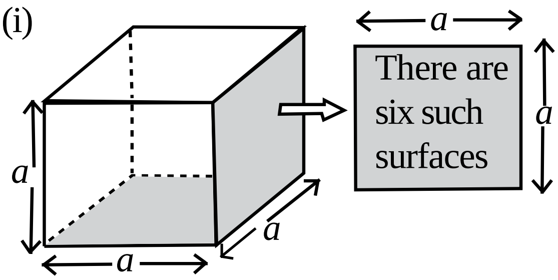
04 The Surface Area of Solids

By studying this lesson you will acquire knowledge on the surface area of,

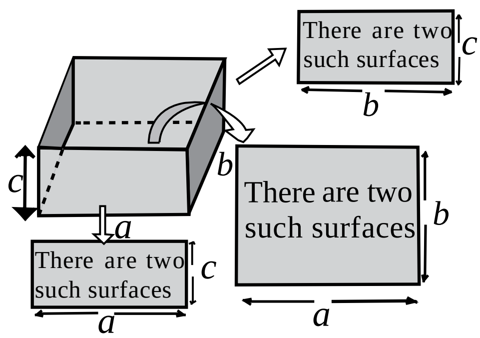
- a right pyramid with a square base
- a cone
- a sphere

Look at the solids given below. They may be very familiar to you. These solids are bounded by plane figures. Let us recall the way of finding the surface areas of these solids. The surfaces of each solid are drawn separately.

(i)



(ii)



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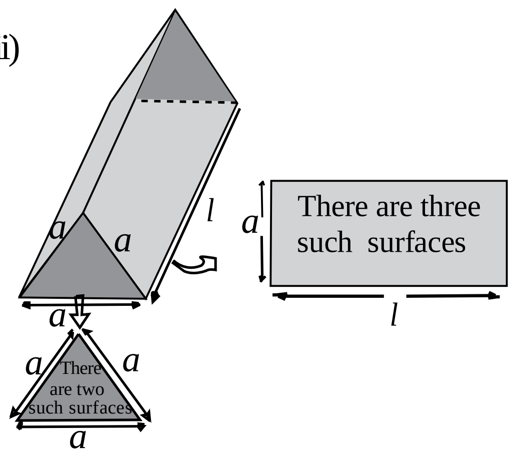

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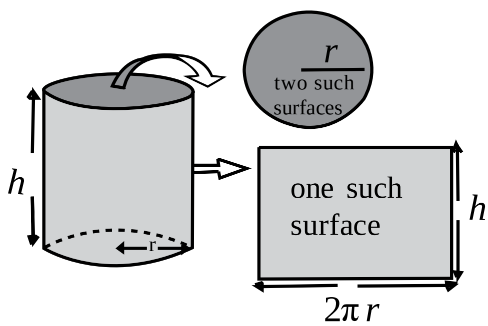
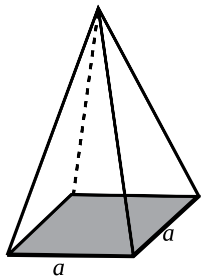
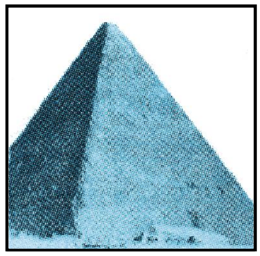


The surface area of a solid can be obtained by finding the surface area of each surface separately

4.1 Surface area of a right pyramid with a square base

This is a right pyramid with a square base. Other than the base, it has four surfaces which are triangular in shape.





In a right pyramid, the top vertex is situated just above the centre of the square base, Therefore all the edges joining this vertex are equal in length.
All the surfaces of the above right pyramid are given below.

