3 Indices and Logarithms II

By studying this lesson you will acquire skills in

- finding the logarithms of number between zero and one.
- multiplying and dividing expressions cointaining numbers between zero and one. (Including powers and roots)
- verifying solutions using the calculator.

Do the exercises given below to recall your knowledge on the multipication and division of numbers using the logarithmic tables.

Exercise 3.1

Simplify the following using logarithmic tables. Verify the answers using the calculator

(1) 1.987×29.75 (2) 99.78×248.89

(3) $49.5 \div 24.8$ (4) $750.2 \div 14.896$

(5) $2.75 \times 3.124 \times 19.875$

(6) $\dfrac{64.85 \times 9.98}{12.78}$ (7) $\dfrac{899.3}{45.01 \times 19.75}$

(8) Estimate the value of $\dfrac{1.75 \times 2.68 \times 3.14}{3.002 \times 1.0275}$ [Simplify the expression using the logarithmic tables and compare the result with the estimated answer.]

(9) The length of a rectangular vegetable bed is 17.75 m and the width is 8.92m. Find the area of the bed using the logarithmic tables. Round off answer to the nearest whole number.

(10) Simplify $\dfrac{22}{7}$ using the logarithmic tables and round off the answer to the third decomal place.

3.1 Finding the logarithm of numbers between 0 and 1

Observe carefully the table given below.

Number	Standard Form	Characteristic
5682	5.682×10^{3}	3
568.2	5.682×10^{2}	2
56.82	5.682×10^{1}	1
5.682	5.682×10^{0}	0
0.5682	5.682×10^{-1}	-1
0.0562	5.682×10^{-2}	...
0.005682	5.682×10^{-3}	...

According to the pattern, find the numbers suitable for the blank places in the characteristic column of the above table.

The numbers suitable for the blank places are -2 and -3. Let us extend this pattern by a few more lines

Number	Scientific notation	Characteristic
0.5682	5.682×10^{-1}	-1
0.05682	5.682×10^{-2}	-2
0.005682	5.682×10^{-3}	-3
0.0005682	5.682×10^{-4}	-4
0.00005682	5.682×10^{-5}	-5

Study the relationship between the number of zeros to the right side of the decimal point adjoining it and the characteristic of the logarithm, in the table given above.

It will be clear to you that the numerical value of the characteristic is greater than the number of zeros, by one.

The characteristic of the logarithm of a number between 0 and 1 is negative. It's numerical value is greater than the number of zeros on the right hand side of the decimal point by one.

Let us find the logarithm of 0.004567.

Since the number of zeros on the right, adjacent to the decimal point is 2,the characteristic is -3. It's mantissa is indicated in the table as 0.6597. Therefore the characteristic and the mantissa of the logarithm of 0.004567 are -3 and 0.6597 respectively.

So, we have a logarithm with its integral part negative and decimal part positive. We use a special notation and specify it by $3.\bar{6}597$. This is read as bar 3, point six, five, nine, seven.

$\therefore \lg 0.004567 = \bar{3}.6597$

Example 1 Find the logarithm of 0.04021.

characteristic is -2

mantissa with respect to 4021 in the table is 0.6043

$\therefore \lg 0.04021 = \underline{\underline{\bar{2}.6043}}$

Exercise 3.2

(1) Write the characteristic of the logarithm of the following numbers

(i) 0.053 (ii) 0.1023 (iii) 0.00030 (iv) 0.0706

(v) 0.209 (vi) 0.5 (vii) 0.001111 (viii) 0.06004

(2) Find the logarithm of each number below.

(i) 0.2543 (ii) 0.7856 (iii) 0.9986 (iv) 0.1023

(v) 0.04586 (vi) 0.00856 (vii) 0.0025 (viii) 0.03409

(ix) 0.07 (x) 0.75 (xi) 0.000125 (xii) 0.2

(3) Explain the error that could arise by writing the logarithm of 0.4567 as 1.6598

3.2 Obtaining the number when the logarithm of a number between 0 and 1 is given. (Finding the antilog)

Here we must find the number corresponding to the given logarithm. This process is called **finding the antilogarithm.**

Example 2

Find the number of which the logarithm is $\bar{2}.4053$.

