....
highest power of *b*	....	....
coefficient of a^3	....	....
coefficient of b^3	....	....
coefficient of a^2b	....	....
coefficient of ab^2	....	....
the pattern of the coefficients	....	....

2. Fill in the blanks.

(i) $(a+b)^3 = a^3 +$ ...... + ... + b^3

(ii) $(x+y)^3 =$ ..... + $3x^2y + 3xy^2$ + ...

(iii) $(p+q)^3 =$ ...... + ... + $3pq^2$ + ...

(iv) $(m+n)^3 =$ .... + $3m^2n$ + ... + ...

(v) $(l-m)^3 = l^3 - 3l^2m$ + ... - m^3

(vi) $(x-y)^3 =$ .... - .... + $3xy^2$ - ......

3. Expand

(i) $(x+p)^3$ (ii) $(t+k)^3$ (iii) $(r+s)^3$

(iv) $(p-q)^3$ (iv) $(c-d)^3$ (iv) $(u-v)^3$

4. Obtain the expansion of $(x - y)^3$, by substituting $(-y)$ for y in the expansion of $(x + y)^3$

5. The length, breadth and height of a container, the shape of which is a cube of side ***a*** cm is increased by ***q*** cm. Find the increase in volume caused by the change.

6.3 Application of the expansions of $(x + b)^3$ and $(x - b)^3$

The cube of a binomial expression can be expanded using the above method.

Example 1 Expand $(x+2)^3$

$$(x+2)^3 = x^3 + 3x^2 \times 2 + 3x \times 2^2 + 2^3$$
$$= \underline{\underline{x^3 + 6x^2 + 12x + 8}}$$

Example 2 Expand $(a-3)^3$

$$(a - 3)^3 = a^3 - 3a^2 .3 + 3 .a. 3^2 - 3^3$$
$$= \underline{\underline{a^3 - 9a^2 + 27a - 27}}$$

Example 3 Expand $(1-y)^3$

$$(1-y)^3 = 1^3 - 3.1^2.y + 3.1.y^2 - y^3$$
$$= \underline{\underline{1 - 3y + 3y^2 - y^3}}$$

This method of expansion of a cube of a binomial expression can be used to find the third power of certain numbers.

Example 4 Find the value of 104^3

$$104^3 = (100+4)^3$$
$$= 100^3 + 3 \times 100^2 \times 4 + 3 \times 100 \times 4^2 + 4^3$$
$$= 1\,000\,000 + 3 \times 10\,000 \times 4 + 3 \times 100 \times 16 + 64$$
$$= 1\,000\,000 + 120\,000 + 4800 + 64$$
$$= \underline{\underline{1\,124\,864}}$$

Example 5 Find the value of 47^3

$$\begin{aligned} 47^3 &= (50-3)^3 \\ &= 50^3 - 3.50^2.3 + 3.50.3^2 - 3^3 \\ &= 125\,000 - 3\times 2500\times 3 + 3\times 50\times 9 - 27 \\ &= 125\,000 - 22\,500 + 1350 - 27 \\ &= 126\,350 - 22\,527 \\ &= \underline{\underline{103\,823}} \end{aligned}$$

Exercise 6.2

01 Expand each of the expressions given below.

(i) $(x+3)^3$ (ii) $(a-2)^3$ (iii) $(b-5)^3$

(iv) $(4+t)^3$ (v) $(5-y)^3$ (vi) $(1+x)^3$

(vii) $(10-r)^3$ (viii) $(1-mn)^3$

02. Expand

(i) $\left(x+\frac{1}{x}\right)^3$ (ii) $\left(n-\frac{1}{n}\right)^3$ (iii) $\left(1-\frac{1}{a}\right)^3$

03. Evaluate using the expansion of the cube of a binomial expression.

(i) 102^3 (ii) 95^3 (iii) 53^3 (iv) 96^3

04. If $a=5$, $b=2$ then find the value of,

(i) $a-b$ (iv) $a^3-b^3=(a-b)(a^2+ab+b^2)$

(ii) (a^2+ab+b^2) (v) $(a+b)(a^2-ab+b^2)$

(iii) $(a-b)(a^2+ab+b^2)$ (vi) $a^3+b^3=(a+b)(a^2-ab+b^2)$

05. i. If $(a+b)=5$ and $ab=6$, evaluate a^3+b^3

ii. If $(a-b)=6$ and $ab=7$, evaluate a^3-b^3

iii. If $x-\frac{1}{x}=\frac{8}{3}$, find the value of $x^3-\frac{1}{x^3}$

For exploration

Study the expansion of gradually increasing powers of $(x+a)$ and the pattern of their coefficients.

expansion of the powers of $(x+a)$			pattern of coefficients
$(x+a)^0$	=	1	1
$(x+a)^1$	=	$x+a$	1 1
$(x+a)^2$	=	$x^2+2ax+a^2$	1 2 1
$(x+a)^3$	=	$x^3+3ax^2+3a^2x+a^3$	1 3 3 1

The above pattern of coefficients is called the Pascal Triangle.

By writing the next steps of the pattern shown on the right hand side,

(i) write the expansion of $(x+a)^4$

(ii) write the expansion of $(x+a)^5$

Study the following factorization.

- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

  $a^3 + b^3 = (a+b)^3 - 3a^2b - 3ab^2$

  $= (a+b)^3 - 3ab(a+b)$

  $= (a+b)[(a+b)^2 - 3ab]$

  $= (a+b)[a^2 + 2ab + b^2 - 3ab]$

  $= (a+b)(a^2 - ab + b^2)$

- By using the expression, $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ show that,

  $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$