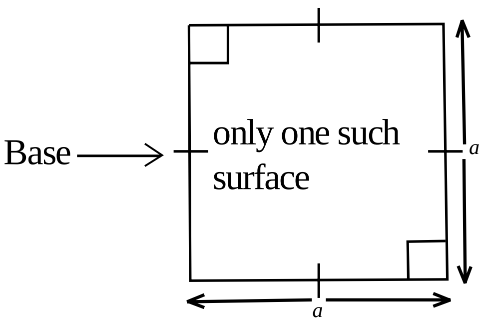


Figure I

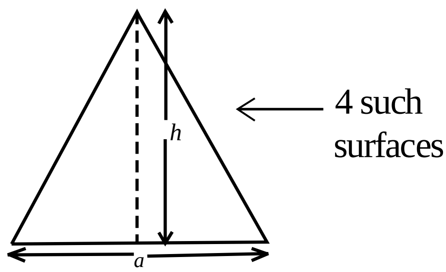


Figure II

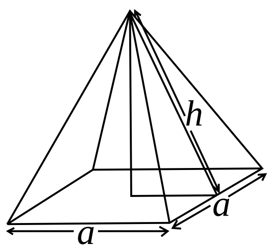
Surface area of figure I and figure II can be calculated as follows.

Area of the base of the pyramid $= a \times a = a^2$

Area of a triangular face $= \frac{1}{2} \times a \times h$

Area of 4 triangular faces $= \frac{1}{2} \times a \times h \times 4$

$= 2ah$



$\therefore$ The total surface area $= a^2 + 2ah$

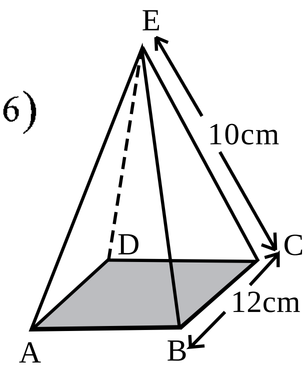
$= \underline{\underline{a(a+2h)}}$

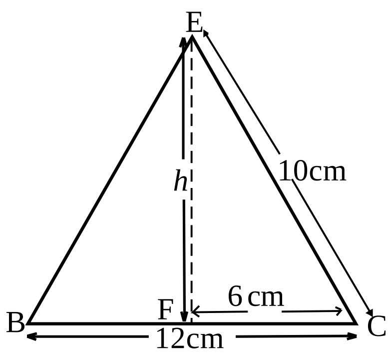
Example 1

A right pyramid of square base is shown in the diagram. The length of a bottom edge is 12 cm. the other edges are 10 cm each. Find the surface area of the right pyramid.

To find the surface area of a triangle, let us first find the height *h*.
Applying Pythagoras' theorem to the triangle EBC

$$h^2 + 6^2 = 10^2$$
$$h^2 = 10^2 - 6^2$$
$$h^2 = (10-6)(10+6)$$
$$= 4 \times 16$$
$$h = \sqrt{4 \times 16}$$
$$h = 2 \times 4$$
$$h = \underline{\underline{8\text{cm}}}$$





Area of a triangular face $= \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm}$

$= 48 \text{ cm}^2$

Area of four triangular faces $= 4 \times 48 \text{ cm}^2 = 192 \text{ cm}^2$

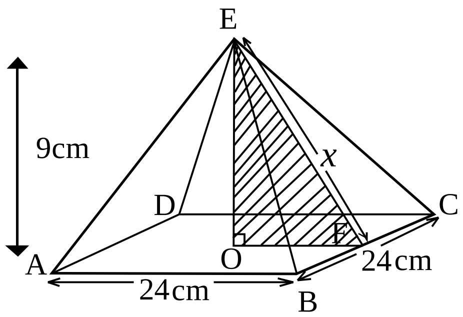
Surface area of the base $= 12 \text{ cm} \times 12 \text{ cm} = 144 \text{ cm}^2$

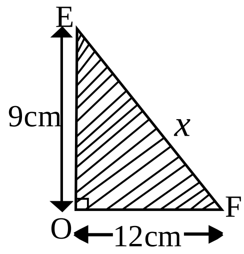
The surface area of the pyramid $= (192 + 144) \text{ cm}^2$

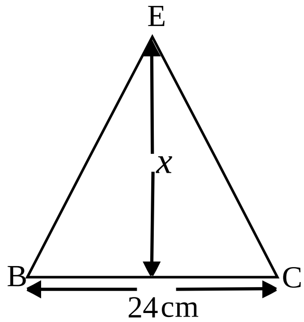
$= \underline{\underline{336 \text{ cm}^2}}$

Example 2

The perpendicular height of a square based pyramid is 9 cm. The length of a side of the base is 24 cm. Find the surface area of the pyramid.