Since the characteristic is -2 in the logarithm, there should be exactly one zero to the right of the decimal point,
The decimal part .4053 corresponds to 2543 in the table.
Therefore the relevant number is 0.02543.

This is written as, **antilog $\bar{2}$.4253 = 0.02543.**

Example 3

Find the antilog of $\bar{3}.7815$.

Sice the characteristic is -3, there should be 2 zeros in the relevant number to the right of the decimal point.

The value corresponding to the decimal part .7815 in the table is 6047.

$\therefore$ antilog $\bar{3}.7815 = 0.006047$

Exercise 3.3

(1) Write the number of zeros to the right hand side of the decimal point in the numbers (antilog) corresponding to each of the following logarithms

(i) $\bar{2}.1435$ (ii) $\bar{3}.6153$ (iii) $\bar{1}.5457$ (iv) $\bar{5}.9763$
(v) $\bar{4}.4871$

(2) Find the antilogarithm of each logarithm.

(i) $\bar{1}.9036$ (ii) $\bar{1}.8772$ (iii) $\bar{2}.7745$ (iv) $\bar{3}.0715$
(v) $\bar{2}.6532$ (vi) $\bar{3}.7086$ (vii) $\bar{3}.9320$ (viii) $\bar{1}.9320$
(ix) $\bar{4}.9320$ (x) $\bar{3}.0043$

3.3 Addition and subtraction of logarithms with negative characteristics

It is important to note that the mantissa is always positive and the characteristic is either negative or non-negative.

Addition of logarithms with a bar.

Example 4

Simplify $\bar{2}.3263 + \bar{1}.4786$.

Decimal parts of the logarithms are positive. In both logarithms the characteristics are negative. Therefore add the decimals and the characteristics separately.

$$\begin{array}{r} \bar{2}.3263 \\ + \quad \bar{1}.4786 \\ \hline \bar{3}.8049 \end{array}$$

$$\bar{2} + \bar{1} = (-2) + (-1) = -3 = \bar{3}$$

Example 5

Simplify $\bar{1}.7251 + \bar{2}.8162$

$$\begin{array}{r} \bar{1}.7251 \\ +\bar{2}.8162 \\ \hline \bar{2}.5413 \end{array}$$

Here, when adding decimals, you are left with a whole number + 1 which goes to the characteristic. This +1 and $\bar{3}$ in the characteristics will add up to $\bar{2}$.

Example 6

Simplify $1.9571 + \bar{2}.7466$

$$1 + \bar{1} + \bar{2} = 1 + (-1) + (-2) = -2 = \bar{2}$$

$$\begin{array}{r} 1.9571 \\ +\bar{2}.7466 \\ \hline 0.7037 \end{array}$$

Here when the decimals are added + 1 will come to the characteristic. Hence the final integer part becomes zero by adding 1 to the characteristics 1 and $\bar{2}$.

$$1 + 1 + \bar{2} = 1 + 1 - 2 = 0$$

Subtracting logarithms with negative characteristics

Example 7

Simplify $\bar{2}.9320 - \bar{1}.7752$

$$\begin{array}{r} \bar{2}.9320 \\ -\bar{1}.7752 \\ \hline \bar{1}.1568 \end{array}$$

Decimals are subtracted in the usual way.

When $\bar{1}$ is subtracted from $\bar{2}$, the result is $\bar{1}$.

$$\bar{2} - \bar{1} = -2 - (-1) = -2 + 1 = -1 = \bar{1}$$

Example 8

Simplify $\bar{1}.5441 - \bar{3}.9058$

$$\begin{array}{r} \bar{1}.5441 \\ -\bar{3}.9058 \\ \hline 1.6383 \end{array}$$

Here, as 9 is to be subtracted from 5, in the decimal part , 1 should be taken from the characteristic. But as it is a negative number that cannot be done.

Therefore, express $\bar{1}.5441$ as $\bar{2} + 1.5441$.

Then the characteristic is changed from $\bar{1}$ to $\bar{2}$. When $\bar{3}$ is subtracted from $\bar{2}$, the result is 1.

Example 9

Simplify $\bar{1}.3054 - 1.9614$

$$\begin{array}{r} \bar{1}.3054 \\ -\ 1.9614 \\ \hline \bar{3}.3440 \end{array}$$

$$\begin{aligned} &\bar{1}.3054 - 1.9614 \\ &= 1 - 1 + \bar{1}.3054 - 1.9614 \\ &= 1 - 1 + (-1) + (0.3054) - 1.9614 \\ &= (-1) + (-1) + 1.3054 - 1.9614 \\ &= -2 + 1.3054 - 0.9614 - 1 \\ &= -2 + 0.3440 - 1 \\ &= -3.3440 \end{aligned}$$

Exercise 3.4

(1) Simplify

(i) $\bar{1}.6042 + \bar{1}.1818$ (ii) $\bar{2}.6355 + \bar{1}.4166$ (iii) $\bar{1}.7340 + \bar{2}.6684$

(iv) $\bar{3}.9890 + \bar{1}.9509$ (v) $1.5911 + \bar{2}.6937$ (vi) $\bar{2}.7356 + 1.6767$

(vii) $\bar{2}.7443 + 1.8062 + \bar{1}.7404$

(2) Simplify

(i) $\bar{3}.7160 - \bar{1}.5729$ (ii) $\bar{2}.6180 - \bar{1}.5428$

(iii) $1.7202 - \bar{2}.9165$ (iv) $\bar{2}.3424 - 1.8156$

(v) $\bar{3}.3874 - \bar{2}.7396$ (vi) $\bar{1}.5922 - \bar{2}.8791$

3.4 Simplification of expressions involving numbers between 0 and 1 using logarithms

Multiplication

Example 10

Simplify 25.87×0.0518.

Let $x = 25.87 \times 0.0518$

By taking logarithms on both sides,

$$lg\ x = lg 25.87 + lg 0.0518$$
$$= 1.4128 + \bar{2}.7143$$

1.4128
$+\bar{2}.7143$
0.1271

$$lg x = 0.1271$$
$$x = antilog\ 0.1271 = 1.34$$
$$x = 1.34$$
$$\therefore 25.87 \times 0.0518 = \underline{1.34}$$

Example 11

Simplify 0.0075×0.125

Let $x = 0.0075 \times 0.125$

$$lg\ x = lg\ 0.0075 + lg 0.125$$
$$= \bar{3}.8751 + \bar{1}.0969$$
$$lg\ x = \bar{4}.9720$$
$$x = antilog\ \bar{4}.9720$$
$$x = 0.0009376$$
$$0.0075 \times 0.125 = \underline{0.0009376}$$

$\bar{3}.8751$
$+\bar{1}.0969$
$\bar{4}.9720$

Example 12

Simplify $15.87 \times 0.01694 \times 0.00275$

Let $x = 15.87 \times 0.01694 \times 0.00275$

$lg\ x = lg\ 15.87 + lg\ 0.01694 + lg\ 0.00275$

$= 1.2007 + \bar{2}.2290 + \bar{3}.4393$

$lg\ x = \bar{4}.8690$

$x = antilog\ \bar{4}.8690$

$x = 0.0007396$

$15.87 \times 0.01694 \times 0.00275 = \underline{\underline{0.0007396}}$

$$\begin{array}{r} 1.2007 \\ \bar{2}.2290 \\ +\bar{3}.4393 \\ \hline \bar{4}.8690 \end{array}$$

Division

Example 13

Simplify $15.78 \div 0.125$

Let $x = 15.78 \div 0.125$

$\lg x = \lg 15.78 - \lg 0.125$

$= 1.1981 - \bar{1}.0969$

$= 2.1012$

$x = anti\ log\ 2.1012$

$x = 126.2$

$\therefore\ 15.78 \div 0.125 = \underline{\underline{126.2}}$

$$\begin{array}{r} 1.1981 \\ -\bar{1}.0969 \\ \hline 2.1012 \end{array}$$

Example 14

Simplify $0.5052 \div 12.84$

Let $x = 0.5052 \div 12.84$

$\lg x = lg\ 0.5052 - lg\ 12.84$

$= \bar{1}.7035 - 1.1086$

$\lg x = \bar{2}.5949$

$x = anti\ log\ \bar{2}.5949$

$x = 0.03935$

$\therefore\ 0.5052 \div 12.84 = \underline{\underline{0.03935}}$

$$\begin{array}{r} \bar{1}.7035 \\ -1.1086 \\ \hline \bar{2}.5949 \end{array}$$

Example 15

Simplify $0.1942 \div 0.8554$

$$\text{Let } x = 0.1942 \div 0.8554$$
$$\lg x = \lg 0.1942 - \lg 0.8554$$
$$= \bar{1}.2882 - \bar{1}.9322$$
$$\lg x = \bar{1}.3560$$
$$x = \text{anti}\log \bar{1}.3560$$
$$x = 0.227$$
$$0.1942 \div 0.8554 = \underline{\underline{0.227}}$$

$$\begin{array}{r} \bar{1}.2882 \\ -\bar{1}.9322 \\ \hline \bar{1}.3560 \end{array}$$