$$x^2 = 9^2 + 12^2$$
$$x^2 = 81 + 144 = 225$$
$$x = 15\text{ cm}$$

Area of a triangular face $= \frac{1}{2} \times 24 \times 15 = 180\text{ cm}^2$

Area of 4 triangular faces $= 180 \times 4\text{ cm}^2 = 720\text{ cm}^2$

Area of the base $= 24 \times 24\text{ cm}^2 = 576\text{ cm}^2$

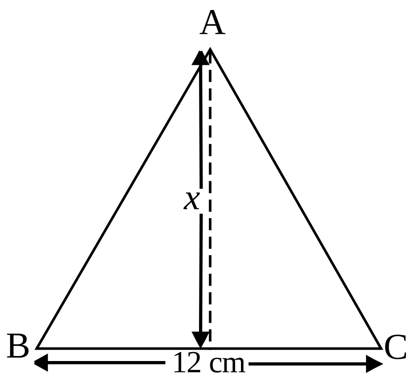
Surface area of the pyramid $= \underline{\underline{1296\text{ cm}^2}}$

Example 3

The surface area of the triangular faces of a square based pyramid is 240 cm^2. If the length of a side of the base is 12 cm. find the height of a triangular face.

Area of a triangular face $= 240 \times \frac{1}{4}\text{ cm}^2$

$= 60\text{ cm}^2$



Area of Δ ABC $= \frac{1}{2} \times 12 \times x$

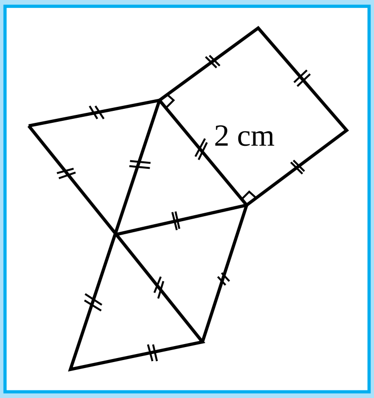
$\therefore \quad \frac{1}{2} \times 12 \times x = 60$

$x = \underline{\underline{10}}$

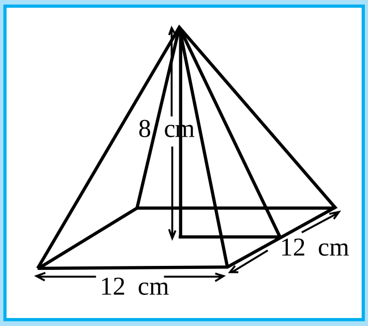
$\therefore$ The perpendicular height = $\underline{\underline{10\text{ cm}}}$

Exercise 4.1

(1) The net of a right pyramid is shown in the diagram. Find the surface area of the pyramid. Use $\sqrt{3} = 1.732$



(2) The diagram of a square based right pyramid is shown below. Find the surface area of the pyramid, if its perpendicular height is 8 cm.

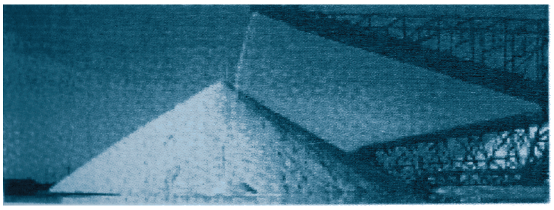


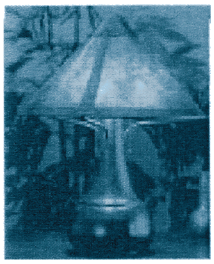
(3) A side of the base of a square based right pyramid is 6 cm and the length of a slant edge is 5 cm. Find the surface area without the area of the base.

(4) Ajith is planning to build a tent in the shape of a pyramid. For the construction, he has 4 sticks each of length 6 m for the base and 4 sticks each of length 5m for the slant edges. Find the area of the cloth needed to cover the pyramid completely.

(5) The surface area of a square based right pyramid is 340 cm^2. A side of the base is 10 cm.
(i) Find the perpendicular height of a triangular face.
(ii) Find the length of a slant edge of a triangular face.

4.2 Surface Area of a Cone



A solid object having a circular base and vertex situated above the centre of the base, is called a right circular cone

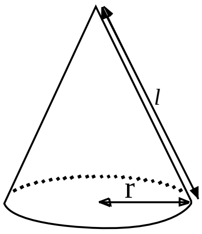


Figure I

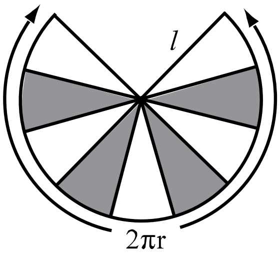


Figure II

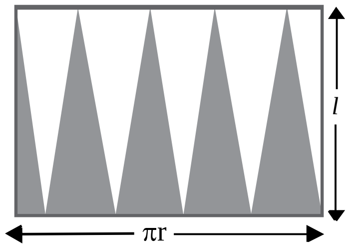


Figure IV

When a cone made of paper as shown in figure 1 is cut with care along the slant edge, and straightened out, then a sector is obtained as shown in figure II. The arc length of the sector is $2\pi r$ (where r is the radius of the circular base) and the slant length is l. Suppose, it is now cut into very small sectors and pasted as shown in figure III. The geometric shape obtained appears to be a rectangle.

Length of the rectangle	$= \pi r$	Units
Width of the rectangle	$= l$	Units
$\therefore$ The area of the rectangle	$= \pi rl$	Square Units
$\therefore$ The area of the curved surface of the Cone	$= \pi rl$	Square Units

The area of the curved surface of a cone $= \pi rl$

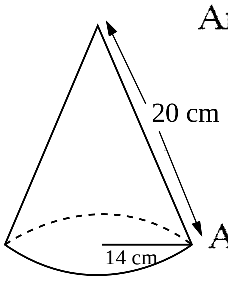
The Total surface area of a Cone = area of the circular base + area of the curved surface

$= \pi r^2 + \pi rl$ Square Units

$= \pi r(r + l)$ Square Units

Example 4

The figure of a cone with a base is given below. Using the given measurements find the surface area of the cone. (Take $\pi = \frac{22}{7}$)



Area of the curved surface of the cone $= \pi rl$

$= \frac{22}{7} \times 14\,\text{cm} \times 20\,\text{cm}$

$= 880\,\text{cm}^2$

Area of the circular base of the cone $= \pi r^2$

$= \frac{22}{7} \times 14\,\text{cm} \times 14\,\text{cm}$

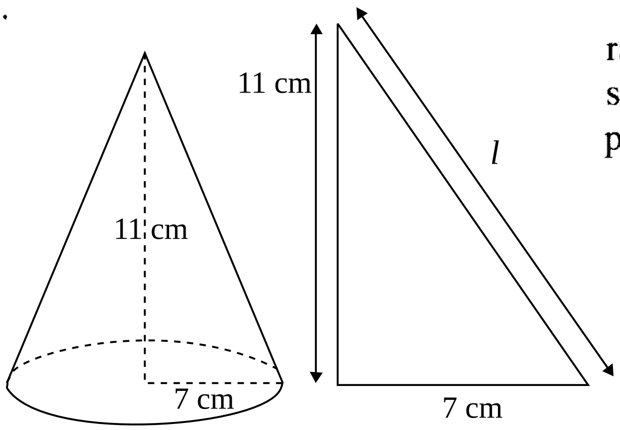
$= 616\,\text{cm}^2$

Total surface area of the cone $= (880 + 616)\text{cm}^2$

$= \underline{\underline{1496\,\text{cm}^2}}$

Example 5

According to the data given in the figure, find the area of the curved surface of the cone.



radius of the base $= 7$ cm
slant length $= l$
perpendicular height $= 11$ cm

According to Pythagoras' theorem,

$$l^2 = 11^2 + 7^2$$
$$l^2 = 121 + 49$$
$$l^2 = 170$$
$$l = \sqrt{170} = 13.03\text{ cm}$$

The area of the curved surface of the cone $= \pi rl$

$$= \frac{22}{7} \times 7 \times \sqrt{170}\,\text{cm}^2$$

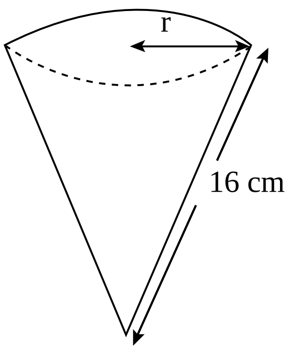
$$= 22\sqrt{170}\,\text{cm}^2$$

$$= 22 \times 13.03\,\text{cm}^2$$

$$= \underline{\underline{286.66\,\text{cm}^2}}$$

Example 6

Find the surface area of a solid cone of slant height 16cm and circumference 66 cm.