3.5 Multiplication and Division using logarithms

Example 16

Simplify $\dfrac{2.578 \times 0.3967}{0.0756}$

$$\text{Let } x = \frac{2.578 \times 0.3967}{0.0756}$$
$$\lg x = \lg 2.578 + \lg 0.3967 - \lg 0.0756$$
$$= 0.4113 + \bar{1}.5985 - \bar{2}.8785$$
$$= 0.0098 - \bar{2}.8785$$
$$\lg x = 1.1313$$
$$x = \text{anti}\log 1.1313$$
$$x = 13.53$$
$$\therefore \frac{2.578 \times 0.3967}{0.0756} = \underline{\underline{13.53}}$$

Exercise 3.5

1. Evaluate using logarithms

(i) 396.2×0.0017 (ii) 0.0083×6.729 (iii) 0.732×0.0615

(iv) $2.8 \times 3.9 \times 0.056$ (v) $2.99 \times 0.73 \times 0.03921$ (vi) $0.034 \times 27 \times 9.08$

(vii) $0.3965 \div 0.0927$ (viii) $0.9391 \div 9.82$ (ix) $2.83 \div 0.0634$

(x) $0.0812 \div 4.93$ (xi) $0.03654 \div 0.4563$ (xii) $0.00957 \div 0.0631$

(2) Evaluate using logarithms. Check the answers using a calculator

(i) $\frac{24.37 \times 3.94}{0.8975}$ (ii) $\frac{0.5621 \times 0.328}{0.4224}$

(iii) $\frac{91.85 \times 0.1985}{11.194 \times 0.1998}$

3.6 Multiplying and dividing logarithms with negative characterisitics by a whole number

We know that the characteristic and the decimal part of a logarithm with bar are negative and positive respectively. Hence we do the mathematical operations for these two parts separately and get the final result by combining these two results together.

Multiplying logarithms with negative characteristics by a whole number

Example 17

Consider the Simplification of $\bar{1}.4624 \times 2$

When the whole number is multiplied by 2, we get, $\bar{1} \times 2 = \bar{2}$

When the decimal part is multiplied by 2, we get , $0.4624 \times 2 = 0.9248$

When added $\bar{2} + 0.9248 = \bar{2}.9248$

$\therefore \bar{1}.4624 \times 2 = \underline{\bar{2}.9248}$

Example 18

Let us simplify $\bar{1}.9741 \times 2$.

When the whole number is multiplied by 2, $\bar{1} \times 2 = \bar{2}$

When the decimal part is multiplied by 2, $0.9741 \times 2 = 1.9482$

When added $\bar{2} + 1.9482 = \bar{1}.9482$

$\therefore \bar{1}.9741 \times 2 = \underline{\bar{1}.9482}$

$\bar{2}.0000$
$+1.\underline{9482}$
$\underline{\bar{1}.9482}$

Example 19
Simplify $\bar{2}.8954 \times 3$.

The whole number $\times 3$ $= \bar{2} \times 3$ $= \bar{6}$

Decimal number $\times 3$ $= 0.8954 \times 3$ $= 2.6862$

When added $= \bar{6} + 2.6862$ $= \bar{4}.6862$

$\therefore \bar{2}.8954 \times 3 = \underline{\underline{\bar{4}.6862}}$

Dividing logarithms with negative characteristics by a whole number

Example 20

Simplify $\bar{2}.8762 \div 2$

Whole number $\div 2$ $= \bar{2} \div 2 = \bar{1}$

Decimal number $\div 2$ $= 0.8762 \div 2 = 0.4381$

When added $\bar{1} + 0.4381 = \bar{1}.4381$

$\therefore \bar{2}.8762 \div 2 = \underline{\underline{\bar{1}.4381}}$

When we divide a characteristic which is negative, if there is a remainder, it cannot be taken to the decimal part as it is a negative number. Therefore we have to change the characteristic so that it is divisible.
Let us see the example below.