$l = 16$ cm, r = ?

Let us find the radius r of the circular base of the cone

Circumference of the circular base of the cone $\}$ $= 2\pi r$

$$= 66$$

$$\therefore 2\pi r = 66$$

$$2 \times \frac{22}{7} \times r = 66$$

$$r = \frac{66 \times 7}{22 \times 2}$$

$$r = \frac{21}{2}$$

$\therefore$the radius = $\underline{\underline{10.5\text{ cm}}}$

The surface area of the solid cone $= \pi rl + \pi r^2$ cm^2

$$= \pi r(l + r)\ \text{cm}^2$$

$$= \frac{22}{7} \times 10.5(16 + 10.5)\,\text{cm}^2$$

$$= \frac{22}{7} \times 10.5 \times 26.5\,\text{cm}^2$$

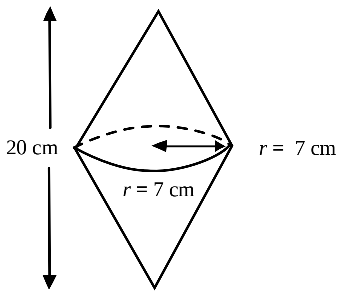
$$= \underline{\underline{874.5\,\text{cm}^2}}$$

Example 7

The area of the curved surface of a cone is 440 cm^2. Its slant height is 20 cm. Find the perimeter of the base.

$$\text{Area of the curved surface} = \pi r l$$
$$= 440 \text{ cm}$$
$$\pi r l = 440 \text{ cm}$$
$$\frac{22}{7} \times r \times 20 = 440 \text{ cm}$$
$$r = \frac{440 \times 7}{22 \times 20}$$
$$r = 7$$
$$\therefore \text{ the radius} = 7\text{cm}$$

$$\text{Perimeter of the base} = 2\pi r$$
$$= 2 \times \frac{22}{7} \times 7$$
$$= \underline{\underline{44}} \text{ cm}$$



An object formed by joining two identical solid cones is shown in the diagram. Find the surface area of the object. The radius of the base of each cone is 7 cm.

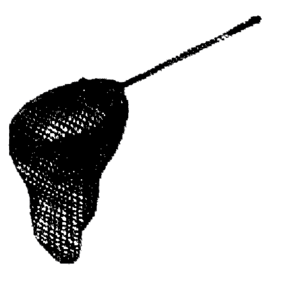
Exercise 4.2

(1) Find the surface area of a solid cone of radius 7 cm and slant height 12 cm.

(2) Nimal wants to find the radius of the circular base of a heap of sand in the shape of a cone. Suggest a method of finding the radius.

(3) The diagram shows a conical butterfly hand net. The radius of the circular frame is 7 cm and the perpendicular height is 24 cm. Find the area of the cloth used to make the hand net.

(Ignore the cloth used for joining)



(4) The circumference of the base of a right conical shaped heap of sand is 528cm. The slant height of the heap of sand is 140 cm Find an approximate value for the area on which the heap of sand is spread. Find the area of the curved surface.

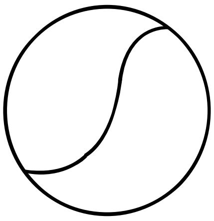
(5) Kamal wishes to make a conical tent. He decides that the height of the tent should be 200 cm, and the radius 1.5 m. Find the area of the canvas required. to make the tent.

(Ignore canvas used for joining)

4.3 Surface Area of a Sphere

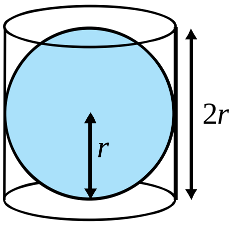
A globe

A soccer ball



A ball

The solids similar to those illustrated in the above diagrams are spheres. **Archimedes** stated that the surface area of a sphere is equal to the area of the curved surface of a cylinder which has the same radius as the sphere and has height equal to the diameter of the sphere.