Example 21

Simplify $\bar{2}.7193 \div 3$

Since the characteristic $\bar{2}$ is not divisible by 3, we express $\bar{2}.7193$ as $\bar{3} + 1.7193$ Then the characteristic is changed from $\bar{2}$ to $\bar{3}$.

Whole number $\div 3$ $= \bar{3} \div 3$ $= \bar{1}$

Decimal number $\div 3$ $= 1.7193 \div 3$ $= 0.5731$

When added $\bar{1} + 0.5731$ $= \underline{\underline{\bar{1}.5731}}$

$\therefore \ \bar{2}.7193 \div 3 = \bar{1}.5731$

(-2) + (-1) + (+1)
= (-3) + (+1)

or

$3\overline{)\bar{3} + 1.7193}$
$\bar{1} + 0.5731$
$= \bar{1}.5731$

Example 22

Consider the simplification $\bar{4}.5899 \div 3$

To make $\bar{4}$ exactly divisible by 3, add $\bar{2}$ and make it $\bar{6}$ and add + 2 to the decimal part to balance the number. The decimal part becomes 2.5899.

Whole number $\div 3 = \bar{6} \div 3 = \bar{2}$

Decimal Part $\div 3 = 2.5899 \div 3 = 0.8633$

$\therefore \bar{4}.5899 \div 3 = \underline{\underline{\bar{2}.8633}}$

$3\overline{)\bar{6} + 2.5899}$
$\bar{2} + 0.8633$
$= \bar{2}.8633$

Exercise 3.6

(1) Simplify

(i) $\bar{2}.1614 \times 2$ (ii) $\bar{1}.2718 \times 3$ (iii) $\bar{1}.7372 \times 3$

(iv) $\bar{2}.9671 \times 3$ (v) $\bar{1}.8692 \times 2$ (vi) $\bar{2}.7993 \times 4$

(vii) $\bar{1}.8820 \times 5$ (viii) $\bar{2}.4786 \times 3$ (ix) $\bar{1}.7959 \times 3$

(x) $\bar{3}.8949 \times 2$

(2) Simplify

(i) $\bar{2}.8949 \div 2$ (ii) $\bar{3}.6513 \div 3$ (iii) $\bar{4}.7364 \div 4$

(iv) $\bar{4}.2504 \div 2$ (v) $\bar{2}.7187 \div 3$ (vi) $\bar{1}.8722 \div 3$

(vii) $\bar{1}.9112 \div 3$ (viii) $\bar{4}.8627 \div 3$ (ix) $\bar{5}.7451 \div 4$

(x) $\bar{1}.8859 \div 4$

3.7 Expressions with powers and roots

We have learnt that $log_a\ m^r = r\ log_a\ m$ under logarithms.

i.e., $\boxed{log_a\ m^r = r\ log_a\ m}$ Accordingly we can write $\log_2 8^3 = 3 \log_2 8$

Example 23 Evaluate 2.895^2

Let $x = 2.895^2$

$lg\ x = 2\ lg\ 2.895$

$= 0.4617 \times 2$

$lg\ x = 0.9234$

$x = antilog\ 0.9234 = \underline{\underline{8.383}}$

$2.895^2 = \underline{\underline{8.383}}$

Example 24 Evaluate $(0.1455)^3$

Let $x = (0.1455)^3$

$lg\ x = 3\ lg\ 0.1455$

$= \bar{1}.1629 \times 3$

$lg\ x = \bar{3}.4887$

$x = antilog\ \bar{3}.4887$

$x = 0.003081$

$\therefore\ 0.1455^3 = \underline{\underline{0.003081}}$

Example 25 Evaluate $\sqrt{0.9726}$

Let $x = \sqrt{0.9726}$

$lg\ x = \frac{1}{2} \times lg\ 0.9726$

$= \frac{1}{2} \times \bar{1}.9880$

$lg\ x = \bar{1}.9940$

$x = antilog\ \bar{1}.9940$

$= 0.9862$

$\therefore\ \sqrt{0.9725} = \underline{\underline{0.9862}}$

$\bar{1}.9880 = \bar{1} + \bar{1} + 1.9880$

$\bar{1}.9880 = \bar{2} + 1.9880$

$\frac{\bar{2} + 1.9880}{2} = \bar{1} + 0.9940$

$= \bar{1}.9940$

Example 26

Evaluate $\sqrt[3]{0.0785}$

$x = \sqrt[3]{0.0785}$

$lg\ x = \frac{1}{3} \times lg\ 0.0785$

$= \frac{1}{3} \times \bar{2}.8949$

$lg\ x = \bar{1}.6316$

$x = antilog\ \bar{1}.6316$

$= \underline{\underline{0.4282}}$

$\therefore \quad \sqrt[3]{0.0785} = \underline{\underline{0.4282}}$

$\frac{\bar{2}.8949}{3} = \frac{\bar{3} + 1.8949}{3}$

$= \bar{1} + 0.6316$

$= \bar{1}.6316$

- The *l*ogarithm of a power of a number can be obtained by multiplaying the logarithm of the number by the relevant power.
- To obtain any root of a number the logarithm of the particular numbe should be divided by the relevant root.

Exercise 3.7

(1) **Evaulate the following using logarithmic tables.**

(i) $(3.985)^2$ (ii) $\sqrt{143.9}$ (iii) $\sqrt{0.01257}$ (iv) $(0.7575)^{\frac{1}{2}}$

(v) $(0.0014557)^{\frac{1}{3}}$ (vi) $\sqrt{40005}$ (vii) $(17.25)^{\frac{2}{3}}$ (viii) $\sqrt[3]{0.1285}$

(ix) $\sqrt[4]{0.0025}$ (x) $(5.002)^2$

(2) Find the logarithms using,

$lg\ 2 = 0.3010$, $lg\ 3 = 0.4771$ and $lg\ 5 = 0.6990$

(i) $lg\ 6$ (ii) $lg\sqrt{2}$ (iii) $lg\ 30$ (iv) $lg\ \sqrt[3]{5}$ (v) $lg\ \frac{2}{3}$

(vi) $lg\ 6^2$ (vii) $lg\ (0.002)^3$ (viii) $lg\ \frac{3}{\sqrt{2}}$ (ix) $lg\sqrt[3]{0.03}$ (x) $lg\ \frac{\sqrt{5}}{\sqrt{3}}$

Evaluating expressions with powers and roots

Example 27 Evaluate $\dfrac{\sqrt{12.42}\times 0.725}{(1.748)^2}$

$$x = \frac{\sqrt{12.42}\times 0.725}{(1.748)^2}$$

$$\lg x = \frac{1}{2}\lg 12.42 + \lg 0.725 - 2\lg 1.748$$

$$= \frac{1}{2} \times 1.0941 + \bar{1}.8603 - 2 \times 0.2425$$

$$= 0.5470 + \bar{1}.8603 - 0.4850$$

$$\lg x = \bar{1}.9223$$

$$x = \text{antilog } \bar{1}.9223 = \underline{\underline{0.8361}}$$

$$\frac{\sqrt{12.42}\times 0.725}{(1.748)^2} = \underline{\underline{0.8361}}$$

	0.5470
+	$\bar{1}$.8603
	0.4073
-	0.4850
	$\bar{1}$.9223

Verify your answer with the help of the calculator.

Example 28 Evaluate $\dfrac{38.54\times(0.0357)^{\frac{1}{3}}}{\sqrt{0.5164}}$

Let $x = \dfrac{38.54\times(0.0357)^{\frac{1}{3}}}{\sqrt{0.5164}}$

$$\lg x = \lg 38.54 + \frac{1}{3}\lg 0.0357 - \frac{1}{2}\lg 0.5164$$

$$= 1.5860 + \frac{1}{3} \times \bar{2}.5527 - \frac{1}{2} \times \bar{1}.7129$$

$$= 1.5860 + \bar{1}.5176 - \bar{1}.8564$$

$$= 1.1036 - \bar{1}.8564$$

$$\lg x = 1.2472$$

$$x = \text{antilog } 1.2472$$

$$= \underline{17.67}$$

$$\therefore \frac{38.54\times(0.0357)^{\frac{1}{3}}}{\sqrt{0.5164}} = \underline{\underline{17.67}}$$

Verify your answer with the help of the calculator.