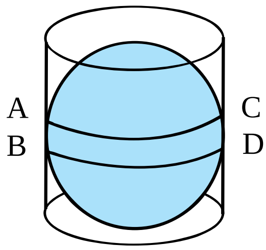
Area of the curved surface of the cylinder $= 2\pi r \times 2r$

$= 4\pi r^2$ square units

$$\boxed{\text{Surface area of a sphere of radius r} = 4\pi r^2 \text{ square units}}$$

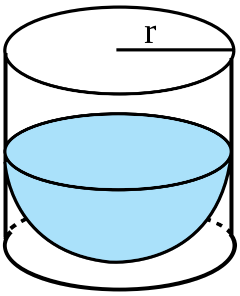
- If the surface area of a sphere is equal to the area of the curved surface of a cylinder, that cylinder is called the **circumscribed cylinder** of the particular sphere.

- **Do you know :**
  When the sphere is circumscribed by the cylinder, the area of the portion separated by AC and BD which are parallel to the circular cross section of the cylinder is equal to the area of the cut portion of the sphere.



The surface area of a hemisphere

The surface area of any portion of a sphere can be obtained by means of it's circumscribed cylinder.



Area of the curved surface of a hemisphere $= 2\pi r \times r$

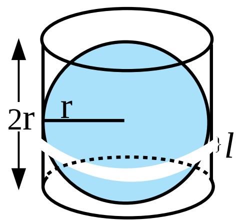
$= 2\pi r^2$ square units

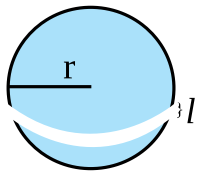
Total surface area of a hemisphere $= \pi r^2 + 2\pi r^2$

$= 3\pi r^2$ square units

- **Find**

  Are the surface areas of the un - shaded parts in the two figures qual ?





Example 8

Find the surface area of a sphere of radius 14 cm. (Take $\pi = \frac{22}{7}$)

Radius of the sphere $= 14$ cm

Surface area of the sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= \underline{\underline{2464 \text{ cm}^2}}$

Example 9

The surface area of a sphere is 154 cm^2. Find the radius of the sphere.

The surface area of the sphere $= 4\pi r^2$

$= 154 cm^2$

$$\therefore \quad 4\pi r^2 = 154$$

$$r^2 = \frac{154}{4\pi}$$

$$r^2 = \frac{154}{4\times\frac{22}{7}}$$

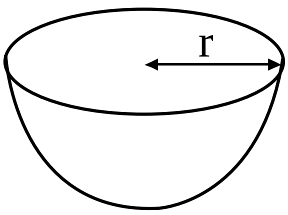
$$r^2 = \frac{154\times7}{22\times4} = \frac{7\times7}{4}$$

$$r = \frac{7}{2}$$

$\therefore$ The radius = $\underline{\underline{3.5 \text{ cm}}}$

Example 10

A hemisphere is shown in the figure. The surface area is 462 cm^2. Find the radius.



The surface area of the curved part of the hemisphere $= 2\pi r^2$

The surface area of the circular part of the hemisphere $= \pi r^2$

Total surface area of the hemisphere $= 3\pi r^2$

$$3\pi r^2 = 462 \text{ cm}^2$$

$$3\times\frac{22}{7}\times r^2 = 462 \text{ cm}^2$$

$$r^2 = \frac{462\times7}{3\times22} \text{ cm}^2$$

$$r^2 = 49 \text{ cm}^2$$

$$r = 7$$

$\therefore$ The radius = $\underline{\underline{7 \text{ cm}}}$

Example 11

A thin copper plate is of length 280 cm and width 220 cm. How many hollow spheres of radius 7 cm can be made using the whole sheet?
(Consider that there was no wastage when melting)

Surface area of a sphere of radius r $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$

$= 616 \text{ cm}^2$

Area of the sheet $= 280 \times 220 \text{ cm}^2$

No. of spheres $= \frac{280 \times 220}{616}$

$= \underline{\underline{100}}$

Exercise 4.3

(1) Find the surface area of a sphere of radius 14 cm.

(2) Find the surface area of a solid sphere of radius 7 cm.

(3) The radius of a sphere is 10 cm. Find the surface area of the circumscribed cylinder of the particular sphere.

(4) Find the area of the surface of a hemispherical solid object of radius7 cm.

(5) The surface area of a sphere is 616 cm^2. Find its radius.